Generalized and Numerically Robust Singularity Correction in TLM Models of Electromagnetic Fields

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Abstract: In this paper we propose a generalized robust method for correcting the coarseness error in TLM simulations involving field singularities. This method is shown to be effective for a variety of structures and material properties. The efficiency and accuracy of this method is demonstrated by comparing it with results from uncompensated TLM computations in connection with Richardson extrapolation for infinitesimal mesh size.

Keywords: TLM, singularity correction, modification of edge cells.

1 Introduction

The modeling of electromagnetic structures with space discrete numerical methods is strongly affected by the presence of field singularities at sharp edges and corners [1]. Regular FDTD and TLM schemes are normally expected to be second-order accurate, but in the presence of singularities the results deteriorate to first-order accuracy. This is due to the inability of the grid to properly resolve the highly non-uniform fields at edges and corners [2] [3] [4]. As a result, the energy stored in the vicinity of the singularities is not properly modeled, resulting in significant errors in the properties of the structure (resonant frequencies, propagation constants, characteristic impedances, S-parameters, etc.). It is thus important to develop and implement corrective measures.

Several approaches have been proposed for dealing with this error [5] [6] [7] [8]. Graded and multi-grid meshing is often applied intuitively to increase the resolution of the grid in the critical regions, but the resulting computational cost is high and often unacceptable. A more intelligent approach is to use basis functions at the discontinuities that resemble the singular fields. Alternatively, the properties of the field space can be modified locally so that the stored energy is correct even though the field itself is poorly resolved. This can take the form of inserted lumped reactive elements, additional transmission line stubs, or local modifications in the constitutive parameters ε and μ. In this paper we propose a simple and robust method for singularity correction in TLM models based on the latter approach. It is numerically robust, independent of the type of singularity (TE or TM), can be automated easily, and has negligible computational impact.

It is important to remember that the singular field distribution is quasi-static since the time derivative of the fields is insignificant compared with their space derivative [9]. While some of the field components become infinite at sharp edges and corners, the energy they contain remains finite. The local character of the singularity fields implies that they are independent of boundary conditions several mesh parameters away. Hence, a single correction will be valid for all external boundary geometries.

2 Description of the proposed method

We have implemented a correction procedure for all types of field singularities at sharp edges and 90˚ corners in 2D and 3D TLM models based on the generalized symmetrical condensed node (GSCN) [10]. The energy storage capacity of the TLM cells in the immediate vicinity of the edge is reduced by lowering the values of their dielectric permittivity and magnetic permeability. Figure 1 shows the four configurations studied in this paper, and Figure 2 shows the location of modified cells.

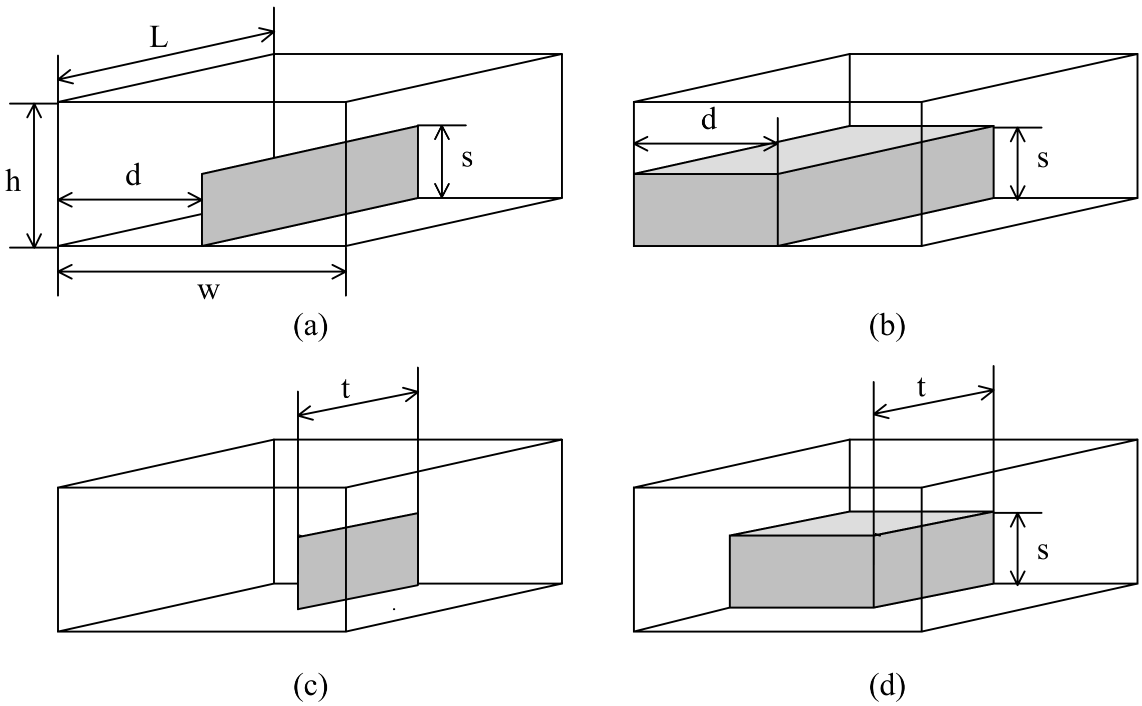

Figure 1 Four types of singularities for which correction coefficients have been determined (a) knife edge (b) 90° edge (c) knife edge corner (d) 90° edge corner

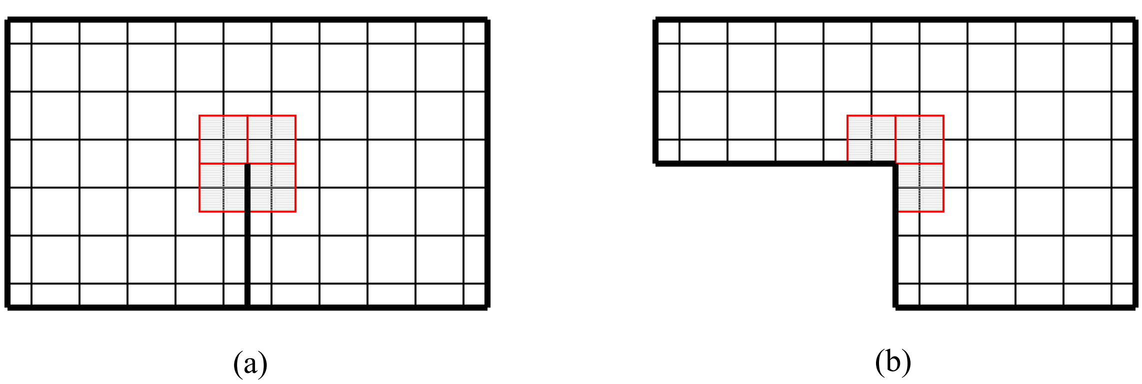

Figure 2 The singularities are corrected by changing the ε and μ in the cells surrounding the edge (a) Four edge cells are modified along the knife-edge (b) Three cells are modified along the 90° edge.

Both constitutive parameters are reduced by the same relative amount in order to preserve the local characteristic impedance of the field space. While the required change in ε and μ can be computed approximately using the known expressions for the quasi-static fields [4], it is straightforward to determine them by optimization. We have thus placed the discontinuities into cavities and modified ε and μ of the edge cells until the resonant frequencies became virtually independent of the mesh size, keeping the discretization small enough for the dispersion error to be negligible. This is illustrated in Figure 3 where the dominant resonant frequency of a cavity with a knife-edge singularity has been drawn over the discretization parameter used to compute that frequency. If the singularity is not corrected, the frequency varies linearly with the mesh size (first order error). However, with proper modification of the edge cells, the computed resonant frequency is practically independent of the mesh size, showing only a slight second-order dispersion error.

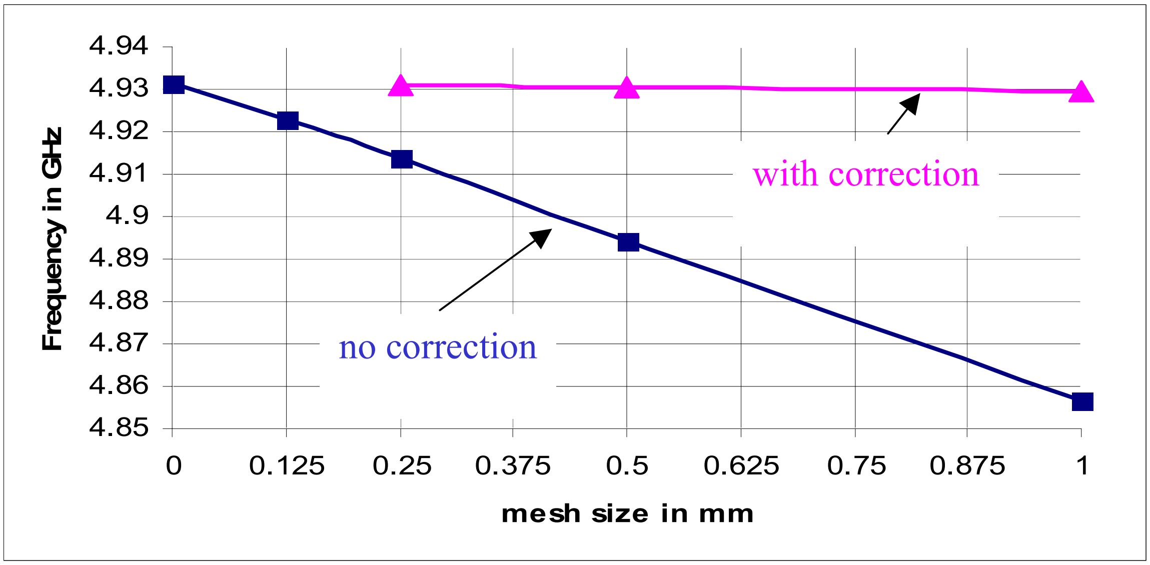

Figure 3 Resonant frequency of a cavity containing a knife-edge singularity, as a function of the mesh size. If the singularity is not corrected, the frequency varies linearly with the mesh size (first order error). However, with proper modification of the edge cells, the computed resonant frequency is practically independent of the mesh size, showing only a slight second-order dispersion error.

Using this optimization process we have obtained the following correction coefficients for the four singularities shown in Figure 1. These correction coefficients are the same when a knife-edge or corner is immersed in materials with arbitrary permittivity and permeability, or when the singularity lies at the interface between different materials as in microstrip lines. The constitutive parameters in each corner cell are simply multiplied by the appropriate correction factor given in Table 1.

Singularity Type	Correction Coefficient for ε	Correction Coefficient for μ
Knife Edge (Figure 1a)	0.808	0.808
90^o Edge (Figure 1b)	0.915	0.915
Knife Edge Corner (Figure 1c)	0.708	0.708
90^o Edge Corner (Figure 1d)	0.875	0.875

Table 1 Correction coefficients for the four types of singularities shown in Figure 1.

3 Implementation of the correction procedure

The theoretical foundation of the Generalized Symmetrical Condensed Node (GSCN) is given by Trenkic et al [10]. At each node, the unconditionally stable time-steps in the presence of different dielectric and magnetic material parameters are:

$$\Delta t_{\varepsilon i} = \frac{\varepsilon_{ri}}{2c}\frac{\Delta j \Delta k}{\Delta i} \qquad \text{and} \qquad \Delta t_{\mu i} = \frac{\mu_{ri}}{2c}\frac{\Delta j \Delta k}{\Delta i} \tag{1}$$

where i, j, k represent x, y, z; ε_{ri} and μ_{ri} are the relative permittivity and permeability in the i-direction, respectively. The time-steps in all three directions, $\{\Delta t_{\varepsilon x}, \Delta t_{\varepsilon y}, \Delta t_{\varepsilon z}, \Delta t_{\mu x}, \Delta t_{\mu y}, \Delta t_{\mu z}\}$, can be obtained via a cyclic permutation of x, y, z over the i, j, k variables in equation (1). The time-step at each node, Δt_{node}, is the minimum of the above local time-steps. The global time-step, Δt, for the mesh is the minimum of Δt_{node} over the whole mesh. The admittance and impedance values for the open and short circuited stubs are:

$$Y_{oi} = 2Y_o\left(\varepsilon_{ri}\frac{\Delta j \Delta k}{\Delta i}\Delta t - 2\right) \qquad \text{and} \qquad Z_{oi} = 2Z_o\left(\mu_{ri}\frac{\Delta j \Delta k}{\Delta i}\Delta t - 2\right) \tag{2}$$

where Y_o and Z_o are the free space admittance and impedance, respectively. Once $\{Y_{ox}, Y_{oy}, Y_{oz}, Z_{ox}, Z_{oy}, Z_{oz}\}$ are obtained, GSCN simulation can be performed efficiently with the scattering algorithm described in [11].

4 Validation of the correction procedure

In this section, simulation results are presented to illustrate the accuracy and efficiency of the proposed correction method. The first example is a knife-edge discontinuity in a rectangular cavity as depicted in Figure 4. A sequence of plain simulations with various mesh resolutions was performed. Furthermore, the structure was also analyzed with the edge correction coefficients given in Table 1. The results are summarized in Table 2 for ease of comparison. Table 3, Table 4 and Table 5 are simulation results obtained with a 90˚ edge, a knife-edge corner and a 90^{o} edge corner, respectively. The validation results indicate that the edge correction method increases the accuracy in the resonant frequency by one order of magnitude in most cases.

	Δl=1 mm	Δl=0.5 mm	Δl=0.25 mm	$\Delta l \to 0$ (extrapolated value)
f (in GHz, without correction)	7.6541	7.7674	7.8235	7.88
Error relative to $\Delta l \to 0$	–2.9%	–1.4%	–0.7%	0
f (in GHz, with correction)	7.8758	7.8776		7.88
Error relative to $\Delta l \to 0$	–0.05%	–0.03%		0

Table 2 Resonant frequency of a rectangular waveguide cavity (L = 15mm, w = 20mm, h = 10mm) containing a knife-edge (d = 10mm, s = 5mm) with and without edge correction.

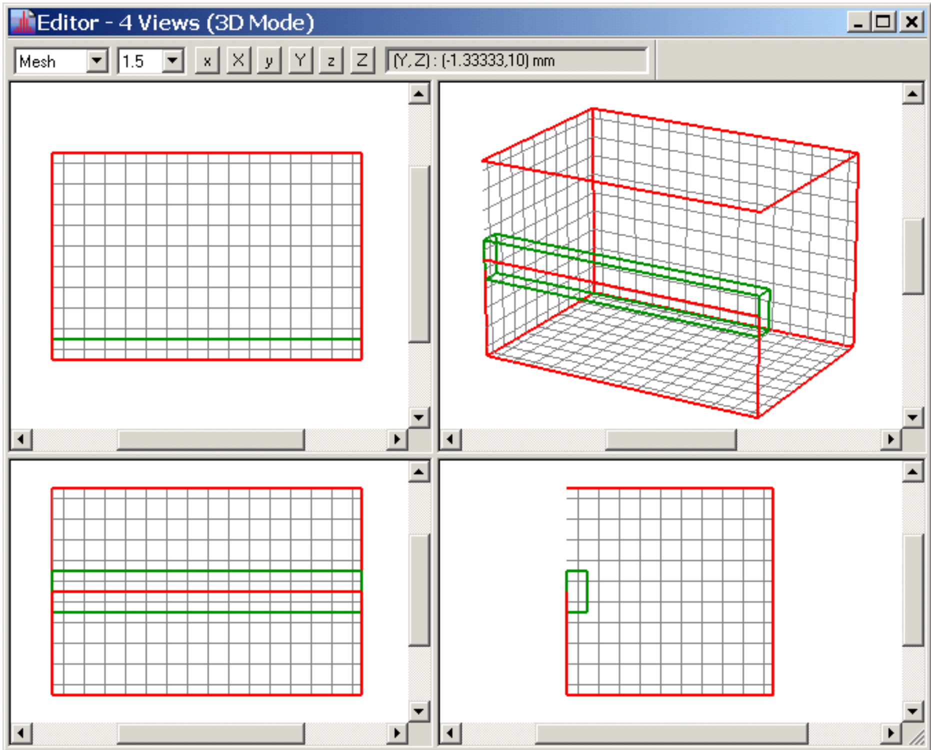

Figure 4 The discretized view of a knife-edge inside a waveguide resonator (due to symmetry property, only one-half the structure is needed to compute the resonant frequency); the green regions identify GSCN cells along the knife edge. The dimensions of the structure are defined in Figure 1 and given in Table 2.

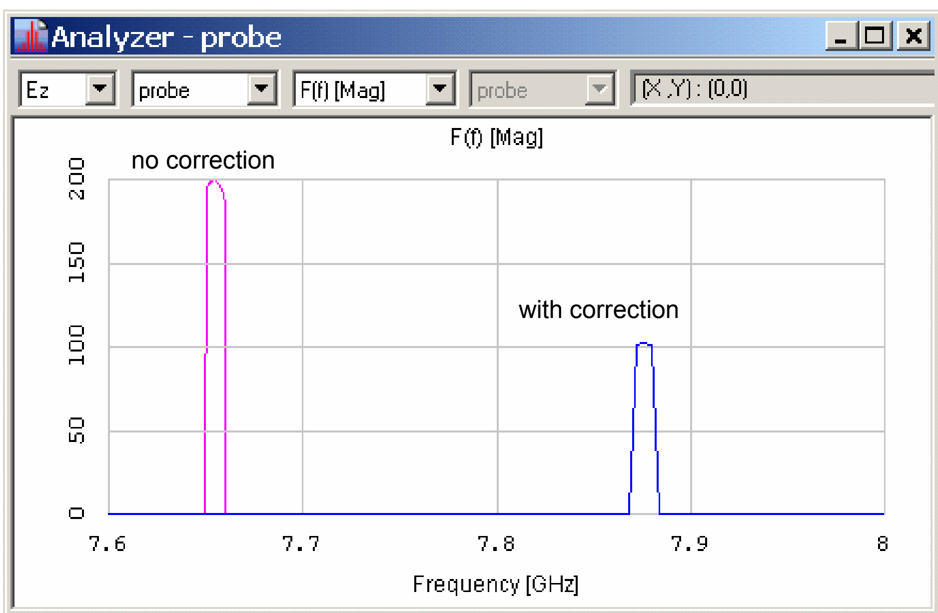

Figure 5 Resonant frequencies, 7.6541 and 7.8758 GHz, obtained with $\Delta l = 1$ mm. The higher resonant frequency was obtained with the edge correction coefficients given in Table 1. The large difference between these results clearly demonstrates the need for singularity correction.

	Δl=0.5mm	Δl=0.25 mm	Δl=0.2 mm	$\Delta l \to 0$ (extrapolated value)
f (in GHz, without correction)	23.0805	23.998	24.1572	24.79
Error relative to $\Delta l \to 0$	–6.9%	–3.2%	–2.6%	0
f (in GHz, with correction)	24.7694	24.7741		24.79
Error relative to $\Delta l \to 0$	–0.08%	–0.06%		0

Table 3 A rectangular waveguide cavity (L=10mm, w=6mm, h=2mm) containing a knife-edge corner (d=3mm, s=1mm, t=5mm)

	Δl=1 mm	Δl=0.5 mm	Δl=0.25 mm	$\Delta l \to 0$ (extrapolated value)
f (in GHz, without correction)	16.9005	17.0653	17.1281	17.17
Error relative to $\Delta l \to 0$	–1.6%	–0.6%	–0.3%	0
f (in GHz, with correction)	17.1592	17.1653		17.17
Error relative to $\Delta l \to 0$	–0.06%	–0.03%		0

Table 4 A rectangular waveguide cavity (L=14mm, w=10mm, h=6mm) containing a 90^o edge (d=5mm, s=3mm)

	Δl=0.5 mm	Δl=0.25 mm	Δl=0.2 mm	$\Delta l \to 0$ (extrapolated value)
f (in GHz, without correction)	29.0281	29.1331	29.1677	29.26
Error relative to $\Delta l \to 0$	–0.8%	–0.4%	–0.3%	0
f (in GHz, with correction)	29.2466	29.2574		29.26
Error relative to $\Delta l \to 0$	–0.05%	–0.01%		0

Table 5 A rectangular waveguide cavity (L=10mm, w=6mm, h=4mm) containing a 90^o edge corner (d=2mm, s=2mm, t=4mm)

5 Conclusion

In this paper we have proposed a simple but accurate and numerically robust singularity correction for GSCN–TLM modeling. The permittivity and permeability inside the cells adjacent to the singularity are modified by a scalar correction factor, which amounts to a quasi-static correction of the electric and magnetic energy stored in the GSCN cells at the singularity. This correction is equally effective for TE, TM and hybrid field excitations of the singularity. Resonant frequency computations incorporating this correction procedure have been compared with data obtained with the regular GSCN method. The comparison shows that the proposed correction method reduces the singularity error by typically one order of magnitude without penalty in terms of computational burden. The effectiveness of the method for the accurate computation of structures with metallic strips (knife edges) or 90^o edges and corners has been clearly demonstrated and is applicable for both homogeneous and inhomogeneous material properties in the singularity region.

6 References

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