Far-Field Synthesis of Sparse Arrays with Cross-polar Pattern Reduction

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Abstract **—** This paper presents a fully deterministic iterative algorithm for the far-field synthesis of antenna arrays with reduction of the cross-polar component. The algorithm synthesizes the excitations as well as the positions of the array elements, providing a sparse geometry. Starting from an initial set of possible positions, the proposed algorithm iteratively solves a sequence of convex optimization problems. At each iteration a suitable objective function is minimized, which allows to reduce the number of radiating elements, among those of the initial set, in presence of constraints on the far-field co-polar and cross-polar patterns. The adopted formulation leads to a second order cone problem (SOCP), which is iteratively solved with CVX, a Matlab-based modeling system developed at the Stanford University and available on the Internet.

Index Terms **—** Co-polar and cross-polar patterns, far-field synthesis, geometrical synthesis, sparse arrays.

I. INTRODUCTION

Since many decades, antenna arrays became very common in several fields of engineering [1, Ch. 11-13]. Due to the presence of a number of radiating elements, they offer to the designer additional degrees of freedom with respect to a single antenna, thus allowing to meet different tasks.

Initially, the synthesis techniques were analytical and suitable for simple geometries, which were fixed a priori, such as linear [2, 3], rectangular [4] and circular [5, 6] arrays. Subsequently, sophisticated synthesis methods were developed, suitable for arrays of arbitrary geometries, capable of satisfying additional constraints. Some of them are deterministic [7, 8], some others are stochastic [9, 10]. Stochastic methods can solve complicated non-linear optimization problems, but are often costly in terms of computational resources. Thus, when available, the deterministic methods should be preferred.

In recent years, the attention of the array engineers has moved to geometrical synthesis algorithms, which increase the number of the degrees of freedom and, more importantly, allow to reduce the number of elements. Also in this context, there are deterministic [11] and stochastic procedures [12].

In this paper we present a totally deterministic approach to the geometrical synthesis of antenna arrays of arbitrary geometry, which also allows to control the cross-polar component of the radiation pattern. The paper is organized as follows. In Section II the problem is formulated and the developed algorithm is described. A numerical example is proposed in Section III to prove the effectiveness of the method. Finally, conclusions are summarized in Section IV.

II. METHOD OF SOLUTION

Given an antenna array consisting of N radiating elements, referred to a Cartesian system $O(x,y,z)$, the far-field radiation patterns at the generic direction ϕ of the xy-plane are:

$$F^{v}(\mathbf{i},\phi)=\sum_{n=1}^{N} i_n f_n^{v}(\phi), \tag{1}$$

where v means "co" or "cr", $\mathbf{i}=[i_1,\ldots,i_N]^T$ is the complex column vector of the excitations, $f_n^{co}(\phi)$ and $f_n^{cr}(\phi)$ are, respectively, the co-polar and cross-polar far-field patterns of the n-th array element.

Given a desired far-field co-polar pattern $F_d(\phi)$, which identifies a main beam region MB and a side lobe region SL, the far-field constraints on the co-polar component can be written as:

$$\begin{cases}|F^{co}(\mathbf{i},\phi)-F_d(\phi)|\le\epsilon & if\ \phi\in MB\\ |F^{co}(\mathbf{i},\phi)|\le\rho^{co}(\phi) & if\ \phi\in SL\end{cases}, \tag{2}$$

where $\rho^{co}(\phi)$ is a real positive function specifying the maximum allowed side lobe level. Analogously, the far-field cross-polar constraint can be written as:

$$|F^{cr}(\mathbf{i},\phi)|\le\rho^{cr}(\phi), \tag{3}$$

where $\rho^{cr}(\phi)$ is a real positive function specifying the maximum allowed cross-polar far-field level.

The synthesis process that we propose consists in: (a) introducing a very dense grid of elements; (b) iteratively determining the excitations of these elements, while satisfying constraints (2) and (3); (c) finally removing those elements that have excitations lower than a given threshold. The last step yields the required sparse array, consisting of a strongly reduced number of elements. In other words, the synthesized excitations and

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positions of the array elements are those of the "surviving" elements.

The method of solution that we are presenting is an evolution of that proposed in [13]. Precisely, Equations (2) and (3) are regarded as the constraints of an optimization problem aimed at reducing (and possibly minimizing) the number of radiating elements. In order to exploit techniques that solve convex optimization problems, an iterative procedure is proposed which, at the $\boldsymbol{k}$-th iteration, solves the following weighted norm-1 minimization problem:

$$\min_{\boldsymbol{i}^k} \sum_{n=1}^{N} \boldsymbol{\alpha}_n^k \left|\boldsymbol{i}_n^k\right| \quad \text{subject to (2) and (3),} \tag{4}$$

where $\mathbf{i}^{\boldsymbol{k}} = [\boldsymbol{i}_1^k, \dots, \boldsymbol{i}_N^k]^{\boldsymbol{T}}$, and the weights $\boldsymbol{\alpha}_n^k$ are given by:

$$\boldsymbol{\alpha}_n^k = \left(\left|\boldsymbol{i}_n^{k-1}\right| + \boldsymbol{\epsilon}\right)^{-1}, \tag{5}$$

where the parameter $\boldsymbol{\epsilon} > \mathbf{0}$ is to be chosen slightly smaller than the smallest non-zero excitation amplitude that one is willing to implement [14]. All the array elements with an amplitude $\left|\boldsymbol{i}_n^k\right| \leq \boldsymbol{\epsilon}$ are switched-off, and contribute to the evaluation of $\left\|\mathbf{i}^k\right\|_{\mathbf{0}}$, which is defined as the number of zero-components of vector $\mathbf{i}^{\boldsymbol{k}}$. With reference to (5), note that a zero excitation at the step $\boldsymbol{k}$, $\boldsymbol{i}_n^k = \mathbf{0}$, might result in a non-zero excitation at the successive step $\boldsymbol{k}+\mathbf{1}$, $\boldsymbol{i}_n^{k+1} \neq \mathbf{0}$. The iterative process is stopped when the number of zero elements does not change in three consecutive iterations. The optimality of the solution is not guaranteed, but numerical examples proved the improvements that can be obtained with respect to usual fixed-grid synthesis algorithms.

The proposed synthesis procedure can be summarized as follows:

1. Define a regular array structure, which will be regarded as the reference array.
2. Make thicker the initial grid to obtain an array with the same geometry but an increased number of possible positions.
3. Choose the parameter ϵ and set $k = 1$.
4. Solve the fixed-grid far-field co-polar synthesis problem described by constraints (2) with one of the algorithms available in the literature: we used the algorithm in [15].
5. Set $k := k + 1$ and evaluate the weights α_n^k by (5).
6. Solve the problem in (4).
7. If $k > 2$ and $\|\mathbf{i}^{k-2}\|_0 = \|\mathbf{i}^{k-1}\|_0 = \|\mathbf{i}^k\|_0$ stop the procedure and consider the vector $\mathbf{i}^k$ as the solution to the problem; else go back to step 5.

In particular, note that the synthesized sparse array consists of only those elements for which $|i_n^k| > \epsilon$. The step 7 gives the excitations $\mathbf{i}^k$ of such elements. In the following section, a numerical example shows the effectiveness of the proposed algorithm.

III. NUMERICAL RESULTS

Starting from the example in [16], we here propose a numerical example that shows the improvements that can be achieved with the presented algorithm in terms of reduction of the number of radiating elements. Note that, constraints on the cross-polar far-field pattern were not considered in [16]. The array used in [16] is shown in Fig. 1 (a) (blue circles). It consists of $N = 54$ radially oriented equally spaced Huygens radiators with the electrical dipoles parallel to the z-axis. The co-polar pattern was defined as the ϑ-component of the electric far-field on the xy-plane ($\vartheta = \pi/2$). The ϕ-components of the electric far-field on the xy-plane were negligible with the electrical dipoles parallel to the z-axis. So, the electrical dipoles were $\pi/20$ radians tilted with respect to the z-axis. This is regarded as the reference array.

As a first step, the method in [16] has been used to solve the far-field co-polar synthesis problem. The synthesized co-polar pattern and the corresponding cross-polar pattern are depicted in Fig. 1 (b).

In order to implement the iterative procedure above described, firstly the inter-element spacing between adjacent elements on the reference array was reduced by a factor 5, leading to a thicker grid of $N = 270$ possible positions. Then, with reference to constraint (2), in the MB region the pattern synthesized with [16] was chosen as the desired far-field pattern $F_d(\phi)$, whereas in the SL region the upper bound of the mask (red line in Fig. 1 (b)) was chosen as the function $\rho^{co}(\phi)$. The function $\rho^{cr}(\phi)$ in (3) was chosen in such a way as to impose a maximum value of -20 dB for the cross-polar component.

According to [13], we set $\epsilon = 10^{-2}$ in (5). Finally, at each step the optimization problem in (4) was solved using [17] in a Matlab code on a laptop with 8GB RAM. The iterative procedure stopped after 5 iterations and required only 217 seconds to give the results. The synthesized sparse array is shown in Fig. 1 (a) (red triangles). It consisted of only 39 elements among the possible 270 elements. Thus, a reduction of 27,8% of radiating elements with respect to the 54 elements of the reference array was achieved. The far-field co-polar and cross-polar patterns resulted to satisfy both constraints (2) and (3) with great accuracy, as is shown in Fig. 1 (c).

Then, we imposed a more stringent requirement on the cross-polar component. Precisely, we chose $\rho^{cr}(\phi)$ in such a way as to obtain, with respect to the co-polar component, reductions of 20 dB in the MB region and 10 dB in the SL region. Also in this case, the proposed method gave patterns satisfying constraints (2) and (3) with very good accuracy (Fig. 1 (d)). The synthesized array consisted of 50 elements ($N_1 = 19$, $N_2 = 31$, reduction of 7.4%). The algorithm required 7 iterations, performed in 373 seconds. The results, in terms of element positions and excitations, are detailed in Table 1.

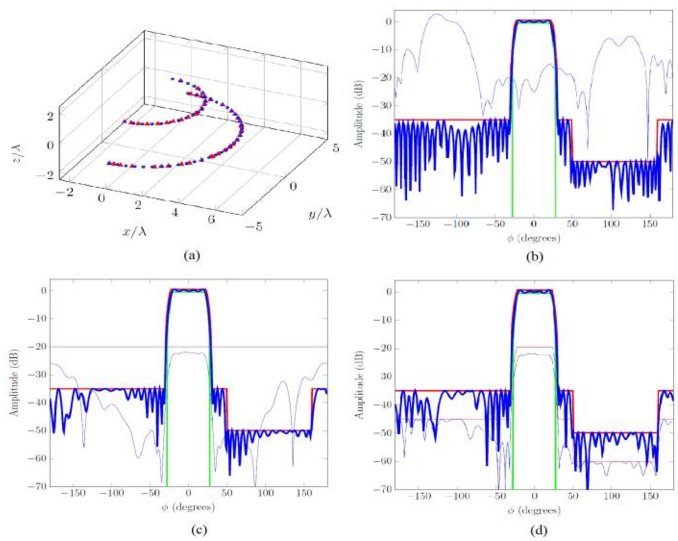

Fig. 1. (a): Geometry of the arrays. Blue circles: elements of the reference array ($N_1 = 20, R_1 = 2.8\lambda,\ z_1 = \lambda;\ N_2 = 34,\ R_2 = 4.6\lambda,\ z_2 = -\lambda$). Red triangles: elements of the synthesized sparse array, consisting of $N = 39$ elements ($N_1 = 14,\ N_2 = 25$, the radii and the z coordinates are as in the reference array). Patterns synthesized with: (b) the method in [16]; (c): the presented method, first example; (d): the presented method, second example. Thick lines: co-polar pattern (blue), upper (red) and lower (green) bounds of the mask. Thin lines: cross-polar pattern (blue) and the function $\rho^{cr}(\phi)$ (red).

Table 1: The results obtained with the proposed approach; positions (φ_n) and excitations (i_n) of the array elements (Angles in degrees. The radii and the z coordinates of the elements can be deduced from the above text.)

	First Example			Second Example				First Example			Second Example		
n	φ_n	$\lvert i_n\rvert$	$\angle i_n$	φ_n	$\lvert i_n\rvert$	$\angle i_n$	n	φ_n	$\lvert i_n\rvert$	$\angle i_n$	φ_n	$\lvert i_n\rvert$	$\angle i_n$
1	-57.27	0.44	-154.66	-90.00	0.11	-19.08	26	-1.60	0.59	111.51	-52.72	0.38	-109.61
2	-48.18	0.50	134.86	-82.73	0.19	-141.98	27	-0.53	0.80	76.60	-44.20	0.37	-141.40
3	-35.45	0.70	28.68	-75.45	0.27	96.96	28	4.79	0.86	91.88	-38.88	0.51	134.37
4	-33.64	0.51	93.37	-66.36	0.11	-2.56	29	33.55	0.24	145.39	-30.35	0.68	123.16
5	-24.54	1.84	4.68	-55.45	0.24	-113.26	30	37.81	0.26	139.26	-22.90	0.34	131.91
6	-11.82	2.17	7.23	-46.36	0.23	129.79	31	41.01	0.37	-172.91	-21.83	0.47	106.97
7	-0.91	2.99	17.22	-39.09	0.31	52.63	32	47.40	0.17	-88.67	-16.51	0.83	77.43
8	10.00	2.62	10.61	-30.00	0.43	40.02	33	53.79	0.30	-41.23	-10.12	1.01	94.74
9	22.73	2.14	-2.30	-20.91	1.10	-11.04	34	60.18	0.18	89.01	-4.79	0.22	139.52
10	33.64	0.60	62.34	-10.00	1.05	19.40	35	64.44	0.25	161.08	-3.73	0.93	104.07
11	35.45	0.52	44.09	-8.18	0.94	-31.85	36	74.02	0.14	-21.75	0.53	1.58	104.45
12	44.54	0.40	140.56	2.73	1.89	29.04	37	80.41	0.08	127.73	6.92	1.69	95.17
13	59.09	0.20	-128.33	4.55	0.64	-47.67	38	84.67	0.09	-142.48	14.38	0.95	81.94
14	88.18	0.02	-91.61	15.45	1.45	6.19	39	90.00	0.09	54.98	20.77	0.49	116.59
15	-90.00	0.11	40.87	22.73	0.65	-48.08	40				28.22	0.50	120.56
16	-86.80	0.12	-93.77	33.64	0.88	47.17	41				29.29	0.16	136.32
17	-74.02	0.17	-16.44	42.73	0.35	134.61	42				35.68	0.67	112.26
18	-62.31	0.29	132.79	55.45	0.30	-118.80	43				39.94	0.34	170.10
19	-59.11	0.35	55.64	80.91	0.13	144.31	44				46.33	0.16	-107.80
20	-45.27	0.25	-124.39	-90.00	0.09	76.53	45				51.66	.40	-79.79
21	-41.01	0.12	-176.46	-86.80	0.16	-78.17	46				62.31	0.36	120.16
22	-37.81	0.16	155.68	-82.54	0.10	148.19	47				72.96	0.15	-21.74
23	-20.77	0.25	111.72	-75.09	0.12	45.20	48				80.41	0.17	101.64
24	-16.51	0.25	72.79	-61.24	0.10	109.42	49				83.61	0.06	-129.23
25	-6.92	0.47	94.09	-56.98	0.32	-3.43	50				87.87	0.10	21.88

IV. CONCLUSIONS

The algorithm proposed in this paper is a fully deterministic iterative procedure able to synthesize sparse arrays of quite arbitrary shapes, including conformal ones. Along with the positions of the elements, the iterative procedure yields their excitations in such a way that a desired far-field co-polar pattern is approximated and the far-field cross-polar pattern does not exceed a prescribed threshold.

The optimality of the solution is not guaranteed, but a numerical example is provided that shows the effectiveness of the presented procedure.

At the knowledge of these authors, the literature does not offer deterministic techniques capable of solving the synthesis problem at hand, for arrays of arbitrary geometry and with the considered constraints.

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