Coupled-Pair Transmission Line Termination Revisited: An MTL Approach

J. Alan Roden Senior Member, *IEEE*

Abstract

In this paper, the proper termination of a coupled stripline geometry is revisited. Termination guidelines are determined by treating the coupled pair and its associated ground reference as a multi-conductor transmission line and subsequently performing a modal analysis of the line. It is shown that terminating the line in its differential impedance alone is not sufficient from an EMC perspective. Results are demonstrated using a modal FDTD implementation of the of the multi-conductor transmission line equations.

Introduction

The differential pair is a dominant transmission line geometry for high-speed data communications systems. Its popularity exists primarily due to its robustness with respect to noise rejection. Commonly, the signals comprising the differential pair are implemented as a set of microstrip or stripline transmission lines which reference a common ground or power plane. Under this scenario the geometry becomes not a simple two conductor coupled line problem but a three-conductor multi-conductor transmission line which support two modes of propagation. Though either or both of these modes may exist on the line, designers typically only properly terminate one of the supported modes. The proper termination of both of the supported modes is the subject of this paper.

The coupled stripline of Figure 1(a) is the simplest of configurations within the multi-conductor transmission line (MTL) class of TEM transmission lines [ⁱ]. In general, a multi-conductor transmission line is comprised of N+1 conductors within a background media ε_r where N is the number of signal conductors in the system. Each of the conductors within the system is coupled to all other conductors within the system through the per-unit-length (PUL) parameters of the line. The PUL parameters are determined using quasi-static techniques from the cross-sectional description of the MTL geometry. The coupled time domain transmission line equations for a *lossless* MTL line are given by

$$\frac{\partial}{\partial z} \mathbf{V}(z,t) = -\mathbf{L} \frac{\partial}{\partial t} \mathbf{I}(z,t)$$
(1.1)

$$\frac{\partial}{\partial z}\mathbf{I}(z,t) = -\mathbf{C}\frac{\partial}{\partial t}\mathbf{V}(z,t)$$
(1.2)

where for a symmetric three-conductor stripline and a homogeneous media

$$\mathbf{C} = \begin{bmatrix} c + c_{12} & -c_{12} \\ -c_{12} & c + c_{12} \end{bmatrix},$$
(1.3)
$$\mathbf{L} = \begin{bmatrix} l & l_{12} \\ l_{12} & l \end{bmatrix}$$
(1.4)

In order to easily analyze the propagation characteristics of a MTL structure, it is necessary to decouple the transmission line equations into the N voltage and current modes which are supported by the line. In general, a rigorous eigenmode analysis is required. For the special case of a three-conductor line with two identical signal conductors, the decomposition procedure is quite simple and the two resulting modes are typically referred to as the even and odd modes of the line [ⁱⁱ] (differential and common mode in the EMC community). These two modes of propagation are distinguished by their current and voltage distributions. For the even mode, equal, symmetric currents exist on the two signal lines with the return current on the reference conductor. For the odd mode, the currents on the two signal lines are anti-symmetric with their mirror image on the reference conductor. The even/odd decomposition for this symmetric three conductor line is defined by the simple relations [i], [ⁱⁱⁱ].

$$\mathbf{V}(z,t) = \mathbf{T}_{\mathbf{v}} \mathbf{V}_{\mathbf{m}}(z,t) \tag{1.5}$$

$$\mathbf{I}(z,t) = \mathbf{T}_{\mathbf{I}}\mathbf{I}_{\mathbf{m}}(z,t)$$
(1.6)

where

$$\mathbf{\Gamma}_{I} = \mathbf{T}_{\mathbf{v}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1\\ -1 & 1 \end{bmatrix}.$$
(1.7)

Substituting (1.5) and (1.6) into (1.1), (1.2) leads to

$$\frac{\partial}{\partial z} \mathbf{V}_m(z,t) = -\mathbf{L}_m \frac{\partial}{\partial t} \mathbf{I}_m(z,t), \qquad (1.8)$$

$$\frac{\partial}{\partial z} \mathbf{I}_{\mathbf{m}}(z,t) = -\mathbf{C}_{\mathbf{m}} \frac{\partial}{\partial t} \mathbf{V}_{\mathbf{m}}(z,t)$$
(1.9)

where

$$\mathbf{L}_{\mathbf{m}} = \begin{bmatrix} l_{odd} & 0\\ 0 & l_{even} \end{bmatrix}, \quad \mathbf{C}_{\mathbf{m}} = \begin{bmatrix} c_{odd} & 0\\ 0 & c_{even} \end{bmatrix}.$$
(1.10)

For the special case of a stripline where the media is homogeneous, the odd and even mode characteristic impedances may be computed directly from (1.10) and the relationship $\mathbf{CL} = \frac{1}{v^2} \mathbf{I}$ where \mathbf{I} is the identity matrix. The even mode impedance is given by

$$Z_{even} = \frac{1}{vC_{even}} = \frac{1}{vC_{11}}$$
(1.11)

whereas the odd mode impedance is given by

$$Z_{odd} = \frac{1}{vC_{odd}} = \frac{1}{v(C_{11} + 2C_{12})}$$
(1.12)

where v is the speed of light in the homogeneous medium. Equations (1.11) and (1.12) are of the same form as given by many microwave reference texts $[^{iv}]$.

Line Termination

The coupled lines described in the previous section can support a combination of even and odd mode signals simultaneously. Clearly, from a functional perspective, one is interested primarily in the odd mode as this mode has more robust transmission characteristics with regards to noise rejection. From a realistic standpoint, however, some even mode energy will exist. The even-mode or common-mode energy can be the result of imperfect drivers or asymmetric routing of the differential pair. While the even mode energy can easily be rejected by differential mode receivers, its existence can lead to significant EMC problems if not treated carefully. Specifically, if the even mode (common mode) is not terminated correctly, a portion of the even mode signal will resonate along the length of the line leading to higher radiated emissions. By implementing proper multi-mode termination, both even and odd mode currents can be terminated and such resonances are avoided.

As is the case with single conductor transmission lines, a multi-conductor transmission line is ideally terminated with the characteristic impedance(s) of the line. Unlike a single line, the characteristic impedance of the multi-conductor line is represented by an N x N characteristic impedance matrix. There are several equivalent formulations for obtaining this matrix, all of which are determined from the constitutive parameters of the line. For the two conductor line discussed here, it is most easily found by first constructing the modal characteristic impedance matrix. This is given in terms of modal capacitances and inductances as

$$\mathbf{Z}_{c}^{m} = \begin{bmatrix} \frac{1}{\nu C_{odd}} & 0\\ 0 & \frac{1}{\nu C_{even}} \end{bmatrix}.$$
 (1.13)

In order to implement a termination circuit, the fully coupled characteristic impedance matrix must be formed from (1.13) and is

$$\mathbf{Z}_{c} = \mathbf{T}_{v} \mathbf{Z}_{c}^{m} \mathbf{T}_{I}^{-1} = \frac{1}{2} \begin{bmatrix} Z_{odd} + Z_{even} & Z_{even} - Z_{odd} \\ Z_{even} - Z_{odd} & Z_{odd} + Z_{even} \end{bmatrix}$$
(1.14)

which is in general a full matrix.

Proper implementation of the above termination is easily accomplished by mapping this termination to a known termination scheme. Specifically, consider the termination scheme shown in Figure 1 (b). The terminal constraints at the load are expressed in matrix form as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} R_1 + R_3 & R_3 \\ R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}.$$
 (1.15)

By equating the elements of (1.15) to those in (1.14) the resistive values of the termination are easily found and are

$$R_1 = R_2 = Z_{odd}, \qquad R_3 = \frac{Z_{even} - Z_{odd}}{2}.$$
 (1.16)

This subject is discussed in detail in [i].

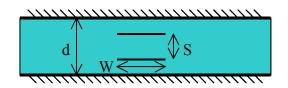
Observations

For a purely differential propagating mode, it is seen by a simple application of Kirchoff's current law at the junction of R_1 , R_2 , and R_3 , that no current will flow in R_3 , thus it is effectively isolated

from the differential mode termination. The common mode termination value, R_3 , diminishes as the coupling between the two lines vanishes, i.e. as $C_{12} \rightarrow 0$. Therefore, in those cases where the two lines of the differential pair are routed separately in an isolated fashion, the inclusion of a common mode termination has a diminishing effect. One hazard of the common mode termination is common impedance coupling [1] which can occur when significant even mode signal is present. In such cases, one must weigh the EMI advantages of this termination scheme against the possible increase in crosstalk voltage.

Example: The Coupled Stripline

To demonstrate the effectiveness of proper coupled-line termination, a simple vertically coupled stripline geometry is studied. The geometry is homogeneous and symmetric thus the even/odd mode field structure discussed previously will be supported. The physical geometry and electrical description of the example is illustrated in Figure 1. The reference plane separation, d, is 12 mils, the signal line separation, S, is 4 mils, and the line width, W, is 6 mils. Each conductor is assumed infinitely thin and perfectly conducting.



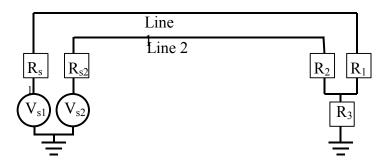


Figure 1 Coupled slotline a) Physical geometry. b) Electrical Circuit

The approximate capacitance matrix is determined analytically using methods described in [iv] and is

$$C = \begin{bmatrix} 83.67 & -55.78\\ -55.78 & 99.16 \end{bmatrix} pF.$$
(1.17)

From (1.13) and (1.17), the modal characteristic impedance matrix found to be

$$Z_c^m = \begin{bmatrix} 49.16 & 0\\ 0 & 245.76 \end{bmatrix} ohms , \qquad (1.18)$$

and the fully coupled characteristic impedance matrix is

$$Z_{c} = \begin{bmatrix} 147.45 & 98.3\\ 98.3 & 147.45 \end{bmatrix} ohms.$$
(1.19)

Electrically, each of the two coupled lines is driven by an independent voltage source. Ideally, these two sources would be of equal magnitude but opposite phase. In such a case, the even mode is not excited. Here, both modes are excited by setting V_{s1} to one volt and V_{s2} to negative one and one half volts. The source waveforms are each step functions with a risetime of 174 picoseconds. The source impedance of each line is set to fifty ohms. The termination values are given by (1.16) and are

$$R_1 = R_2 = 49.16 \text{ ohms}, \quad R_3 = 98.3 \text{ ohms}.$$
 (1.20)

The MTL equations given in (1.1), (1.2) are easily solved using the FDTD technique with exact termination and load conditions [^v]. These coupled equations are solved using a time step governed by the Courant Friedrich Lewy (CFL) condition which requires $\Delta_t \ll \frac{\Delta_z}{v}$ where Δ_z is the spatial discretization of the MTL line, Δ_t is the time step at which the solution is advanced, and v is the maximum velocity of propagation along the line. For a transmission line within a homogeneous media, a 'magic' time step exists and is given by $\Delta_t = \frac{\Delta_z}{v}$. In this special case, Δ_z may be chosen arbitrarily and the resulting solution is *exact*. This is the case for the homogeneous stripline. Therefore an accurate and extremely efficient solution is available.

The results of the even mode termination scheme of (1.16) using the values given in (1.20) are shown in Figure 2. Also shown here is the more commonly used differential termination scheme where R₁ and R₂ are equal to Z_{odd}, and R₃ is set to zero ohms. The differential mode signal proved to be perfectly terminated regardless of the common mode termination. As is demonstrated here, significant ringing exists in both the voltage and current waveforms as a result of the improper termination of the common mode signal. Clearly, the oscillating current will increase the radiation of the topology.

Conclusion

In this paper the proper termination of symmetric coupled transmission lines was demonstrated and validated. Though the example presented was a homogeneous stripline, the same theory is applicable to coupled microstrip lines and in a more limited sense to shielded twisted-pair transmission lines. Though functional currents may be terminated using a simple two resistor scheme, proper common mode termination requires a slightly more complicated topology where a third resistor to ground is used. It was shown that this termination provided a more robust termination from an EMC standpoint since common mode current resonances were eliminated.

Finally, it is important to note that the common mode current discussed in this work is not equivalent to the highly radiating antenna currents with which EMI engineers often do battle. In this work, the currents and voltages are purely TEM in nature thus their radiation will usually be small compared to true antenna mode currents.

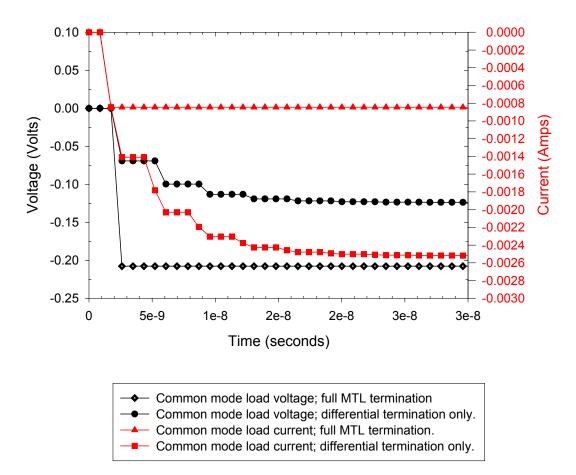


Figure 2 Common mode load voltage $(V_1+V_2)/2$ and Load Current $(I_1+I_2)/2$ for the homogeneous coupled stripline geometry of Figure 1 (b).

References

- [1] Clayton R. Paul, *Analysis of Multiconductor Transmission Lines*, John Wiley Interscience, NY, 1994.
- [2] Clayton R. Paul, Introduction to Electromagnetic Compatibility, John Wiley & Sons, NY, 1992.
- [3] David M. Pozar, Microwave Engineering, John Wiley & Sons, NY, 1998.
- [4] Clayton R. Paul, "Incorporation of Terminal Constraints in the FDTD analysis of Transmission Lines", *IEEE Trans. On Electromagnetic. Compatibility*, May, 1994.