Time Domain Models of Negative Refractive Index Metamaterials

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Abstract

The physics and wave properties of materials with negative refractive index have been studied extensively in recent years. However, computationally efficient numerical models for such media are not readily available to designers who wish to create novel components that incorporate such materials. In addition to being numerically efficient and robust, they must incorporate the dispersive behavior of the refractive index and the wave impedance of metamaterials and allow for the imposition of realistic boundary geometries and properties. In this paper we present a 3D numerical model that is based on the distributed node TLM network in which reactive elements are embedded. The resulting periodic structure supports backward waves and constitutes an artificial medium with negative phase velocity and negative refractive index. When embedded in a time domain electromagnetic field simulator, this model provides a versatile simulation tool for metamaterial design. Since such a transmission line model is realizable it also provides a framework for the actual realization of 3D metamaterials. We will derive the theoretical foundations for the model and show some validating simulation results that demonstrate its capabilities and potential.

Index Terms — Metamaterials, artificial dielectrics, backward waves, focusing, left-handed media (LHM), TLM method, negative permittivity, negative permeability, negative refractive index, periodic structures.

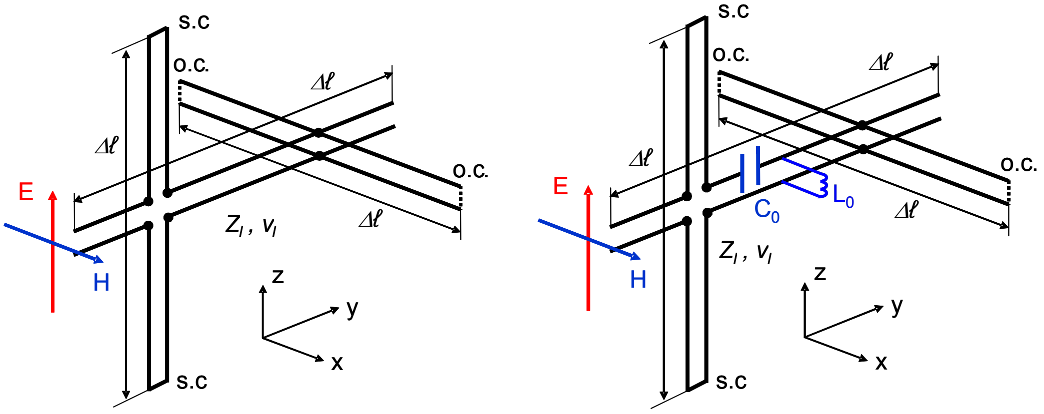
1 Introduction

The consequences of formally introducing negative values for the permittivity and permeability in Maxwell's equations were first discussed in the late 1960's by Victor Veselago [1]. He concluded that such properties would result in negative phase velocity and a negative index of refraction. In regard to the realization of such materials, Veselago pointed out that μ and ε could only be negative if they were frequency-dispersive, which means that one could not create any substances that would have constant negative constitutive parameters over a wide range of frequencies. More than 30 years passed before such media – now referred to as metamaterials – were first realized. One type consisting of periodic arrays of thin wire cylinders and split-ring resonators was proposed by researchers around Pendry [2] and Smith [3], [4]. These resonant periodic structures have frequency-dispersive negative refractive index properties over a relatively narrow bandwidth. Non-resonant negative index metamaterials based on periodically loaded transmission line networks were proposed independently in 2002 by Iyer and Eleftheriades [5], [6], Caloz and Itoh. [7] and Oliner [8]. The wave properties of such high-pass structures were already discussed by Ramo, Whinnery, and Van Duzer [9] who noted that they support backward waves with a phase velocity opposite to the group velocity and the Poynting vector. To realize a 1D structure of this type, Caloz and Itoh [7] periodically loaded a microstrip line with series capacitors and shunt inductors; Eleftheriades and his group [5], [6] soldered discrete series capacitors and shunt inductors into a two-dimensional microstrip transmission line mesh. Their theoretical analyses and experimental studies confirmed that such loaded transmission line networks have indeed wave properties similar to those predicted by Veselago [1]. However, truly 3D structures based on this principle have not yet been realized to date.

The 3D electrodynamic model of negative refractive index metamaterials proposed in this paper is a conceptual extension of these 1D and 2D periodically loaded transmission line structures. We have recently developed and implemented two computational TLM models of metamaterials. The first model [10] is a periodically loaded two-dimensional shunt node model that emulates the microstrip structures proposed by Caloz and Itoh, [7] and Iyer and Eleftheriades [5], [6]. The second model [11], [12] is based on the 3D SCN (Symmetrical Condensed Node) TLM and employs an inter-cell connection algorithm that models the reactive periodic elements embedded in the network. Unfortunately, the 3D SCN based model is not physically realizable, and we are thus proposing a new 3D distributed node model which can actually be manufactured. This paper deals exclusively with the computational properties of the distributed node TLM model. Its realization and manufacturing will be presented at the 2005 IEEE MTT Microwave Symposium [15].

2 Proposed 3D Distributed Node TLM Model of Metamaterials

The 3D distributed TLM node [13] consists of three shunt and three series nodes per cell - oriented in the three coordinate directions - that connect the lossless transmission line or link line sections of the mesh. It is, in fact, a transmission line network equivalent of the FDTD Yee cell. It well known [14] that the low-frequency wave properties of this TLM network are those of the axial equivalent cell, shown in Fig. 1a for z-polarized plane wave propagation in the y-direction. Due to symmetry the transverse link lines form reactive stubs that periodically load the main link line. Following the concept of previous publications [5]-[8] we can realize backward wave properties by embedding a series capacitance C_0 and a shunt inductance L_0 into the cell as shown in Fig. 1b.



(a) (b)

Fig. 1. (a) Equivalent cell for z-polarized plane wave propagation in y-direction. (b) Insertion of a series capacitance C_0 and a shunt inductance L_0 into the axial equivalent cell create a backward wave structure. The exact position of the elements is not critical and they could be integrated into the nodes as additional stubs.

Assuming that the link lines have an inductance L'_ℓ per unit length and a capacitance C'_ℓ

per unit length, the characteristic link line impedance and phase velocity of the unloaded network (Fig 1a) will be:

$$Z_\ell = \sqrt{\frac{L'_\ell}{C'_\ell}} \quad \text{and} \quad v_\ell = \frac{1}{\sqrt{L'_\ell C'_\ell}} \ , \tag{1}$$

while the network phase velocity v_n and the Bloch impedance Z_n will be given by

$$v_n = \frac{1}{\sqrt{2L'_\ell 2C'_\ell}} = 0.5 v_\ell \quad \text{and} \quad Z_n = \sqrt{\frac{2L'_\ell}{2C'_\ell}} = Z_\ell \, . \tag{2}$$

This network models a medium characterized by a permeability μ_n, a permittivity ε_n and a refractive index n_n that are related to the link line properties as follows:

$$\mu_n = 2L'_\ell \, ; \varepsilon_n = 2C'_\ell \, ; Z_n = \sqrt{\frac{\mu_n}{\varepsilon_n}} = Z_\ell \, ; v_n = \frac{1}{\sqrt{\mu_n \varepsilon_n}} = \frac{v_\ell}{2} \, ; \; n_n = \frac{c}{v_n} = \frac{\sqrt{\mu_n \varepsilon_n}}{\sqrt{\mu_0 \varepsilon_0}} = \sqrt{\frac{4L'_\ell C'_\ell}{\mu_0 \varepsilon_0}} = 2\frac{c}{v_\ell} \, . \tag{3}$$

where c is the velocity of light in free space. In order to create a negative refractive index n_m we must determine the values of C_0 and L_0 such that the *total* series impedance Z_s and total shunt admittance Y_p of the cell become both negative. These are given by

$$Z_s = j2\omega L'_\ell \Delta\ell + \frac{1}{j\omega C_0} = j\omega(2L'_\ell \Delta\ell - \frac{1}{\omega^2 C_0}) = j\omega\mu_m \Delta\ell \tag{4}$$

$$Y_p = j2\omega C'_\ell \Delta\ell + \frac{1}{j\omega L_0} = j\omega(2C'_\ell \Delta\ell - \frac{1}{\omega^2 L_0}) = j\omega\varepsilon_m \Delta\ell \, . \tag{5}$$

μ_m and ε_m are the effective permeability μ_m and permittivity ε_m of the metamaterial that this unit cell models. The embedded elements C_0 and L_0 are now expressed in terms of the desired metamaterial properties using (4) and (5) in which we replace the terms $2L'_\ell$ and $2C'_\ell$ by the permeability μ_n and ε_n of the host network given in (3):

$$C_0 = -\frac{1}{\omega^2 \Delta\ell(\mu_m - \mu_n)} \quad \text{and} \quad L_0 = -\frac{1}{\omega^2 \Delta\ell(\varepsilon_m - \varepsilon_n)} \, . \tag{8}$$

ω is the design frequency at which the desired negative values of μ_m and ε_m are to be realized. To ensure that the low frequency approximation is be valid, the mesh parameter $\Delta\ell$ must be much smaller than the spatial wavelength in the metamaterial (typically less than 1/10th) over the operating frequency range.

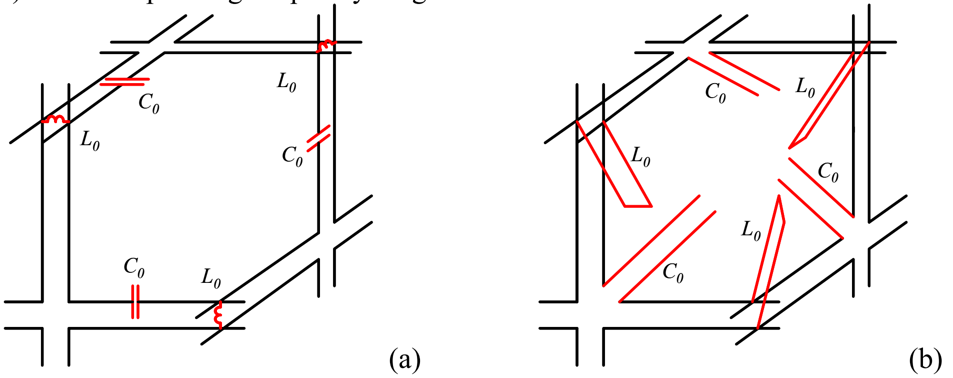


Fig. 2 (a) Extension of the loaded cell to 3D (b) Incorporation of the reactances into the nodes in the form of reactive stubs suitable for the TLM impulse scattering formulation.

The final step towards the creation of a 3D metamaterial model is to combine loaded axial equivalent cells into a full 3D distributed node network as demonstrated in Fig. 2. The embedded reactances are conveniently incorporated into the series and shunt nodes in the form of equivalent open (capacitive) and short-circuited (inductive) stubs (Fig. 2b).

Conclusion

A 3D distributed node computational TLM model of negative refractive index metamaterials has been presented. It is an extension of the reactively loaded transmission line models of negative refractive index materials proposed in the literature [5] – [8]. The embedded elements can be represented by additional stubs. These can, in turn, be included in the TLM algorithm by modifying the scattering procedure for the node.

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