On the Iteration of Surface Currents and the Magnetic Field Integral Equation

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Abstract - In an effort to mathematically validate the convergence properties of various surface current-iterative methods, the magnetic field integral equation is analyzed for its contraction mapping properties. The analysis is performed first on the general integral operators and then on the matrices representing the discrete forms of the integral operators associated with the different iterative methods. The contraction mapping properties are determined by investigating the spectral radius of each linear operator. Conditions for the verification and validation of these iterative methods are provided, along with mathematical checks for the existence of spurious modes and the existence of internal resonance.

1. Introduction

Throughout the past decade, a variety of surface current-iterative methods [1-6] have been proposed to solve electromagnetic scattering and radiation problems involving the Magnetic Field Integral Equation (MFIE) and related integral equations. Most of these methods were developed as alternatives to the traditional Method of Moments (MoM) [7,8] solution, which involve the inversion of a dense (*most often complex*) matrix. While each of the methods showed some computational advantages, each also introduced a new issue of concern: that of convergence. For the most part, the convergence of these iterative methods has been shown by numerical example, but little mathematical analysis has been offered for the understanding of why these methods converge to the correct solution. Oftentimes convergence is simply claimed when the delta change in iterations falls below some pre-set limit. In the following paper, the MFIE is examined to explain why these surface current-iterative methods have been successfully iterated to convergence.

In previous work Kaye *et al.* [1] found that iterative solutions to the MFIE for the surface current would always yield convergence for Perfectly Electrically Conducting (PEC) bodies which where divided into two parts: a side illuminated by the incident field, and a shadowed side. Coupled integral equations were written for each side. The iterative process started on the illuminated side by iterating from the Physical Optics (PO) [9] current to an improved value. The process then went to the shadowed side where the improved current on the illuminated side was used with an initial estimate (*most often zero*) for the shadowed side current to obtain an improved shadowed side current. The process continued by returning to the illuminated side to obtain further improvement for the illuminated side current and then returned to the shadowed side to obtain further improvement to the shadowed current. The process was halted when the delta change in the solution fell below some pre-set limit.

Reuster & Thiele [2] found that the same iterative procedure could be used for PEC cavities where the aperture of the cavity was treated as the illuminated side and the cavity walls were treated as the shadowed side. The iterative process began by taking the aperture field to be that produced by an external plane wave illumination. The aperture field was then used to find the total magnetic field, via iteration, along the cavity walls. The field along the cavity walls was then used to update the total field at the aperture via iteration. The improved knowledge of the aperture field was then used to obtain a further update of the field along the cavity walls, which in turn was used to update the aperture field. Again, the process was halted when the delta change in the solution fell below some pre-set limit.

Obelleiro-Basteiro *et al.* [3] demonstrated a method similar to the iterative method proposed in [2] where the PO approximation was used to simplify the iterative procedure. Numerical results are presented which demonstrate the convergence and accuracy of the method, but no mathematical analysis is provided as to why the method converges. Collins & Skinner [4] demonstrated an iterative method for calculating the scattering from perturbed circular dielectric cylinders by using equivalent currents along the perimeter of the cylinder. Their paper alludes to the mathematical properties of the method's convergence, but no mathematical analysis is presented. Reuster, *et al.* [5] showed by numerical example that convergence could be obtained by directly iterating the entire currents on a PEC body if the PO approximation was used as an initial condition; however, little mathematical analysis was provided. Finally, Hodges & Rahmat-Samii [6] present an advanced iterative method for large PEC bodies consisting of both wires and closed surfaces. Their iterative method involves the Electric Field Integral Equation (EFIE) as well as the MFIE and the PO approximation. Again, no mathematical analysis is provided as to why the method converges, and convergence is determined when the delta change in the solution falls below some pre-set limit.

Essentially, all of these surface current-iterative methods are fixed-point iterative problems [10-12] where the MFIE or a related integral equation is manipulated (*usually by observing physical characteristics associated with the particular problem*) to develop an iterative scheme which is "hopefully" a contraction mapping. The establishment of a contraction mapping is the unique feature that guarantees that the iterative method will converge in a monotonic mean-square sense. In the work that follows, two formulations of the magnetic field integral equation (*Maue's formulation and the Total Field formulation*) are analyzed for their potential contraction mapping characteristics. The effect of the discrete form operator size on the contraction mapping properties of each formulation is studied, and a mathematical check is presented for the existence of spurious modes and the existence of internal resonance. Finally, a general iterative scheme, which involves subdividing the integral operator for the creation/insurance of contraction mappings, is presented. This general iterative scheme is related to the surface current-iterative methods [1-6] that were discussed earlier.

2. Contraction Mapping Analysis of the MFIE

For simplicity a 2D Transverse Electric (TE) PEC scattering problem, as shown in Figure 1, is chosen as a baseline model for analysis. This particular scattering problem allows the normally complex 3-dimensional vector integral equation to be reduced to a 2-dimensional scalar integral equation which still maintains all of the properties associated with the general vector integral equation. Hence, no loss in generality is experienced in working with the reduced integral equation. The derivation of both Maue's equation and the Total Field equation proceeds as follows.

For 2D electromagnetic radiation and scattering problems, where the magnetic field is parallel to the geometry of interest, the MFIE may be stated as [1,13]:

$$-H_z^I(\bar{r}) = \frac{j\beta}{4}\int_C H_z^T(\bar{r}')h_1^{(2)}(\beta|\bar{r}-\bar{r}'|)\cos\phi\, dl + \frac{1}{2}H_z^T(\bar{r}) \quad (1)$$

Where C is the simple closed curve (*in the x-y plane*) describing the PEC body of interest,

$$\lim_{\bar{r}' \to \bar{r}} \frac{j\beta}{4}\int_C H_z^T(\bar{r}')h_1^{(2)}(\beta|\bar{r}-\bar{r}'|)\cos\phi\, dl = 0$$

, and $\frac{1}{2}H_z^T(\bar{r})$ is the principle value of the integral. From (1), both Maue's formulation and the Total Field formulation for the MFIE may be obtained as follows.

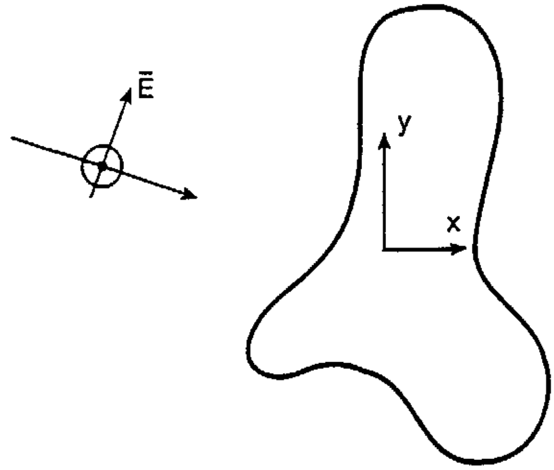


Figure 1 – 2D TE PEC Scattering Problem

Maue's Formulation. Maue's formulation is obtained from (1) by subtracting the integral operator from both sides of (1) and then scaling (1) by a factor of two.

$$H_z^T(\bar{r}) = -2H_z^I(\bar{r}) + \frac{-j\beta}{2}\int_C H_z^T(\bar{r}')h_1^{(2)}(\beta|\bar{r}-\bar{r}'|)\cos\phi\, dl \quad (2)$$

Total Field Formulation. The Total Field formulation is obtained from (1) by subtracting the integral operator from both sides of (1) and then adding the principle value to both sides of (1).

$$H_z^T(\bar{r}) = -H_z^I(\bar{r}) + \frac{-j\beta}{4}\int_C H_z^T(\bar{r}')h_1^{(2)}(\beta|\bar{r}-\bar{r}'|)\cos\phi\, dl + \frac{1}{2}H_z^T(\bar{r}) \quad (3)$$

Note that the only significant difference between Maue's formulation and the Total Field formulation is the location of the principle value of the integral. It will be shown that the location of the principle value has a major effect on the contraction mapping properties of the MFIE.

Integral equations (2) and (3) may be solved using fixed-point iteration, provided that these integral formulations are contraction mappings. By definition, contraction mappings may be iterated directly to a unique solution with a guarantee of mean squared-monotonic convergence [10-12]. This convergence is independent of the initial guess used to initiate the iterative process. The general test for a contraction mapping proceeds as follows.

Let ψ_1 and ψ_2 represent two approximations for ψ in an equation of the following form, where L is any linear operator and F is a constant forcing function

$$\psi = L(\psi) + F\,. \tag{4}$$

Then (4) is a contraction mapping and may be solved directly using fixed-point iteration (*with the guarantee of mean squared monotonic convergence*) if and only if

$$|L(\psi_2 - \psi_1)| \leq \kappa\,|\psi_2 - \psi_1|, \qquad 0 \leq \kappa < 1\,. \tag{5}$$

Applying the above test to Maue's formulation yields (6).

$$|H_z^T(\bar{r})_2 - H_z^T(\bar{r})_1| \geq \left|\frac{\beta}{2}\int_C \left[H_z^T(\bar{r}')_2 - H_z^T(\bar{r}')_1\right] h_1^{(2)}(\beta|\bar{r}-\bar{r}'|)\cos\phi\, dl\right| \tag{6}$$

After simplification, using the triangle inequality theorem, the contraction-mapping test can be obtained for Maue's formulation (7).

$$\int_C |h_1^{(2)}(\beta|\bar{r}-\bar{r}'|)\cos\phi|\, dl \leq \frac{2}{\beta} \tag{7}$$

Similarly, the contraction-mapping test can be obtained for the Total Field formulation (8).

$$\int_C |h_1^{(2)}(\beta|\bar{r}-\bar{r}'|)\cos\phi|\, dl \leq \frac{4}{\beta} \tag{8}$$

Note that (7) and (8) are purely functions of the problem's geometry and the wavelength of interest ($\beta = 2\pi/\lambda$). The requirements for convergence are independent of the initial conditions used in the iterative process. Also note that the convergence condition for Maue's equation is more difficult to satisfy than the convergence condition for the Total Field equation. This is a direct result of the removal of the principle value from the integral operator in Maue's formulation. While both (7) and (8) may be shown true for a particular geometry, there does not exist a general proof for arbitrary PEC scattering bodies. Hence, further investigation into the contraction mapping properties of both Maue's formulation and the Total Field formulation is continued on a discretized/numerical level.

3. Contraction Mapping Analysis of the Discretized MFIE

The inability to work with (7) and (8) results in the need to approximate the integral operators in (2) and (3) with systems of linear equations. Applying a pulse-basis point-patching MoM expansion [7] to (2) and (3) results in the following approximate expansions for Maue's formulation and the Total Field formulation. Similar analysis can be performed for more advanced MoM expansions [8].

$$H_z^T(\bar{r}_m) = -2H_z^I(\bar{r}_m) + \frac{-j\beta}{2}\sum_{m\neq n}\Delta_n H_z^T(\bar{r}_n) h_1^{(2)}(\beta|\bar{r}_m - \bar{r}_n|)\cos\phi_{mn} \tag{9}$$

$$H_z^T(\bar{r}_m) = -H_z^I(\bar{r}_m) + \frac{-j\beta}{4}\sum_{m\neq n}\Delta_n H_z^T(\bar{r}_n) h_1^{(2)}(\beta|\bar{r}_m - \bar{r}_n|)\cos\phi_{mn} + \frac{1}{2}H_z^T(\bar{r}_m) \tag{10}$$

Expressing (9) and (10) in matrix from yields (11) and (12), respectively.

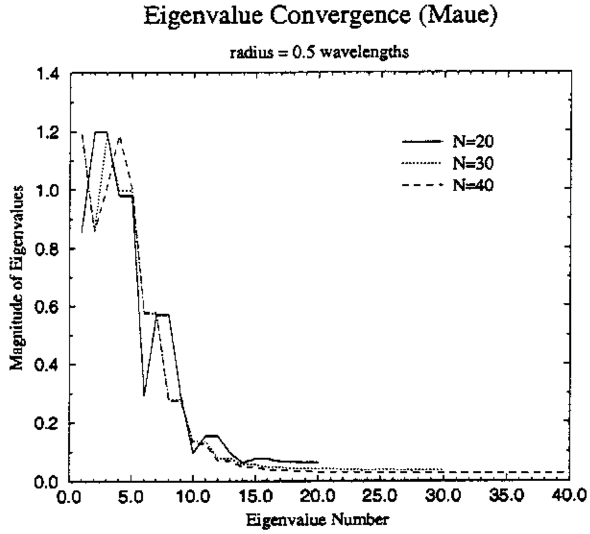
$$\begin{bmatrix} \\ H_z^T(\bar{r}_m) \\ \\ \end{bmatrix} = \begin{bmatrix} 0 & \frac{-j\beta}{2}\Delta_n h_1^{(2)}(\beta|\bar{r}_m - \bar{r}_n|)\cos\phi_{mn} & & & \\ & 0 & & & \\ & & 0 & & \\ & & & 0 & \\ \frac{-j\beta}{2}\Delta_n h_1^{(2)}(\beta|\bar{r}_m - \bar{r}_n|)\cos\phi_{mn} & & & & 0 \end{bmatrix} \begin{bmatrix} \\ H_z^T(\bar{r}_n) \\ \\ \end{bmatrix} - \begin{bmatrix} \\ 2H_z^I(\bar{r}_m) \\ \\ \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} \\ H_z^T(\bar{r}_m) \\ \\ \end{bmatrix} = \begin{bmatrix} 1/2 & \frac{-j\beta}{4}\Delta_n h_1^{(2)}(\beta|\bar{r}_m - \bar{r}_n|)\cos\phi_{mn} & & & \\ & 1/2 & & & \\ & & 1/2 & & \\ & & & 1/2 & \\ \frac{-j\beta}{4}\Delta_n h_1^{(2)}(\beta|\bar{r}_m - \bar{r}_n|)\cos\phi_{mn} & & & & 1/2 \end{bmatrix} \begin{bmatrix} \\ H_z^T(\bar{r}_n) \\ \\ \end{bmatrix} - \begin{bmatrix} \\ H_z^I(\bar{r}_m) \\ \\ \end{bmatrix} \tag{12}$$

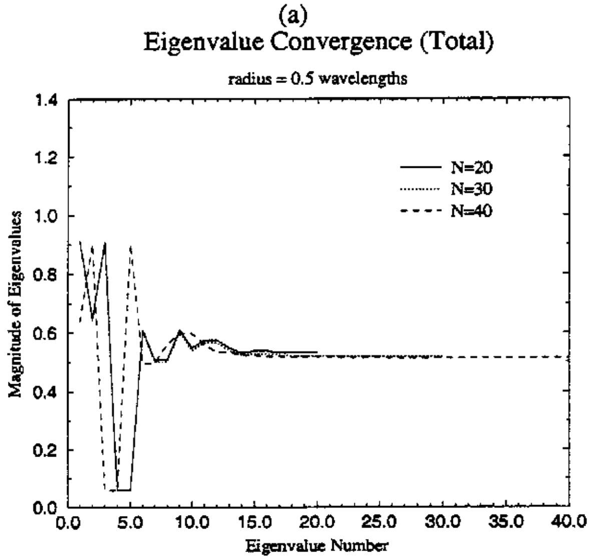
Note that the principle value of the integral appears as the diagonal elements of the matrix equation associated with the Total Field formulation (12) while the principle value of the integral is incorporated into the forcing function of the matrix equation associated with Maue's formulation (11). This is a direct result of the removal of the principle value from the integral operator in Maue's formulation.

It is now possible to analyze (11) and (12) for their contraction mapping properties in a fashion similar to the analysis performed in Section 2.0. For matrix equations in the form of (11) and (12), the matrix equation represents a contraction mapping if and only if the spectral radius of the matrix operator ρ is less than one [10,11]. The spectral radius of a matrix is defined as the magnitude of the largest eigenvalue of the matrix. In general, eigenvalues are the most difficult of all matrix characteristics to compute, and for this reason an advanced matrix analysis package was utilized to perform the following study. The matrix analysis package is LAPACK, and was developed for Linear Operator analysis. This software package is in the public domain, and may be obtained via the Internet.

While the magnitudes of the eigenvalues associated with a given MoM expansion are typically functions of the number of elements used in the expansion, it may be shown that the eigenfunctions of a given MoM expansion converge in a fashion similar to the convergence of the MoM solution. Figures 2a and 2b show the magnitude of the complex eigenvalues for a 0.5 λ radius PEC circular cylinder using 20, 30, and 40 basis functions.



(a)

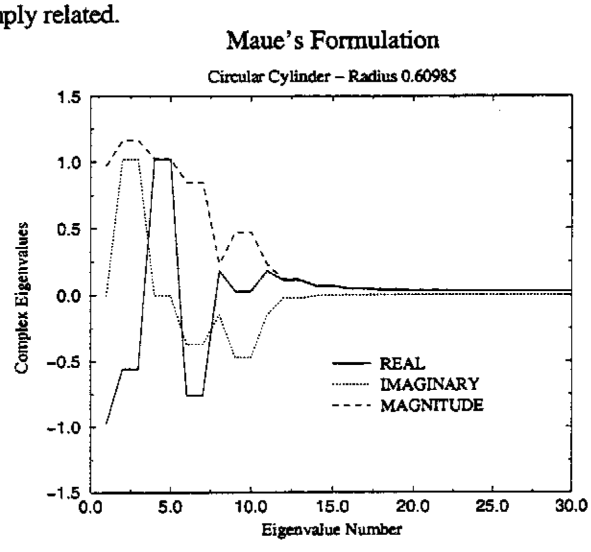


(b)

Figure 2 – Eigenvalue Convergence

For both Maue's formulation and the Total field formulation, convergence occurs at approximately 30 basis functions *(which is approximately 8 basis functions per wavelength)*. This is consistent with MoM expansions for a smooth surface [7]. It should be noted that no new eigenvalues occur above 30 basis functions, which equates to no new information being gained by increasing the number of basis functions being used. Also, note that the eigenvalues for the Total field equation tend toward 0.5 (*for large eigenvalue numbers*) and the eigenvalues for Maue's formulation tend toward 0.0 (*for large eigenvalue numbers*). These two limiting eigenvalues directly correspond to the diagonal values of their respective integral expansion (11) and (12).

Figures 3a-3d show plots of the complex eigenvalue versus the eigenvalue number for 2-dimensional circular cylinders with radii of 0.60985 λ and 0.38275 λ, respectively. These radii were chosen because they correspond to the TE01 and TM01 cutoff frequencies for circular waveguides [13]. Circular cylinders with these radii are historically difficult to solve numerically because of internal resonance problems. It should be noted that in both cases Maue's equation has eigenvalues with magnitudes greater than one, and the Total Field equation has eigenvalues with magnitudes less than or equal to one. This implies, for these particular cases, direct iteration of Maue's equation will not yield convergence and direct iteration of the Total Field equation may yield convergence (*the largest eigenvalue has to be strictly less than one to guarantee convergence*). In addition, for radii that correspond to the cutoff frequencies of the TE0n and TM0n circular waveguide modes, both the Total Field equation and Maue's equation will have eigenvalues of value one. It is these eigenvalues (*of value one*) which are responsible for the historically documented resonance problems. It also should be noted that Total Field equation has a zero eigenvalue for the 0.60985 λ case (*figure 3b*). This zero eigenvalue can cause a spurious mode to appear in the solution. While eigenvalues of value zero are not generally problems for direct iterative solutions, eigenvalues of values one or greater are problems, and lead to divergent solutions. It is this problem of eigenvalues of value one or greater that the previous surface current-iterative methods [1-6] are indirectly solving. Section 4 presents a general method for directly dealing with the problem of eigenvalues of value one or greater. Furthermore, it may be shown that the eigenvalues for Maue's equation and those for the Total Field equation are simply related.



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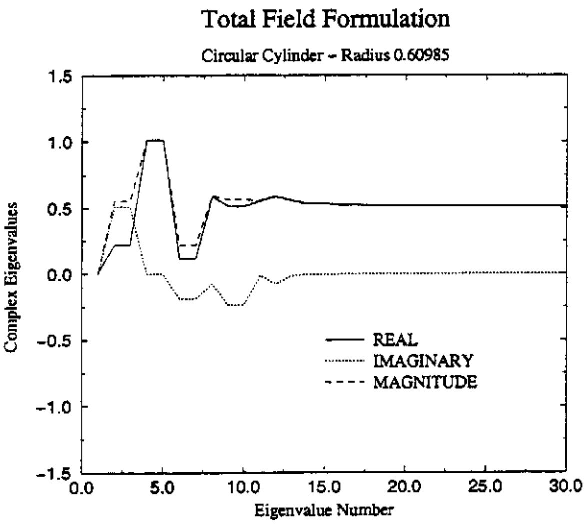

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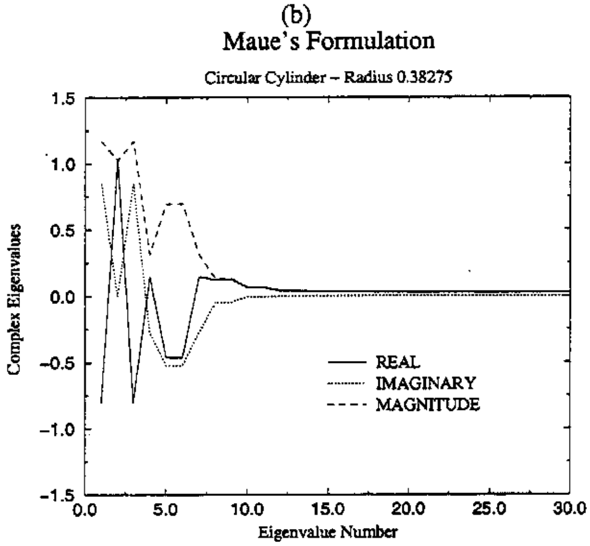

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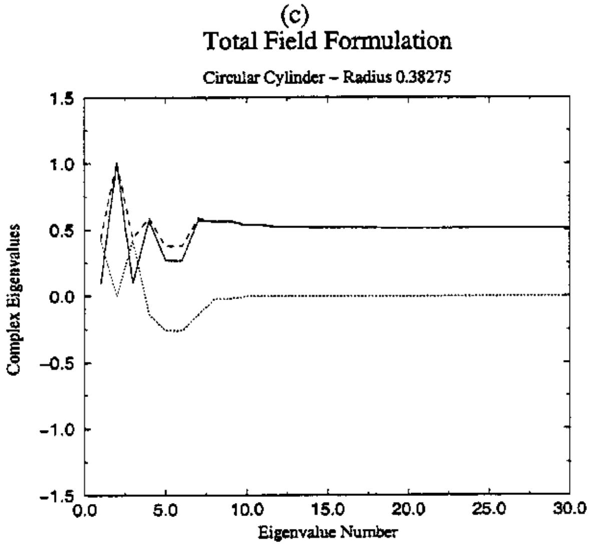


(d)

Figure 3 – Complex Eigenvalue Analysis

Write Maue's equation from (2) in operator form as

$$H_z^T = L^m(H) - 2H_z^I \tag{13}$$

and the Total Field equation from (4) as

$$H_z^T = L^T(H) - H_z^I, \tag{14}$$

where the operators L^T *and* L^m are related by

$$L^T = \frac{1}{2}I + \frac{1}{2}L^m \tag{15}$$

and [I] is the identity matrix.

Let λ^m, v^m be an eigenvalue and eigenvector pair for L^m, then v^m is an eigenvector for L^T corresponding to an eigenvalue $\lambda^T = \frac{1}{2}[I + \lambda^m]$. To see this, assume

$$L^m(v^m) = \lambda^m v^m. \tag{16}$$

It follows that

$$\begin{aligned} L^T(v^m) &= \frac{1}{2}[I + L^m]v^m \\ &= \frac{1}{2}[v^m + L^m(v^m)] = \frac{1}{2}[I + \lambda^m]v^m = \lambda^T v^m \end{aligned} \tag{17}$$

By inspection of the curves in Figures 2 and 3, it may be seen that the above relationship is true for large eigenvalue numbers. By noting the complex nature of the eigenvalues in Figure 3 it may also be seen that the above relationship is true for the small eigenvalue numbers.

4. Methods of Insuring a Contraction Mapping

As can be seen from Figures 2 and 3, not all of the 2D TE PEC scattering problems contain a spectral radius that is less than one. In general, the spectral radius of the matrix associated with the Total Field formulation is less the spectral radius of the matrix associated with Maue's formulation. However, geometrical cases still exist where the spectral radius of the matrix associated with the Total Field formulation is greater than or equal to one (*figure 3b and 3d*). For these situations, the resulting system of linear equations can not be solved using a direct implementation of the fixed-point iterative method, and attempts to do so will result in a rapid divergence in the solution vector. For this case, where the spectral radius of the matrix associated with the particular magnetic field formulation is greater than one, a modified version of the fixed-point iterative method must be applied if a contraction mapping like-iterative solution is to be obtained.

In their work [1,14-16] Thiele *et al.* showed that it was possible to apply the method of fixed-point iteration to a 2D TE PEC scattering problem by subdividing the scattering body into two pieces which they defined as the illuminated side and the shadow side. While their work was restricted to a single subdivision of Maue's formulation, their method is shown here to be completely general and may be applied to any of the magnetic field formulations for any number of arbitrary subdivisions provided the largest eigenvalue of each subdivision is less than unity.

We can express the hybrid iterative method (HIM) [1] as (18) where the sub-matrix $[L_{11}]$ is associated with the illuminated side, the sub-matrix $[L_{22}]$ is associated with the shadowed side, and the sub-matrices $[L_{12}]$ and $[L_{21}]$ are associated with the coupling between the illuminated and shadowed sides. The iterative method used in [1] to solve (18) is represent in (19).

$$\begin{bmatrix} H_z^T(\bar{r}_m) \end{bmatrix} = \begin{bmatrix} [L_{11}] & [L_{12}] \\ [L_{21}] & [L_{22}] \end{bmatrix} \begin{bmatrix} H_z^T(\bar{r}_m) \end{bmatrix} - \begin{bmatrix} 2H_z^I(\bar{r}_m) \end{bmatrix} \tag{18}$$

$$\begin{bmatrix} H_z^T(\bar{r}_m)^1 \end{bmatrix} = \begin{bmatrix} [L_{11}] & [0] \\ [0] & [I] \end{bmatrix} \begin{bmatrix} H_z^T(\bar{r}_m)^0 \end{bmatrix} - \begin{bmatrix} 2H_z^I(\bar{r}_m) \\ [0] \end{bmatrix},$$

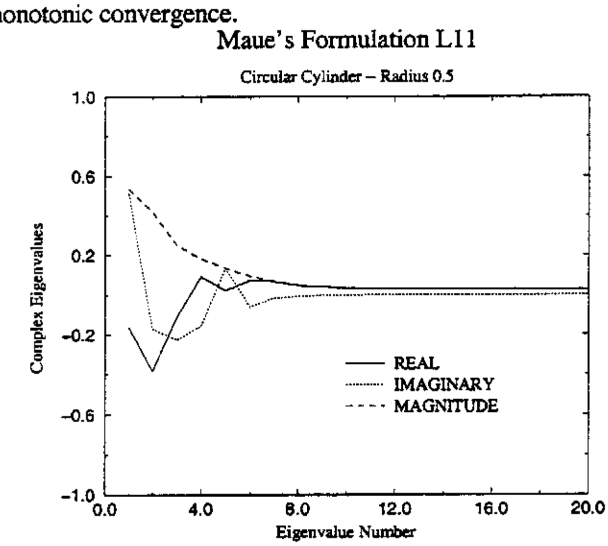
$$\begin{bmatrix} H_z^T(\bar{r}_m)^2 \end{bmatrix} = \begin{bmatrix} [I] & [0] \\ [L_{21}] & [I] \end{bmatrix} \begin{bmatrix} H_z^T(\bar{r}_m)^1 \end{bmatrix} - \begin{bmatrix} [0] \\ 2H_z^I(\bar{r}_m) \end{bmatrix},$$

$$\begin{bmatrix} H_z^T(\bar{r}_m)^3 \end{bmatrix} = \begin{bmatrix} [I] & [0] \\ [0] & [L_{22}] \end{bmatrix} \begin{bmatrix} H_z^T(\bar{r}_m)^2 \end{bmatrix} - \begin{bmatrix} [0] \\ 2H_z^I(\bar{r}_m) \end{bmatrix},$$

$$\begin{bmatrix} H_z^T(\bar{r}_m)^4 \end{bmatrix} = \begin{bmatrix} [I] & [L_{12}] \\ [0] & [I] \end{bmatrix} \begin{bmatrix} H_z^T(\bar{r}_m)^3 \end{bmatrix} - \begin{bmatrix} 2H_z^I(\bar{r}_m) \\ [0] \end{bmatrix},$$

$$\begin{bmatrix} H_z^T(\bar{r}_m)^0 \end{bmatrix} = \begin{bmatrix} H_z^T(\bar{r}_m)^4 \end{bmatrix}. \tag{19}$$

This four step iterative method was found to always converge without internal resonance problems, although a formal proof was never given [1]. If we examine the eigenvalues of the resulting sub-matrices, it is apparent why the method converged and why it had no internal resonance problems. Figures 4a-d show the eigenvalues for the HIM matrices $[L_{11}]$, $[L_{12}]$, $[L_{21}]$ and $[L_{22}]$ of a circular cylinder with radius 0.5 wavelengths. The eigenvalues for each of the four matrices are all less than one; hence, each of the four sub-iterations is itself a contraction mapping. Since the four sub-iterations are nested in a linear fashion, then by the triangle inequality theorem the total iterative procedure is itself a contraction mapping. Thus, the system of linear equations may be iterated directly to a unique solution with a guarantee of mean squared-monotonic convergence [10-12]. It is extremely important to note that while the four sub-matrices in [1] were originally defined in terms of the system's excitation (*illuminated and shadowed sides*), the contraction mapping characteristics of the system are independent of the system's excitation. All that is important is that the matrix is partitioned such that the largest eigenvalue of each sub-matrix is less than one. If this case is true, then by the definition of a contraction mapping, the system of linear equations may be iterated directly to a unique solution with a guarantee of mean squared-monotonic convergence.



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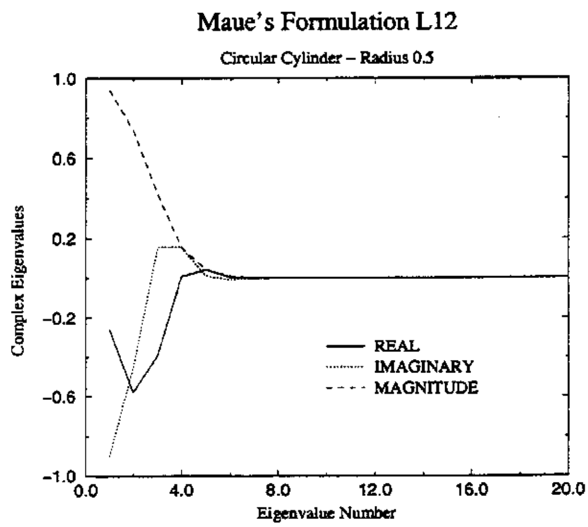

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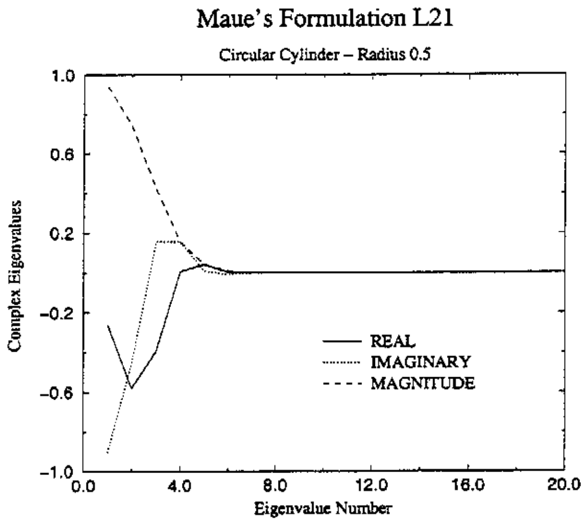

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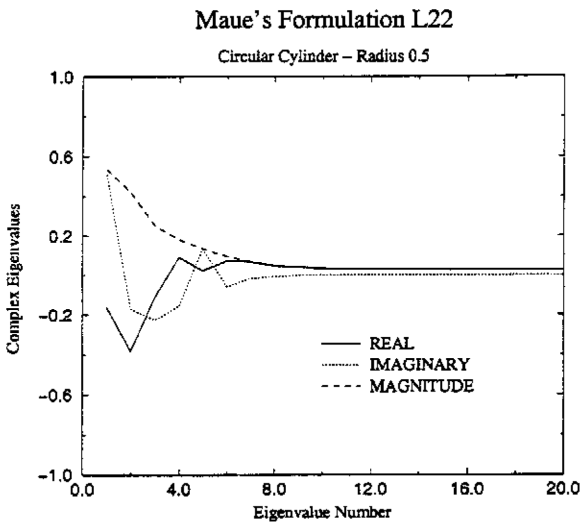
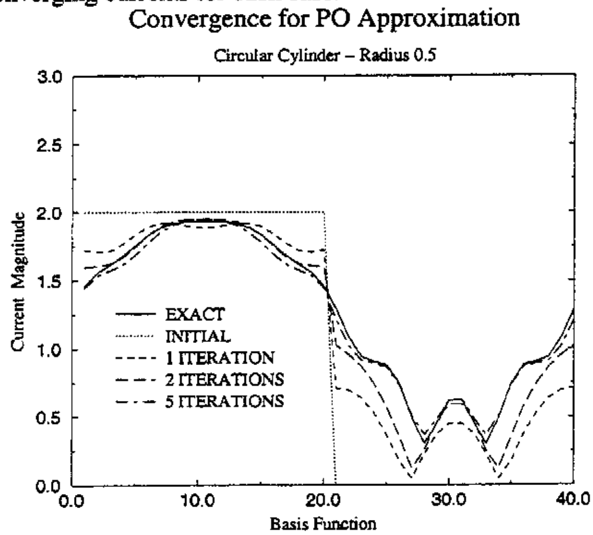


(d)

Figure 4 – Eigenvalue Analysis the Hybrid Iterative Method

While the guaranty of convergence is independent of excitation function and the initial guess used to start the iteration procedure, the rate of convergence is not. In fact the rate of convergence is heavily dependent upon the initial guess used to start the iteration procedure. To illustrate this point, the current distribution on a 0.5 λ radius circular cylinder is solved for an incident plane wave using two different initial guesses. For this particular example, the Total Field formulation was utilized directly since all of its eigenvalues are less than one. The first initial guess used was the traditional PO approximation used in [1-6], and the second initial guess used was the negated reflection of the PO approximation. The PO approximation is considered to be a very good initial guess, while the negated reflection of the PO approximation is considered to be a very poor initial guess. In both cases, the exact solution was obtained directly using MoM. Figures 5a and 5b show the converging currents for each case.



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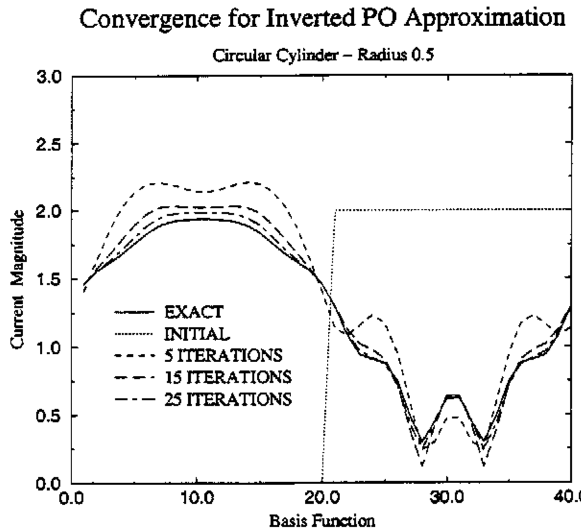

(b)

Figure 5 – Effects of Initial Guess on Convergence Rate

Figure 5a shows these currents for an initial guess that is the usual physical optics estimate of the current; twice the incident field on the illuminated side and zero on the shadow side. Note the vast improvement in the solution after one iteration, and the nearly converged result after five iterations. Figure 5b shows the currents for an initial guess of zero on the illuminated side and minus twice the incident field on the shadowed side; the "opposite" of the physical optics estimate. Note while the contraction mapping nature of the iteration procedure overcomes the poor initial guess, the rate of convergence is significantly slower, approximately five times slower than for the previous guess.

5. Summary and Conclusions

Maue's formulation and the Total Field formulation of the MFIE have been analyzed as to their suitability for iterative solutions. It was shown that both formulations could be directly iterated to a unique solution with a guarantee of mean squared-monotonic convergence provided the eigenvalues of their corresponding matrix expansions were less than one. This convergence is independent of the initial guess used to initiate the iterative process. The guarantee of monotonic mean square convergence is a direct result of the contraction mapping properties of Maue's formulation and Total Field formulation. The conditions under which these formulations have contraction mapping properties were developed in Section 2 for the integral forms and in Section 3 for the discrete forms.

Section 3 showed how the eigenvalues of the matrix representation could be used to determine the convergence characteristics of a matrix used in the iterative solution process. For the cases presented, the eigenvalue plots showed a preference for the Total Field formulation over Maue's formulation since the latter tended to have larger eigenvalues than the former. Eigenvalues of value greater than unity indicate that the solution will not have contraction mapping properties. Eigenvalues of value equal to one correspond to internally resonant cases. Eigenvalues of value equal to zero can allow the existence of spurious modes. In Section 4, it was noted that the previously published Hybrid Iterative Method [1] converged because the division of the geometry into two parts produced (for the cases examined) sub-matrices whose eigenvalues were less than unity, thereby creating a series of contraction mappings. The authors believe that the convergence obtained in the other direct surface current-iterative methods [2-6] discussed occurred because of a similar phenomenon.

Lastly, the material in this paper shows how to use either formulation of the MFIE successfully to obtain an iterative solution. Also, iterative solutions to the MFIE have the potential to substantially reduce the required computational time for large PEC bodies provided that the iterative process is initiated with a "good" initial guess, such as the PO approximation. Finally, additional computational speed can be obtained for extremely large PEC bodies by the direct parallelization of the iterative methods discussed in Sections 3 and 4.

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