Electromagnetic Scattering by a System of Dielectric Spheres Coated With a Dielectric Shell

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Abstract

Analytical solution is derived to the problem of scattering of electromagnetic plane wave by an array of dielectric spheres each coated with a dielectric shell. The incident, scattered and transmitted electric and magnetic fields are expressed in terms of the vector spherical wave functions. The vector spherical translation addition theorem is applied to impose the boundary conditions on the surface of various layers. Numerical results are computed and presented graphically for the radar cross sections of several configurations of spheres system with multi dielectric layers.

1. Introduction

Many authors have studied the scattering of electromagnetic plane wave by a dielectric sphere coated with a dielectric shell. Aden and Kerker [1] obtained analytical expressions to the scattering of electromagnetic plane wave by a dielectric sphere coated with a concentric spherical shell of different dielectric materials, while Scharfman [2] presented numerical results for the special case of a small electrical radius (ka<1) dielectric coated conducting sphere. It was found in those early studies that the presence of dielectric coatings leads to substantial increase in the backscattering cross section for an appropriate choice of the dielectric constant and thickness of the coating relative to that of uncoated sphere. Further, Wait has extended the solution to the case of scattering by a radially inhomogeneous sphere [3], while a numerical solution using the method of moments obtained by Medgyesi-Mitschang and Putnam for the case of dielectric-coated concentric sphere [4]. More recently, an exact solution of electromagnetic plane wave scattering by an eccentric multilayered sphere was developed by Lim and Lee [5]. Numerous papers on the scattering from systems of spheres of various natures in close proximity have been treated by numerous researchers [6-11].

Up to now, there has been no analytical or numerical solution to the problem of scattering of electromagnetic plane wave by an array of conducting spheres each coated with a dielectric layer. In this paper, we extend the solution of scattering by two dielectric spheres covered with a dielectric shell [9] to the case of scattering by a system of dielectric spheres each covered with a dielectric shell. The solution to this problem has many practical applications since, for example, it may be used to study the scattering by complex objects simulated by a collection of spheres [12], and it may also be used to check the accuracy of numerical solutions.

From the design point of view, the backscattering cross section of an array of N dielectric coated spheres can be controlled to exploit multiple resonances by optimizing the multivariables of the system. These include the size and location of each sphere, number of dielectric layers coating each sphere as well as the thickness and relative dielectric constant of each layer as already done for conducting cylinders [13].

2. Formulation of the Problem

Consider a linear array of N dielectric spheres each coated with a dielectric shell and having different radii and unequal spacing with centers lying along the z axis, as shown in Fig. 1. Electromagnetic plane wave of unit electric field intensity, whose propagation vector $\overline{K}$ lies in the x-z plane and makes an angle α with the z-axis, is assumed to be incident on the spheres. Its incident electric and magnetic fields are

$$\overline{E}_i = e^{j\overline{k}\cdot\overline{r}}\ \hat{y} \tag{1}$$

$$\overline{H}_i = -\frac{1}{\eta} e^{j\overline{k}\cdot\overline{r}} (\cos\alpha\,\hat{x} - \sin\alpha\,\hat{z}) \tag{2}$$

with k being the wave number, $\hat{x}, \hat{y},$ and $\hat{z}$ are the unit vectors along the *x*, *y* and *z* axes, respectively, and η is the surrounding medium intrinsic impedance. The incident electric and magnetic fields may be expanded in terms of spherical vector wave functions around the center of the p^{th} sphere as

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$$\bar{E}_i(r_p,\theta_p,\phi_p)=\sum_{n=1}^{\infty}\sum_{m=-n}^{m=n}[P_p(m,n)\bar{N}^{(1)}_{mn}(r_p,\theta_p,\phi_p)+Q_p(m,n)\bar{M}^{(1)}_{mn}(r_p,\theta_p,\phi_p)] \tag{3}$$

$$\eta\bar{H}_i(r_p,\theta_p,\phi_p)=j\sum_{n=1}^{\infty}\sum_{m=-n}^{m=n}[P_p(m,n)\bar{M}^{(1)}_{mn}(r_p,\theta_p,\phi_p)+Q_p(m,n)\bar{N}^{(1)}_{mn}(r_p,\theta_p,\phi_p)] \tag{4}$$

where $\bar{M}^{(1)}_{mn}$ and $N^{(1)}_{mn}$ are the spherical vector wave functions of the first kind representing incoming waves associated with the spherical Bessel function, while $P_p(m,n)$ and $Q_p(m,n)$ are the incident field expansion coefficients defined in [7-8,14]. The field in the region II can be also expressed in terms of the vector spherical wave functions of the first and third kinds. Hence the electric and magnetic fields may be written as

$$\begin{aligned}\bar{E}_{II}(r_p,\theta_p,\phi_p)=\sum_{n=1}^{\infty}\sum_{m=-n}^{m=n}[&A'_{Ep}(m,n)\bar{N}^{(1)}_{mn}(r_p,\theta_p,\phi_p)\\&+A''_{Ep}(m,n)\bar{N}^{(3)}_{mn}(r_p,\theta_p,\phi_p)\\&+A'_{Mp}(m,n)\bar{M}^{(1)}_{mn}(r_p,\theta_p,\phi_p)\\&+A''_{Mp}(m,n)\bar{M}^{(3)}_{mn}(r_p,\theta_p,\phi_p)]\end{aligned} \tag{5}$$

$$\begin{aligned}\eta\bar{H}_{II}(r_p,\theta_p,\phi_p)=j\sum_{n=1}^{\infty}\sum_{m=-n}^{m=n}[&A'_{Ep}(m,n)\bar{M}^{(1)}_{mn}(r_p,\theta_p,\phi_p)\\&+A''_{Ep}(m,n)\bar{M}^{(3)}_{mn}(r_p,\theta_p,\phi_p)\\&+A'_{Mp}(m,n)N^{(1)}_{mn}(r_p,\theta_p,\phi_p)\\&+A''_{Mp}(m,n)\bar{N}^{(3)}_{mn}(r_p,\theta_p,\phi_p)]\end{aligned} \tag{6}$$

where $A'_{pE}\,(m,n)$, $A'_{pM}\,(m,n)$, $A''_{pE}\,(m,n)$, and $A''_{pM}\,(m,n)$ are the field expansion coefficients, while $\bar{M}^{(3)}_{mn}$ and $\bar{N}^{(3)}_{mn}$ are the vector spherical wave functions of the third kind representing outgoing waves associated with the spherical Hankel function. The subscripts *E* and *M* denote transverse magnetic (TM) and transverse electric waves (TE), respectively. The field in region I of the pth sphere may be written in terms of the vector wave functions of the first kind, i.e.,

$$\bar{E}_I(r_p,\theta_p,\phi_p)=\sum_{n=1}^{\infty}\sum_{m=-n}^{m=n}[A_{EP}(m,n)\bar{N}^{(1)}_{mn}(r_p,\theta_p,\phi_p)+A_{MP}(m,n)\bar{M}^{(1)}_{mn}(r_p,\theta_p,\phi_p)] \tag{7}$$

$$\bar{H}_I(r_p,\theta_p,\phi_p)=\sum_{n=1}^{\infty}\sum_{m=-n}^{m=n}[A_{EP}(m,n)\bar{M}^{(1)}_{mn}(r_p,\theta_p,\phi_p)+A_{MP}(m,n)\bar{N}^{(1)}_{mn}(r_p,\theta_p,\phi_p)] \tag{8}$$

where A_{EP} and A_{MP} are the unknown transmitted coefficients. Finally, the scattered electric and magnetic fields from the pth sphere are expanded as

$$\bar{E}^s(r_p,\theta_p,\phi_p)=\sum_{n=1}^{\infty}\sum_{m=-n}^{m=n}[A_{Ep}(m,n)\bar{N}^{(3)}_{mn}(r_p,\theta_p,\phi_p)+A_{Mp}(m,n)\bar{M}^{(3)}_{mn}(r_P,\theta_P,\phi_P)] \tag{9}$$

$$\eta\bar{H}^s(r_p,\theta_p,\phi_p)=j\sum_{n=1}^{\infty}\sum_{m=-n}^{m=n}[A_{Ep}(m,n)\bar{M}^{(3)}_{mn}(r_p,\theta_p,\phi_p)+A_{Mp}(m,n)\bar{N}^{(3)}_{mn}(r_P,\theta_P,\phi_P)] \tag{10}$$

where $A_{EP}\,(m,n)$, $A_{MP}(m,n)$ are the unknown scattered field coefficients. To express the scattered fields from the qth sphere in the coordinate system of the pth sphere, we apply the spherical vector translation addition theorem for translation along the z-axis [15], i.e.,

$$\bar{M}^{(3)}_{mn}(r_q,\theta_q,\phi_q)=\sum_{v=1}^{\infty}[A^{mv}_{mn}(d_{pq})\bar{M}^{(1)}_{mv}(r_p,\theta_p,\phi_p)+B^{mn}_{mv}(d_{pq})\bar{N}^{(1)}_{mn}(r_P,\theta_P,\phi_P)] \tag{11}$$

$$\bar{N}^{(3)}_{mn}(r_q,\theta_q,\phi_q)=\sum_{v=1}^{\infty}[A^{mv}_{mn}(d_{pq})\bar{N}^{(1)}_{mv}(r_p,\theta_p,\phi_p)+B^{mn}_{mv}(d_{pq})\bar{M}^{(1)}_{mn}(r_P,\theta_P,\phi_P)] \tag{12}$$

where $A^{mn}_{mv}(d_{pq})$ and $B^{mn}_{mv}(d_{pq})$ are the translation coefficients of the spherical vector translation addition theorem. To determine the unknown scattered field coefficients, we apply the boundary conditions on the various interfaces, i.e.,

$$\bar{r}_p\times\left(\bar{E}_i(b_p,\theta_p,\phi_p)+\sum_{p=1}^{N}\bar{E}^s(b_p,\theta_p,\phi_p)\right)=\bar{r}_p\times\bar{E}_{II}(b_b,\theta_p,\phi_p) \tag{13}$$

$$\bar{r}_p\times\left(\bar{H}_i(b_p,\theta_p,\phi_p)+\sum_{p=1}^{N}\bar{H}^s(b_p,\theta_p,\phi_p)\right)=\bar{r}_p\times\bar{H}_{II}(b_p,\theta_p,\phi_p) \tag{14}$$

$$\bar{r}_p\times\bar{E}_{II}(a_p,\theta_p,\phi_p)=\bar{r}_p\times\bar{E}_I(a_p,\theta_p,\phi_p) \tag{15}$$

$$\bar{r}_p\times\bar{H}_{II}(a_p,\theta_p,\phi_p)=\bar{r}_p\times\bar{H}_I(a_p,\theta_p,\phi_p) \tag{16}$$

Substituting the appropriate field expansion expressions in equations (13) to (16), and applying the orthogonality properties of spherical vector wave functions and eliminating the transmission coefficients we obtain

$$A_{EP}(m,n)=v_n(\rho_p)\,P_P(m,n)+\sum_{\substack{q=1\\q\neq p}}^{N}\sum_{v=1}^{\infty}[A^{mv}_{mn}(dpq)A_{EP}(m,v)+B^{mv}_{mn}(dpq)A_{MP}(m,v)] \tag{17}$$

$$A_{Mp}(m,n)=u_n(\rho_p)\,Q_p(m,n)+\sum_{\substack{q=1\\q\neq p}}^{N}\sum_{v=1}^{\infty}[A^{mv}_{mn}(dpq)A_{Mp}(m,v)+B^{mv}_{mn}(dpq)A_{Ep}(m,v)] \tag{18}$$

where $v_n(\rho_p)$ and $u_n(\rho_p)$ are the electric and magnetic scattered field coefficients for a single dielectric sphere coated with a dielectric layer [1,9]. Equations (17) and (18) may be written in matrix form for the purpose of computing the scattered field coefficients, i.e.

$$\bar{A}=\bar{L}+T\,\bar{A} \tag{19}$$

where $\bar{A}$ and $\bar{L}$ are column matrices for the unknown scattered and incident field coefficients, respectively, and T is a square matrix which contains the translation addition coefficients.

Once the scattered field is computed from equation (19), the normalized bistatic cross section can be obtained as in [16].

3. Numerical Results

In order to check the validity of our computer program, several numerical tests were conducted and the results compared favorably with previously published results [7-8,11]. These tests included the limiting cases of (i) an array of dielectric spheres obtained by setting kb$\approx$ka , ε_{IIr} =1 or $\varepsilon_{IIr} = \varepsilon_{Ir}$ (ii) an array of conducting spheres each coated with a single dielectric layer obtained by setting $\varepsilon_{Ir} = \infty$ and (iii) an array of conducting spheres obtained by setting $\varepsilon_{Ir} = \infty$ and kb$\approx$ka or ε_{IIr} =1.

In this paper, we presented numerical results for different sphere arrays to show the dependence of the radar cross section on various parameters characterizing the geometry, material properties, and incidence angles. Fig. 2 shows the normalized bistatic cross section versus the scattering angle θ for a system of three identical spheres in the E and H-planes. The electrical radii of the outer and inner spheres are ka=2.0 and kb=2.5, respectively, while the electrical separation between successive spheres is kd=7.0, and the relative dielectric permittivity of the inner dielectric layer is 3.0 and the outer is air. The purpose of this comparison is to check the accuracy of the computer code for the dielectric sphere case [8] as a special case of the dielectric spheres except the relative dielectric permittivity of the dielectric layer is set equal to unity. The parameters of Fig. 3 are similar to Fig. 2 except that the dielectric layer has a value of 2. We can see that the number of resonances in E plane is increased. Figs. 4 and 5 have the same parameters as in Fig. 3 except that the number of spheres is increased to five and eight, respectively. We can see that the number of resonances also increases with the number of spheres.

Fig. 6 shows the normalized backscattering cross section versus the electrical distance (kd), which ranges from 8 (touching) to 15.5 for end fire incidence and the number of spheres is five. The electrical radii of outer and inner spheres are ka=4.0 and kb=3.0, repectively, while the relative dielectric permittivity of the inner dielectric layer is 3.0 and for the outer layer is 2. Fig. 7 is similar to Fig. 6 except the number of spheres is increased to 8. We can see that the location of the maximum peaks did not change by increasing the number of spheres for both cases. Furthermore, the magnitude of the normalized backscattering cross section at the maximum peaks increased with increasing number of spheres.

In Figs. 8 and 9 we have plotted the normalized backscattering cross as a function of the angle of incidence α, which ranges from 0 to 90 degrees for a system of three and eight spheres. The electrical radii of the outer and inner spheres are ka=1.5 and kb=1.0, repectively, while the relative permittivity of the inner dielectric layer is 4 while for the outer layer is 3 and the electrical separation between the centers of the spheres is 3.0 (touching).

4. CONCLUSIONS

We have obtained an analytic solution of the problem of scattering by an array of dielectric spheres each coated with a dielectric shell. The boundary conditions are satisfied at various interfaces with the help of the vector translation addition theorem. The system of equations was written in matrix form while the scattered field coefficients were obtained by matrix inversion. Numerical results were presented for different numbers of spheres, angles of incidence, electrical separation, and relative dielectric constant. For the general case of spheres orientation, the reader may find more details in [8].

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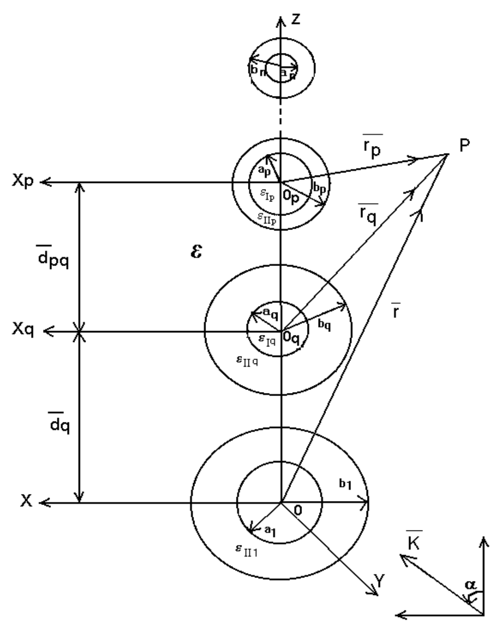

Fig. 1: Geometry of the scattering problem.

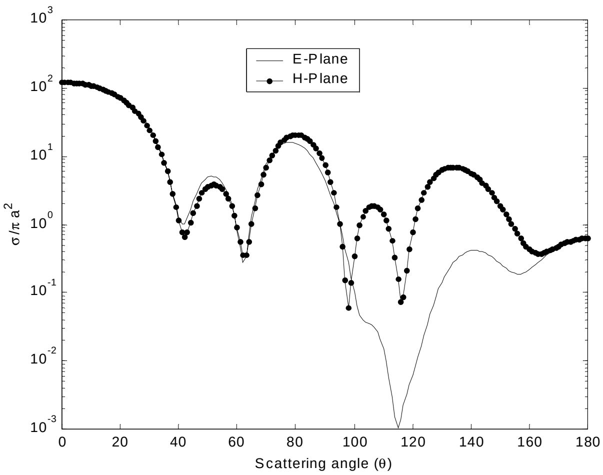

Fig. 2: Normalized bistatic cross section patterns for three identical dielectric spheres each covered with dielectric layer with ka=2.0, kb=2.5, kd=7.0, α=0, ε_{Ir}=3.0, and ε_{IIr}=1.0. In the E-plane ($\phi = \pi/2$) and H-plane ($\phi = 0$).

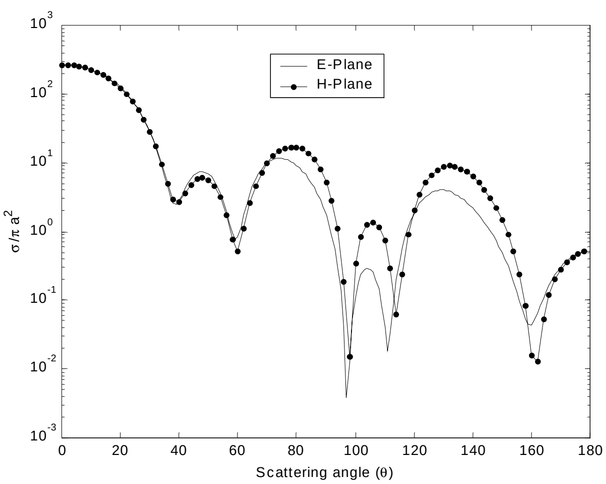

Fig. 3: Normalized bistatic cross section patterns for three identical dielectric spheres each covered with dielectric layer with ka=2.0, kb=2.5, kd=7.0, α=0, ε_{Ir}=3.0, and ε_{IIr}=2.0.

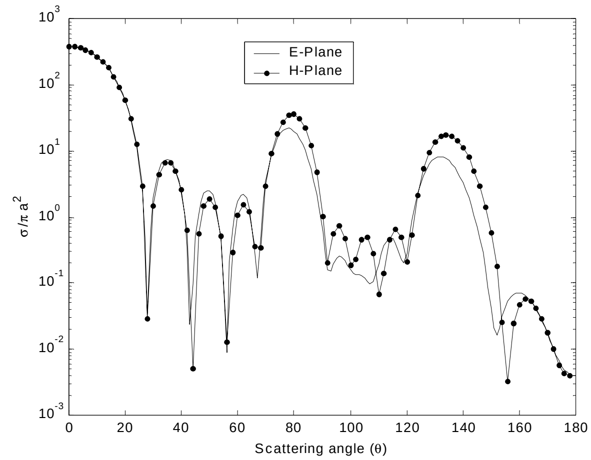

Fig. 4: Normalized bistatic cross section patterns for five identical dielectric spheres each covered with a dielectric layer with ka=2.0, kb=2.5, kd=7.0, α=0, ε_{Ir}=3.0, and ε_{IIr}=2.

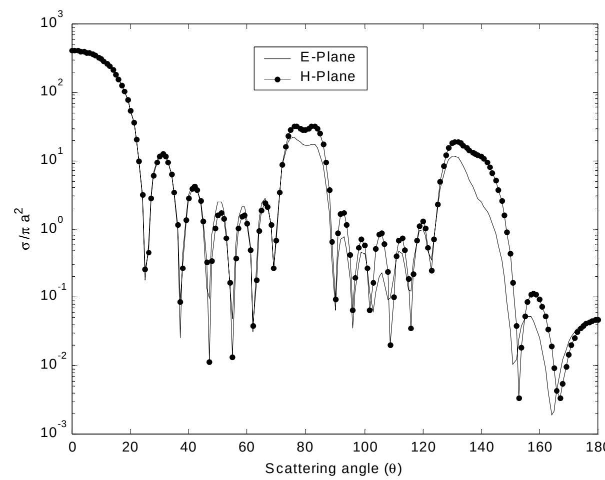

Fig. 5: Normalized bistatic cross-section patterns for eight identical dielectric spheres each covered with a dielectric layer with ka=2.0, kb=2.5, kd=7.0, α=0, ε_{Ir}=3.0, and ε_{IIr}=2.0.

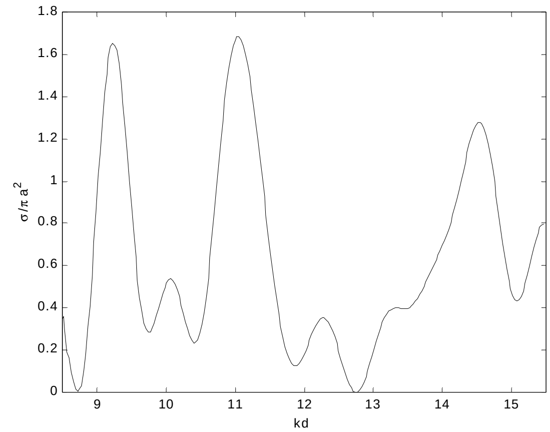

Fig. 6: Normalized backscattering cross section versus electrical separation (kd) for end-fire incidence and a linear array of five identical dielectric spheres each covered with a dielectric layer with: ka=4.0, kb=3.0, $\alpha = 0.0$, ε_{Ir} =3.0, and ε_{IIr} =2.0.

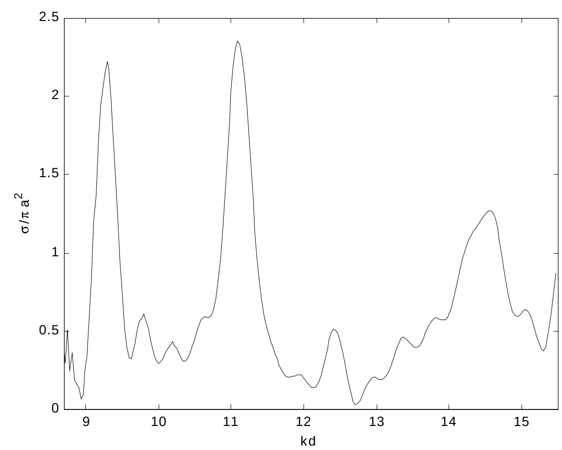

Fig. 7: Normalized backscattering cross section versus electrical separation (kd) for end-fire incidence and a linear array of eight identical dielectric spheres each covered with a dielectric layer with: ka=4.0, kb=3.0, $\alpha = 0.0$, ε_{Ir} =3.0, and ε_{IIr} =2.0.

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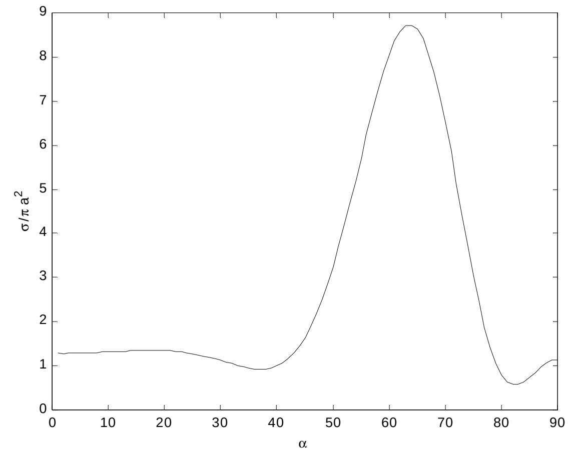

Fig. 8: Normalized backscattering cross section versus aspect angle α for a linear array of three identical dielectric spheres each covered with a dielectric layer with ka=1.5, kb=1.0, kd=3.0, ε_{Ir} =4.0, and ε_{IIr} =3.0.

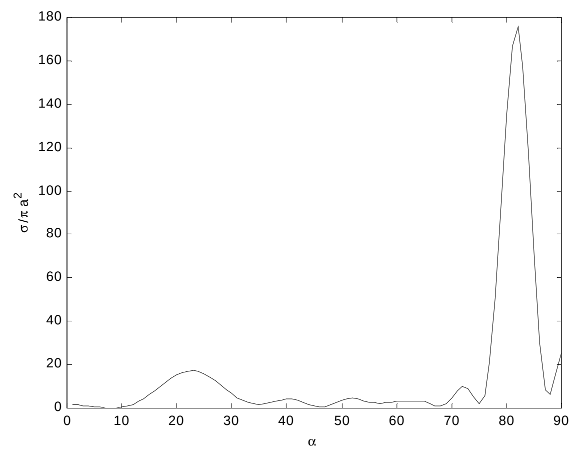

Fig. 9: Normalized backscattering cross section versus aspect angle α for a linear array of eight identical dielectric spheres each covered with a dielectric layer with ka=1.5, kb=1.0, kd=3.0, ε_{Ir} =4.0, and ε_{IIr} =3.0.

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