Point-Matched Time Domain Finite Element Method Applied to Multi-Conductor Electromagnetic Transients Analysis

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Abstract – A point-matched time domain finite element method (TDFE) applied to the analysis of electromagnetic transients in multi-conductor transmission line network is presented. The TDFE method solves the partial differential transmission line equations based on a semi-discrete approximation using the finite element method and leap-frog scheme. Some oscillations can occur due to the stability condition of the technique. Using the modal analysis these oscillations can be eliminated. A surge propagation in a simple multi-conductor network is presented and the results are compared with the Electromagnetic Transients Program (EMTP) simulations.

1. INTRODUCTION

A transient is the situation that occurs when the initial stability of the system is disturbed and the system is forced to settle in other stability condition. They are usually of short duration and decreasing amplitude with respect to time, space or both. In engineering practice, studies of these conditions are of increasingly importance.

The transient response of transmission line networks can be found by accounting for all interactions between forward and backward traveling waves as a result of discontinuities or disturbances. The evolution of the waveforms on the lines can be computed by solving a system of linear nodal equations in discrete time steps.

Some transmission line problems cannot be adequately modeled by the equivalent circuit approach. For transient scattering applications the point-matched time domain finite element (TDFE) method has been used [1-10] with great advantages. One of the great difficult with this approach is the errors due to the stability condition necessary to obtain the numerical solution (the others are the boundary conditions) which can introduce high frequency oscillations in the simulations.

This paper presents the modal analysis applied to the point-matched time domain finite element method for multi-conductor transmission line transient problems. In this approach, the propagation of disturbance on multi-conductor transmission line is simulated numerically in the modal domain by solving a n-dimensional boundary value problem at each time step. Results are presented for a simple network and compared with the Electromagnetic Transients Program (EMTP).

2. TRANSMISSION LINE EQUATIONS

The time domain formulation of multi-conductor transmission line problems (in the TEM approximation) can be described by a system of partial differential equations in (x,t) [11,12]

$$\begin{aligned} -\frac{\partial v(x,t)}{\partial x} &= Ri(x,t) + L\frac{\partial i(x,t)}{\partial t}, \\ -\frac{\partial i(x,t)}{\partial x} &= Gv(x,t) + C\frac{\partial v(x,t)}{\partial t}, \end{aligned} \tag{1}$$

where $v(x,t)$ and $i(x,t)$ are column vectors of the phase voltages and currents and R, G, L and C are the resistance, conductance, inductance and capacitance matrices per unit length respectively. There are n voltage and current equations describing the system and they are correspondingly increased in their number of terms to accommodate the couplings between the conductors. The simultaneous solution of these equations reveals n modes of propagation with, in the general case, each having its own velocity. Analytical solutions can be obtained for simple cases (e.g., lossless and distortionless lines) [11].

3. FINITE ELEMENT METHOD

Using the finite element method in the solution of time-dependent problems, the spatial approximation is considered first and the time approximation next. Such a procedure is commonly known as *semidiscrete approximation* (in space) [13]. The finite element method requires the line to be subdivided into a finite number of regions called elements. Each element has points called interpolation nodes. This allows the voltage v and the current i to be written in the form

$$
\begin{aligned}
v(x,t) &= \sum_{i=1}^{M} \phi_i(x) V_i(t), \\
i(x,t) &= \sum_{j=1}^{N} \psi_j(x) I_j(t),
\end{aligned}
\tag{2}
$$

where M and N are the number of nodes of the finite element segments, and $\phi_i(x)$ and $\psi_j(x)$ are basis functions which interpolate the voltage and current within each element, defined as

$$
\begin{aligned}
\phi_i(x) &= \begin{cases} 1 & x = x_i \\ 0 & \text{at other nodes,} \end{cases} \\
\psi_j(x) &= \begin{cases} 1 & x = x_j \\ 0 & \text{at other nodes.} \end{cases}
\end{aligned}
\tag{3}
$$

The two nodes of a first-order finite element are located such that each voltage element contains an interpolation node for the current and each current element contains an interpolation node for the voltage. Only the interpolation functions associated with the adjacent nodes contribute to the summation in (2). Hence, substitution of (2) into (1), using (3) yields

$$
\begin{aligned}
RL^{-1} I_j + \frac{dI_j}{dt} &= -L^{-1} \sum_{a=1}^{2} \frac{d\phi_a}{dx} V_a(t) \\
& \qquad j = 1,2,\ldots,N, \\
GC^{-1} V_i + \frac{dV_i}{dt} &= -C^{-1} \sum_{a=1}^{2} \frac{d\psi_a}{dx} I_a(t) \\
& \qquad i = 1,2,\ldots,M.
\end{aligned}
\tag{4}
$$

In this equation, $\phi_a = \phi_a(x)$, $\psi_a = \psi_a(x)$, V_a represents the value of the two voltage nodes adjacent to the current I_j, and I_a represents the value of the two current nodes adjacent to the voltage node V_i.

This completes the semidiscrete finite-element formulation of (1), resulting in a set of ordinary differential equations in time. The second step is the time approximation, using the leap-frog scheme [3,4,14]. In the leap-frog scheme, the time derivatives in (4) are approximated by the Euler method, where the current time derivative is computed one-half time step before the voltage time derivative. Substituting the independent current and voltage terms in equation (4) by the interpolated values at $t=n+½$ and $t=n-½$, the following equations are obtained

$$
\begin{aligned}
I_j^{n+1/2} &= Z_1(2L - R\Delta t) I_j^{n-1/2} - \frac{2\Delta t}{\Delta x} Z_1 \sum_{a=1}^{2} \frac{d\phi_a}{dx} V_a^n \\
& \qquad j = 1,2,\ldots,N, \\
V_i^{n+1} &= Z_2(2C - G\Delta t) V_i^n - \frac{2\Delta t}{\Delta x} Z_2 \sum_{a=1}^{2} \frac{d\psi_a}{dx} I_a^{n+1/2} \\
& \qquad i = 1,2,\ldots,M,
\end{aligned}
\tag{5}
$$

where $Z_1 = (2L + R\Delta t)^{-1}$ and $Z_2 = (2C + G\Delta t)^{-1}$. The approximation functions $\phi_i(x)$ and $\psi_j(x)$ depend on the type of element (number of nodes). Using $\phi_i(x)$ and $\psi_j(x)$ as first-order interpolation polynomials, (5) reduces to the following final equations [3,4]

$$
\begin{aligned}
I_j^{n+1/2} &= (2L - R\Delta t) Z_1 I_j^{n-1/2} - \frac{2\Delta t}{\Delta x} Z_1 \left(V_{j+1}^n - V_j^n\right) \\
& \qquad j = 1,2,\ldots,N, \\
V_i^{n+1} &= (2C - G\Delta t) Z_2 - \frac{2\Delta t}{\Delta x} Z_2 \left(I_i^{n+1/2} - I_{i-1}^{n+1/2}\right) \\
& \qquad i = 2,\ldots,N+1.
\end{aligned}
\tag{6}
$$

4. STABILITY CRITERION

The solution to the leap-frog scheme approximation represented by (6) is stable [1,14] if

$$
u \frac{\Delta t}{\Delta x} \leq 1, \tag{7}
$$

where u is the wave propagation speed. This implies that the wave must not propagate more than one subdivision in space during one time step. Condition (7) can be analyzed using the step response of a single-conductor system shown in Figure 1 [5].

Using a fixed time step (Δt), for space discretization (Δx) greater than Δt, the solution for the transmission line differential equations is incorrect. For the matrix system presented in (6) the errors obtained with the TDFE method applied to the solution of the partial differential equations are due to a unique value of space discretization used for different wave propagation speed.

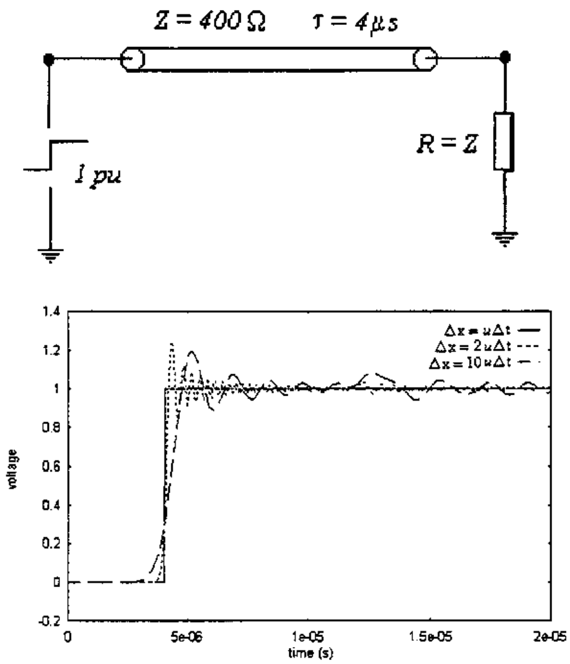

Figure 1 - Stability criterion

5. MODAL ANALYSIS

Wedephol [15] and Bickford [12] show that it is possible to normalize (1) and thereby replace them by a similar set of equations free of mutual terms. The problem then reduces to the solution of n single-phase equations of the same general form [16]. Just as the solution of the single-circuit wave equation leads to the mode of propagation and relationship between current and voltage (wave speed propagation and surge impedance) for the voltage and current waves on the single circuit, so the solution of these equations yields the n modes of propagation for the multiple conductor system.

Rewriting (1) in the frequency domain, one can obtain

$$\frac{\partial^2 v(x,w)}{\partial^2 x} = (ZY)v(x,w)$$
$$\frac{\partial^2 i(x,w)}{\partial^2 x} = (YZ)i(x,w) \tag{8}$$

where $Z = R + jwL$, $Y = G + jwC$. The approach used to decouple each one of those equations is similar to diagonalize either ZY or YZ [17]. In the diagonalization process, two transformation matrices are needed: matrix Q for the currents ($I_{phase} = QI_{mode}$) and matrix P for the voltages ($V_{phase} = PV_{mode}$). P and Q are the solutions of the eigenproblems

$$P^{-1}(ZY)P = \gamma^2$$
$$Q^{-1}(YZ)Q = \gamma^2 \tag{9}$$

where γ_i is the i-th eigenvalue and the columns of P and Q are the eigenvectors of ZY and YZ respectively. The transformation matrices are theoretically complex and frequency-dependent. With a frequency-dependent transformation matrix, modes are only defined at the frequency at which the transformation matrix is calculated. Then the concept of converting a multi-conductor line into decoupled single-conductor lines (in the modal domain) cannot be used over the entire frequency range. It is possible to find an approximate transformation matrix which is real and constant. The errors of this approximation vary with frequency. They are small in one particular region and large in other regions, depending on how the approximation is chosen. However, the problem of how to choose this constant transformation matrix remains.

There is a class of conductor configuration in which the process of diagonalization is greatly simplified. It is called balanced system. A balanced transmission line is defined as a line where all diagonal elements of Z and Y are equal among themselves, and all off-diagonal elements are equal among themselves. Balanced lines have a useful property, that is, the transformations to decouple their differential equations are independent of the particular system. There are several well-known transformations for balanced lines: symmetrical components, Clark's transformation, Karrenbauer's transformation, among others. For Karrenbauer's transformation

$$P = Q = \begin{pmatrix} 1 & 1 & 1 & . & . & 1 \\ 1 & 1-n & 1 & . & . & 1 \\ 1 & 1 & 1-n & . & . & 1 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 1 & 1 & 1 & . & . & 1-n \end{pmatrix} \tag{10}$$

where N is the number of conductors. For lossless high frequency approximation one can show that (10) is a good approximation and can be used to solve some problems.

6. NUMERICAL EXAMPLE

A performance test is considered using the circuit shown in Figure 2. This simple geometry was chosen for two reasons. First, its boundary conditions are stable (open ended lines and simultaneous switch closing) and easy to compute. Secondly, a balanced lossless three-phase transmission line is used for simplification of the modal analysis application.

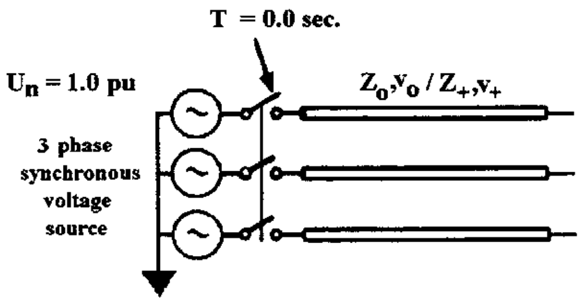

Figure 2 - Example circuit

Table I shows the switching surge modal parameters for the system of Figure 2. A switching surge is computed considering that the switches close at time $t=0$, for maximum voltage at phase 1. Figure 3 shows the voltage at the end of the line (phase 2) where high-frequency oscillations can be seen due to the different wave propagation speed.

Table I - Modal parameters

mode	Impedance (Ω)	Velocity (m/s)
zero	966.98	2.58E8
positive	362.59	2.30E8

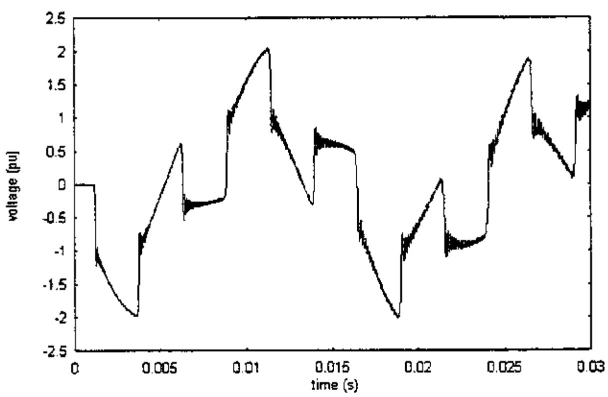

Figure 3 - TDFE method

Figure 4 shows the results for the numerical simulation using the modal analysis. Using three equivalent decopled single-conductor lines the oscillations due to the stability criterion are eliminated. For comparison, Figure 5 shows the same results obtained from the EMTP simulation (Microtran version [18]).

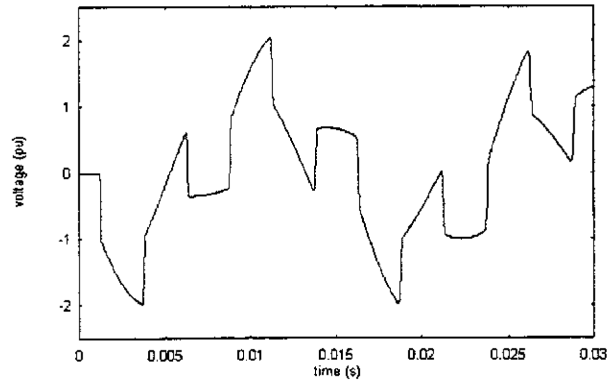

Figure 4 - TDFE method with modal analysis

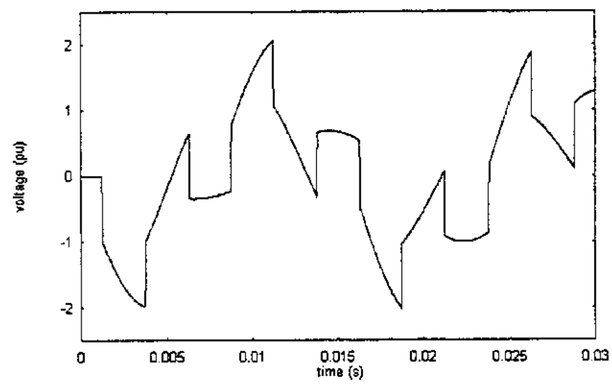

Figure 5 - EMTP simulation

7. CONCLUSIONS

The point-matched time domain finite element method applied to the numerical solution of the multi-conductor transmission line partial differential equations is stable but some high frequency oscillations can occur due to the different line speed wave propagation.

Applying the modal analysis to the numerical solution, the simulation errors are corrected. Some simplifications were made to obtain the transformation matrices. Work is in progress to obtain better results by modifying the constant-frequency approximate transformation matrices.

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