Design of a Planar, Concentric Coil for the Generation of a Homogeneous Vertical Magnetic Field Distribution

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Abstract **─** This paper describes an algorithm to design a planar, concentric coil that generates a homogeneous vertical magnetic field distribution, with a target self inductance. The algorithm consists of several steps. The first step is the random generation of a large set of concentric coils, taken into consideration a number of boundary conditions such as the outer dimensions and the radius of the conducting wire. The optimal coil is found from this set after a twofold selection, namely on the self inductance value and on the uniformity of the vertical magnetic field, which was evaluated using a cost function. The algorithm is straightforward and based on closed expressions, which leads to a rapid execution of the procedure. We evaluated the algorithm on a representative reference configuration. The best coil of the large set was selected and the quasi-uniform field distribution of this coil was confirmed by experimental verification.

Index Terms **─** Magnetic field, planar coil, self inductance, uniform field distribution.

I. INTRODUCTION

In several growing domains such as inductive wireless powering [1-4], Radio Frequency IDentification (RFID) [5] and Near Field Communication (NFC) [6], two inductively coupled coils are used to transfer energy and to exchange information over a short-range, wireless link. In inductive wireless powering systems, the energy transfer dominates the design, while for RFID or NFC the emphasis is on the data transfer. In both approaches, an increased homogeneity of the magnetic field is a favorable property. For example, if the transmitter coil for an inductive wireless power transfer system generates a more homogeneous field, it enhances the flexibility of positioning the receiver coil [7]. This can be beneficial for the charging of low-power consumer electronics such as smartphones [8-10], but also for higher power applications, such as the on-road charging of electrical vehicles while they are driving [11, 12]. An increased homogeneity of the field is also for RFID and NFC applications beneficial: it increases the readout region and efficiency of information exchange [13], which is an advantage for, e.g., moving receivers. An example application is the registering of the RFID tags of cattle at an automatic feeding installation [14].

Besides the flat field distribution, it is also important to take the self inductance into account. If one wants to replace an existing coil with an optimized coil with a more homogeneous field, it will have a minimal impact on the rest of the circuitry if the self inductance is not modified.

In this study, we will develop an algorithm to design a concentric, fully planar, transmitter coil that leads to a high and quasi-uniform magnetic field distribution with a specific self inductance value. The result of the algorithm is a set of dimensional parameters that can be used directly to design the coil. Limitations imposed by, e.g., the wire radius, are taken into account.

Interesting work on the design of coils for the generation of quasi-uniform magnetic field generation has been published in recent years by various authors. Kim et al [15] designed a planar coil of multiple loops connected in series and parallel. They mix the current direction in each loop in forward and in reverse, thus generating a more uniform magnetic field distribution. The same alternate winding design is applied by Zhang and Chau [16] to continuously charge a moving receiver. They apply different transmitter coils next to each other with alternating current directions, to minimize the gap in the field distribution between different transmitter coils. In this way they create a more uniform magnetic field which allows for an enhanced energy transfer performance around the gap between the coils.

Casanova et al. [17] designed a dedicated coil of 20 cm by 20 cm for inductive wireless power transfer. The coil design is a flat spiral. The geometry is determined by sweeping all possible parameters, evaluating the magnetic field for all possibilities, and choosing the design that leads to the most flat field distribution using an objective function. An analogous procedure was used by Yinlian et al. [18] to design two coil structures that realize in parallel a flat field distribution.

Liu and Hui [19] have patented a technique, based

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on a hybrid coil structure. It consists of a concentric coil and a spiral winding. A number of limitations are posed here, since the spacing between the spiral coil loops must be constant. It is also observed that the spiral coil is not located in the same plane of the concentric coil, which makes the structure inherently non-planar, which can be considered less attractive for most applications due to the increased height of the coil. The same applies for the work of Lee et al. [20] who proposed a technique to bend a rectangular coil further away from the receiving coil at the edges of this structure. By realizing a more uniform magnetic field distribution, they increased the overall transfer efficiency up to 44% compared to a conventional coil, regardless of the location of the receiving coil.

Waffenschmidt [7] developed an excellent algorithm to determine the turn distribution of a planar coil to generate a specified homogeneous field. In the first step, his algorithm calculates a current distribution from a specified magnetic field. In the second step, the corresponding turn distribution to this current distribution is derived. Using the generalized minimal residual method, this algorithm can be extended to more coil turns [21]. Azpúra [22] proposed a semi-analytical method for designing a coil that generates a more uniform field. Starting from the Biot-Savart law, he analytically determines the field distribution of a start configuration. He then uses an iterative algorithm based on TABU search to improve the homogeneity of the field. The number of turns of each coil and the thickness of wire is adjusted at each iteration until the required field is reached.

An important difference between our algorithm and the aforementioned references is that we impose a certain, by the user determined, self inductance for the planar coil. In that way, our improved coil will have a minimal impact on the rest of an existing circuitry since the self inductance is not modified. Previous related work always managed to increase the homogeneity of the field by, among others, changing the inductance. Moreover, as a result of the availability of closed expressions for the field distribution and self inductance, our algorithm can be evaluated in a very short time.

This paper is organized as follows. In Section II, the expressions for the field evaluation, the self inductance and the homogeneity of the field for a circular, concentric, planar coil are elaborated. These expressions are used extensively in the algorithm that is described in detail in Section III, which describes in detail the flow in order to obtain the planar coil with the best field distribution and a specific target self inductance. The algorithm is demonstrated on a representative example in Section IV, where the results are also verified experimentally. The main results are summarized in the conclusions (Section V).

In this study, we limit our attention to a concentric coil. The reason is that when a certain degree of freedom is offered to the user to position its electronic device, the most flexible solution is the one where there is no rotational preference direction. This obviously leads to a concentric configuration.

II. FIELD DISTRIBUTION AND INDUCTANCE CALCULATION

The vertical magnetic field generated by a circular current loop with radius R and current I is available in a closed form. If the origin of the coordinate set is chosen at the center of the circular loop, with the loop in the *xy*-plane, Equation (1) is applicable [23]:

$$H_z(r,z) = \frac{I}{2\pi}\frac{1}{\sqrt{(r+R)^2+z^2}} \cdot \left\{\mathcal{K}(k) - \frac{r^2-R^2+z^2}{(r-R)^2+z^2}\mathcal{E}(k)\right\}, \quad (1)$$

with

$$k = \frac{2\sqrt{rR}}{\sqrt{(r+R)^2+z^2}}. \quad (2)$$

Here, r and z are the cylindrical coordinates of the observation point, while $\mathcal{K}(k)$ and $\mathcal{E}(k)$ are the complete elliptic integrals of the first and second kind respectively. When multiple, concentric loops are present, as shown on Fig. 1, the field is found as the sum of the different loop contributions. Without loss of generality (due to linearity), we normalize the field at a current of 1 A. We suppose that the same current is flowing counterclockwise (top view) in the different loops. Equation (3) thus describes the vertical magnetic field generated by N concentric coils with different radii R_n:

$$H_z(r,z) = \frac{1}{2\pi}\sum_{n=1}^{N}\frac{1}{\sqrt{(r+R_n)^2+z^2}} \cdot \left\{\mathcal{K}(k_n) - \frac{r^2-R_n^2+z^2}{(r-R_n)^2+z^2}\mathcal{E}(k_n)\right\}, \quad (3)$$

with

$$k_n = \frac{2\sqrt{rR_n}}{\sqrt{(r+R_n)^2+z^2}}. \quad (4)$$

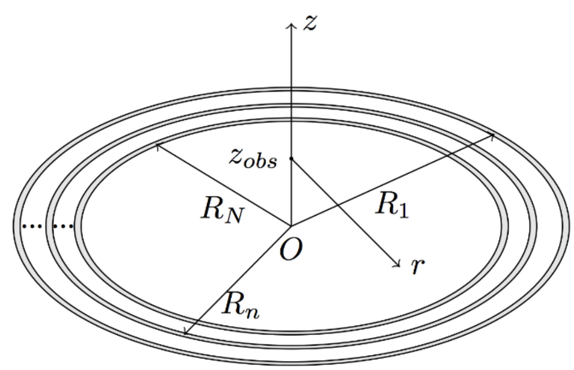

Fig. 1. Configuration under study with the corresponding conventions.

Since we want to design a planar, concentric coil with a certain target self inductance L, we need a closed expression for this lumped parameter. If the radius of the wire r_w is much smaller than the radius of the loop R_n, the self inductance L can be approximated by [24]:

$$L = \sum_{n=1}^{N} \mu_0 R_n \left[\ln\left(\frac{8R_n}{r_w}\right) - 2\right] + \sum_{i=1}^{N}\sum_{j=1}^{N} M_{i,j}(1-\delta_{i,j}), \quad (5)$$

with

$$M_{i,j} = \mu_0\sqrt{R_i R_j}\left[\left(\frac{2}{k_{i,j}} - k_{i,j}\right)\mathcal{K}(k_{i,j}) - \frac{2}{k_{i,j}}\mathcal{E}(k_{i,j})\right], (6)$$

and

$$k_{i,j} = \frac{2\sqrt{R_i R_j}}{R_i + R_j}. \quad (7)$$

$\delta_{i,j}$ is the Kronecker delta (zero for $i \neq j$ and 1 if $i = j$). In order to have a planar structure, the radial difference between any two different loops should be at least two times the radius of the wire r_w:

$$|R_i - R_j| \geq 2r_w \text{ for } i \neq j. \quad (8)$$

Based on previous expressions, we can construct a planar, circular concentric coil with a certain self inductance L. Once the dimensions of the different loops are chosen, the field at any height z_{obs} can be evaluated making use of expression (3). The goal is however, that an optimized coil with a specific self inductance L is defined, which leads to a homogeneous vertical field distribution at a specific observation height z_{obs}. In order to quantify the homogeneity of the field in an interval between r_{min} and r_{max}, we define the following cost function v:

$$v = \int_{r_{min}}^{r_{max}} \left|\frac{\mathrm{d}H_z(r,z_{obs})}{\mathrm{d}r}\right| \mathrm{d}r = \int_{r_{min}}^{r_{max}} |\mathrm{d}H_z(r,z_{obs})|. \quad (9)$$

It is clear that v will be minimal for flat and thus, homogeneous field distributions between r_{min} and r_{max}.

III. ALGORITHM

In our algorithm, we choose the maximal outer dimension of the coil, being $R_1 + r_w$, fixed. It has been proven [25] that the *distribution* of the magnetic field for a circular current loop is independent on the outer radius R. Indeed, expressions (1) and (3) can be rewritten as a function of two dimensionless parameters, the normalized radius and the normalized height, with R as the normalization variable [25]. R then acts as a scaling factor and does not modify the distribution.

The goal of the algorithm is to find a concentric, planar configuration with a given exterior dimension $R_1 + r_w$ and a given self inductance L that generates a homogeneous field distribution. If one wants to design a planar, concentric coil with a specific inductance using as less wire length as possible, it can be derived from expression (5) that all current loops must be located at the utmost exterior. We will call this configuration the reference coil with exterior dimension $R_1 + r_w$ (Fig. 2 shows such a coil with $N_{ref} = 10$ number of loops). Figure 3 shows the typical magnetic field distribution for such a configuration. One notices that the magnetic field in the center is lower than near the edges. It is clear that in order to realize a more uniform vertical field distribution, we have to increase the field in the central region. If we consider a second configuration where we fill the entire central region with current loops (i.e., a "disk configuration"), we obtain a magnetic field that is higher in the center than at the outer edges (Fig. 3). In this latter case, the different loops are closely stacked to each other and the different values (for n from 1 to N) are:

$$R_n = r_w + (N-n)2r_w. \quad (10)$$

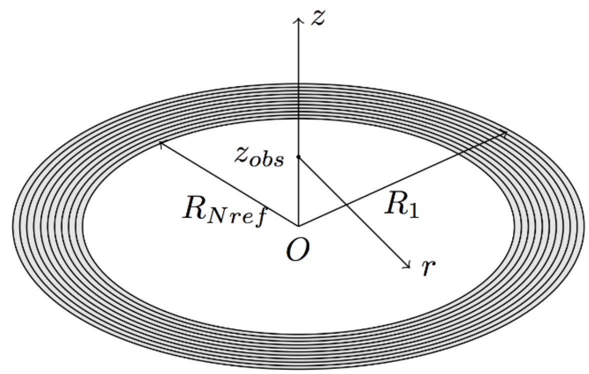

Fig. 2. Reference coil with all loops at the utmost exterior.

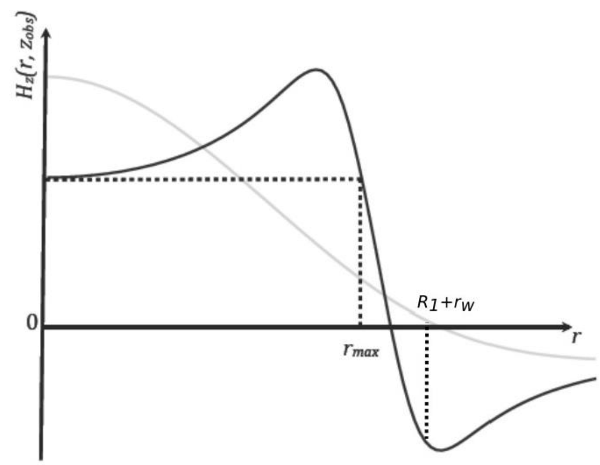

Fig. 3. Typical field distributions: all loops at the outer edge (black curve) and all loops lowest possible radius (gray curve).

Both typical field distributions are also found by other authors, e.g., [7]. Based on the distributions of Fig. 3, we can now propose a strategy to obtain a more homogeneous field distribution, with a given exterior dimension $R_1 + r_w$ and given self inductance L_{ref}. If we consider the reference coil and simply add extra loops in the central region, we obtain a more homogeneous field distribution. However, this would increase the self inductance. Therefore, in order to keep the self inductance fixed to the self inductance of the reference coil, we also will have to decrease the number of loops at the outer edge. One can see, based on Equations (5) and (8), that the total number of loops of the optimized coil N_{opt} will be larger than N_{ref}.

The goal of the algorithm is to define a planar, concentric coil that leads to a significant more uniform z-oriented magnetic field. Therefore, we have to define a lateral region where this condition has to be realized, i.e., an interval over which the cost function v (Equation (9)) is to be evaluated. Since we want a field as homogeneous as possible over an area as large as possible, we choose the value of r_{min} to be 0, and let the value of r_{max} be determined by the reference coil. We define r_{max} as the radial distance where the decaying field becomes smaller than the value found in the center in the case of the reference coil, as illustrated graphically on Fig. 3. Above this radius r_{max}, the variation of the field is no longer relevant.

For the input of the algorithm, the following parameters are given by the user: the required self inductance L_{ref}, the radius of the wire r_w, the outer dimension of the planar coil $R_1 + r_w$ and the vertical distance z_{obs} where the magnetic field is to be observed. The algorithm is split in two steps. First, a set of concentric coils is created that have a self inductance that is close to the value of the required self inductance L_{ref}. In the second step, the configuration within this set that delivers the most uniform $H_z(r, z_{obs})$ (where r runs from 0 to r_{max}) is determined.

On Fig. 4, the flowchart of the algorithm is shown. At the start, the user inputs the values for L_{ref}, r_w, $R_1 + r_w$ and z_{obs}. From these data, the algorithm calculates for all possible reference coils (i.e., coils with all loops exterior) the corresponding inductance, based on expression (5). It then selects the coil with the inductance closest to L_{ref}. We call this the reference coil with number of loops N_{ref}. From this reference coil, the value of r_{max} is calculated (using Equation (3) and the given observation height z_{obs}).

The next task to be executed is the generation of a large set of planar coils. Therefore, the number and radii of the loops is modified and a random generator determines a new set of radii. The limitation on the random generator is that the difference of the values of the radii R_n must be at least $2r_w$ in order to keep the structure planar. The random generator returns uniformly distributed random numbers in the specified interval. We define N_{ext} as the number of loops that is kept at the most outside position and N_{int} as the number of loops that have a radius smaller than $R_1 - 2N_{ext}r_w$. These loops are thus located in the central part of the coil. Figure 5 shows an example, where N_{ext} equals 8 and N_{int} is 3. The total number of loops N_{opt} is of course $N_{ext} + N_{int}$. The only limitation towards the radii of the inner loops is that they must differ by at least $2r_w$. The difference between the interior loop radii values can vary.

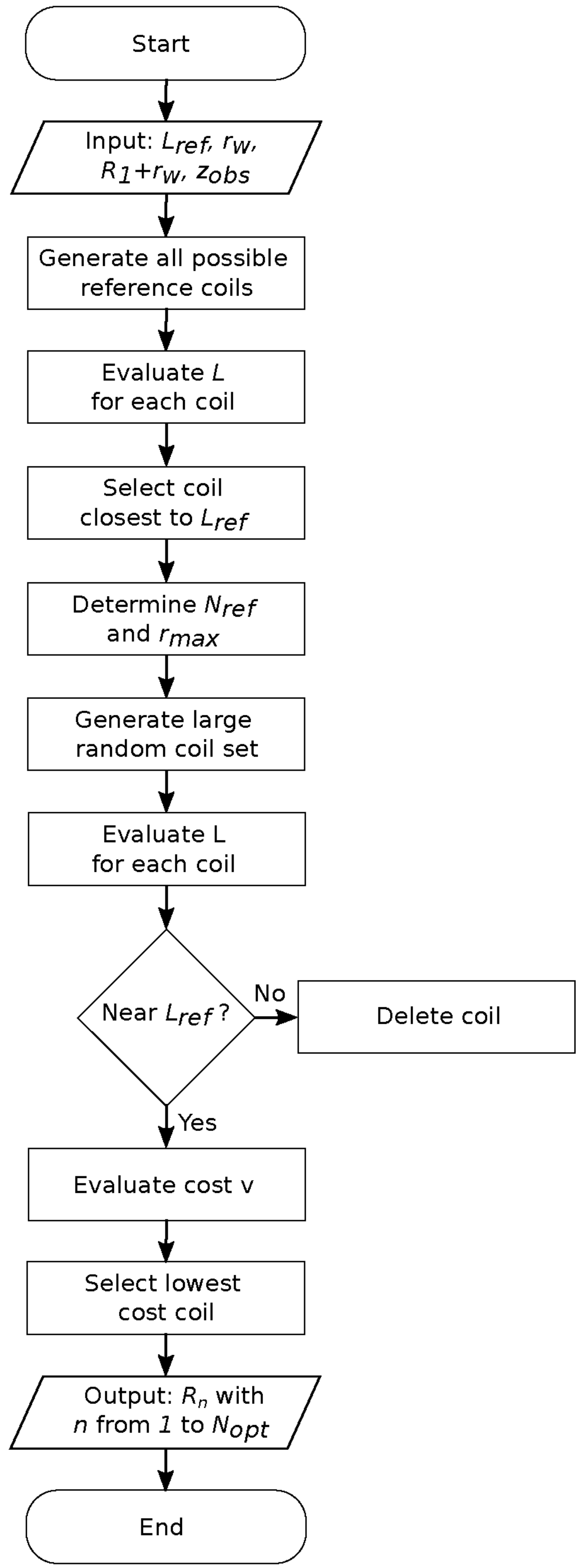

Fig. 4. Flowchart of the algorithm.

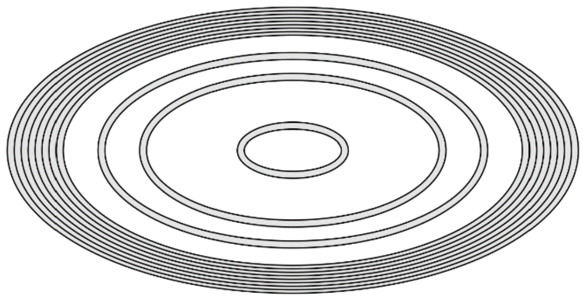

Fig. 5. Planar concentric coil with $N_{ext} = 8$ and $N_{int} = 3$.

Then the self inductance L of each coil is calculated by using the closed expression (5). Based on the calculated self inductance values, we decide which coils can be deleted and which ones are selected for further evaluation. The user has a certain degree of freedom here, but a relative difference of a couple of percent seemed a reasonable choice. For example, the Agilent 42851 Precision LCR meter has, depending on the measurement conditions, an inductance measurement accuracy of about 2%. Therefore, $L \in [0.98\,L_{ref}, 1.02\,L_{ref}]$, which allows a maximum relative difference of 2%, are the boundaries we will choose in Section IV. At this point of the procedure, we have a large set of concentric coils with a self inductance that is close to the user specified L_{ref}.

The next step is the evaluation of the cost function v (expression (9)) for the remaining coils to find the current loop distribution that leads to the most uniform field for r between 0 and r_{max}. For the numerical evaluation of v, we approximate the integral of Equation (9) by a finite sum, as described by equation (11). Here, r_0 equals 0, r_{M+1} equals r_{max} and M is large. We have observed that this sum converges rapidly as M approaches several hundreds of evaluation points between 0 and r_{max}:

$$v \approx \sum_{m=0}^{M} |H_z(r_{m+1}) - H_z(r_m)|. \qquad (11)$$

Since this corresponds with the midpoint Riemann sum, the order of magnitude of the error as a consequence of this approximation is $\mathcal{O}((r_{max}/M)^2)$. In order to find the design with the most uniform vertical field distribution at the height z_{obs}, the coil with the smallest cost value v is selected. The outcome of the procedure delivers us all required parameters of the optimized planar coil: the number of loops N_{opt}, with the corresponding radii R_n.

IV. EXAMPLE AND EXPERIMENTAL VERIFICATION

To illustrate the algorithm, we apply it on a representative example. The dimensions of the set-up are based on the “power transmitter design A2” [26] from the Qi-standard that allows free positioning. We have constructed a reference coil with the following dimensions: $2r_w$ equals 0.83 mm (a wire that was available for measurement verification), $R_1 + r_w$ is 40 mm and the number of loops N_{ref} is 10. The exact numbers are not dedicated to our method, it rather allows us to focus the attention on the main, representative results. An observation height (z_{obs}) of 5 mm and frequency of 100 kHz was chosen, which are representative for the wireless charging of low power electronic devices [27] (with the exception of biological implants where larger distances and frequencies are more typical).

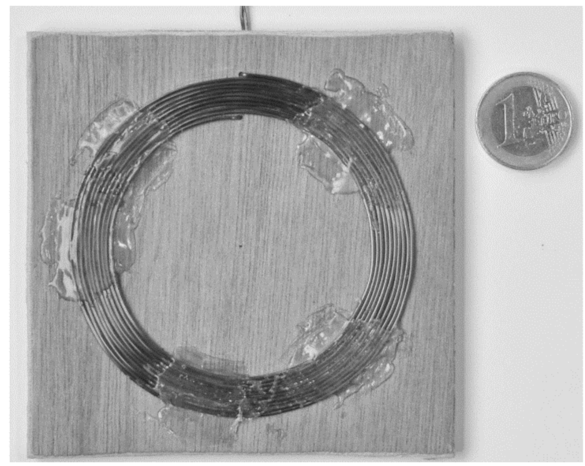

Fig. 6. The reference coil.

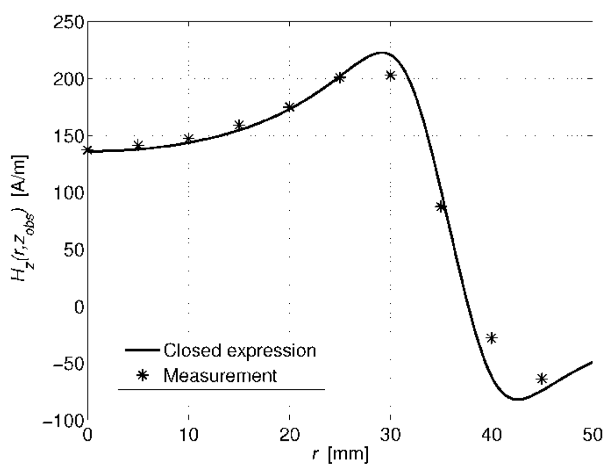

Fig. 7. Vertical magnetic field of the reference coil at the observation height z_{obs} of 5 mm.

A photograph of this coil is depicted on Fig. 6. The reflecting zones are caused by the glue to keep the loops at the same position. In order to have an estimate on the size of both coils, a one euro coin (that has a diameter of 23.25 mm) is also shown on the picture.

Application of the self inductance formula (Equation (5)) leads to a value of 13.24 µH for L_{ref}. The vertical magnetic field distribution is shown on Fig. 7, calculated with closed expression (3), where the excitation current equals 1 A. For the rest of this study, the results are normalized to a current of 1 A. The distance r_{max}, as defined in the description of the algorithm, corresponds to a value of 34.9 mm in this specific configuration. As mentioned above, the variation of the field above this radius r_{max} is not relevant.

On Fig. 7, experimental data is also shown. We have used the GM08 Hand-held Gaussmeter from Hirst Magnetic Instruments Ltd. to measure the vertical magnetic field. Very good correspondence is obtained between the analytical formulation and the measured data. Now that we have defined the reference coil and verified its vertical field distribution, we can start the algorithm with the goal to find a planar concentric coil with a much more uniform field distribution at z_{obs}= 5 mm for r between 0 and r_{max}. In order to compare the results with our reference coil, we will require that the new planar coil has the same self inductance L_{ref} as our reference coil (i.e., 13.24 μH), with a maximum relative difference of ± 2%.

In total, we evaluated 10^5 different coils (interior radii random generated) with values for N_{ext} of 6 to 9, where the maximal value of $N_{ext} + N_{int}$ was 15. For larger values of $N_{ext} + N_{int}$, many small loops were at the interior in order to achieve the same inductance. This increases the field at the center too much so that field homogeneity cannot be reached anymore. With this set, 7488 coils (thus about 7.5%) passed the test of being close to the original L_{ref}, with a maximal relative difference of ± 2%. From these 7488 coils, the smallest v-number was 97.1 A/m, where the v-number of the reference coil was 179.7 A/m.

The coil that was selected has a theoretical self inductance L_{opt} of 13.27 μH, which deviates only 0.23% from the L_{ref} of 13.24 μH. The optimized coil has 9 exterior loops (N_{ext}) and 3 interior loops (N_{int}). The values of R_1 to R_9 are straightforward, since they are part of the exterior loops. R_n equals in our case $40 - 0.83/2 - (\mathrm{n} - 1)0.83$ mm, where n runs from 1 to 9. For the three other, interior loops we have a value of 26.41 mm for R_{10}, 20.80 mm for R_{11} and 12.95 mm for R_{12}. We fabricated this optimized coil, as can be seen on Fig. 8.

As a first verification, we measured the self inductance of our coils using an Agilent 42851 Precision LCR meter. We measured a value of 12.20 μH and 12.42 μH for respectively the reference coil and the optimized coil. Two observations can be made. In the first place, the measured self inductance deviates from the theoretical predictions based on the formula (5). On the one hand, this theoretical prediction is an approximation. On the other hand, one also has to take into account that due to practical reasons, it is not always possible for the entire circumference to reduce the distance between any two adjacent loops to zero. This will thus lead to some loss of self and mutual inductance. However, the most important observation, apart from the value itself, is that the two measured self inductances are close to each other, within a relative difference of 2%.

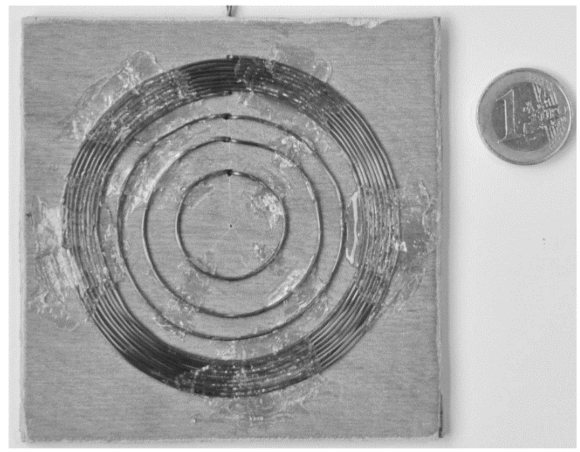

Fig. 8. The optimized coil.

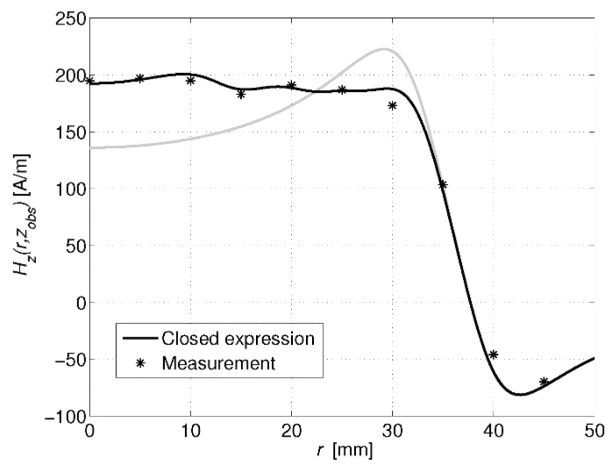

Fig. 9. Vertical magnetic field of the optimized coil at the observation height z_{obs} of 5 mm. The gray line shows the vertical magnetic field for the reference coil as comparison.

On Fig. 9, we have shown the field distribution of the coil with the minimal cost v, with the corresponding measurement result. One can see that the field itself, is indeed much more uniform over the r-region between 0 and r_{max}, which means that the cost v is a good selection criterion. A second observation, is that the field values are also high over the entire region, it is only at the outer regions of the optimized coil that the field values are smaller than the reference coil field values. On Fig. 9, a gray line with the result of the reference coil is added to facilitate the comparison. Remark that by minimizing the cost *v*, we actually only targeted to obtain a uniform field, without directly imposing conditions on the value of this field. It turns out that a minimized cost also leads to a high field value. Table 1 summarizes the results for the reference and optimized coil.

Table 1: Overview of different parameters for the reference and optimized coil

	Reference Coil	Optimized Coil
N_{ext}	10	9
N_{int}	0	3
$L_{theoretical}$	13.24 μH	13.27 μH
$L_{measured}$	12.20 μH	12.42 μH
v	179.7 A/m	97.1 A/m

The algorithm for this example was executed in the numerical environment Matlab® on a modest computer, i.e., a MacBook Pro with an Intel Core i7 processor of 2.6 GHz with 4 GB 1600 MHz DDR3 SDRAM. It took 57.6 s for the algorithm to produce the end results for this representative example. The short algorithm time is due to the available closed-form expressions at the different steps. The time-consuming part is almost entirely attributed to the fact that many (10^5) coils need to be evaluated.

Because the algorithm chooses the best coil (with regard to the cost function *v* from a large coil set), the procedure always converges and produces a useful result. The user implementing our method has to be aware of two possible issues:

- Since the algorithm selects the optimal coil from a random generated coil set, the exact same solution will not be reproduced when the program is executed more than once. However, if the user sets the number of generated coils high enough (10^5 in our example), the end result will lead to as equally homogeneous magnetic field distribution, well within any practical measurement errors. Given the fast execution time of the algorithm on a modest machine, the user will not experience any significant obstacles by choosing a very large coil set.
- The novelty of this algorithm, compared to previous work, is that it imposes a by the user chosen self inductance L of the coil. This allows the user to replace an existing coil with the calculated coil, allowing a more homogeneous field with minimal impact on the rest of the network. The algorithm requires an interval for this self inductance L, for example a relative difference of 2% as chosen in this example. However, it might be beneficial to choose a larger interval, thus allowing more freedom to generate a uniform field distribution. Depending on the application, the user can, by circuital simulation, study the influence of varying the coil inductance and adapt the interval accordingly.

VI. CONCLUSION

In this paper, an algorithm to design a concentric, planar coil that leads to a homogeneous vertical magnetic field distribution is described. The novelty of this approach is that the procedure takes a certain target self inductance into account, so that the optimized coil can easily replace another coil without impact on the rest of the circuitry. As a consequence of the availability of closed expressions for the field distribution and self inductance, a very large set of random generated coils can be evaluated in a very short time. The result of the algorithm leads to all parameters required to realize the optimized planar coil, taken into account the radius of the wire in order to keep the structure planar. The method is applied on a representative coil. The results were verified experimentally and excellent correspondence was obtained.

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