Exact Transient Field of a Horizontal Electric Dipole with Double Exponential Excitation on the Boundary Two Dielectrics

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Abstract **─** In this paper, the exact formulas are derived for the time-domain electromagnetic field radiated by a horizontal electric dipole with double exponential excitation along the boundary between two different dielectrics. The transient field components consist of two lateral pulses with the amplitude attenuation factor ρ^{-2}. In particular, the two lateral pulses travel along the boundary in Regions 1 and 2 with different velocities.

Index Terms **─** Double exponential excitation, exact formulas, horizontal electric dipole.

I. INTRODUCTION

The frequency-domain electromagnetic field radiated by horizontal and vertical electric dipoles on or near the plane boundary between two dielectrics like air and earth or sea water and rock, had been investigated widely for its useful applications in subsurface and closed-to-the-surface communication, radar, and geophysical prospecting and diagnostics [1-11]. This problem is well summarized in the monograph by King, Owens, and Wu [11]. In addition, the time-domain properties and applications of the transient field radiated by a dipole source on the plane boundary between two dielectrics have also been treated by many investigators [12-22]. In pioneering work by Van der Pol [12], the detailed analysis was carried out on the transient field of a vertical electric dipole with a delta-function excitation on the boundary between two half-spaces by invoking the Hertz potential. The important work on the exact formulas for the transient field components generated by a vertical electric dipole with delta-function excitation on the boundary between two dielectrics, was addressed by Wu and King [18]. Lately, the exact solution on the lateral electromagnetic pulses due to horizontal and vertical dipoles with delta-function excitation and Gaussian pulse excitation on the boundary between two dielectrics, was subsequently obtained in [19][20]. In the past decade, with the extensions of the work by Wu and King [18], some progresses were made on the exact and approximated solutions for the transient field of a dipole source with delta and Gaussian excitations on the boundary of two different media [21-24].

It is well known that both the lighting electromagnetic pulse source and the nuclear electromagnetic pulse source are a typical double exponential excitation source. It is seen that the double exponential excitation source is more popular comparing with the source with delta-function excitation, and that with Gaussian excitation. Evidently, it is important to treat the exact solution on the transient field of a dipole source excited by double exponential currents. In what follows, we will attempt to obtain the exact formulas for the transient field components radiated by a horizontal electric dipole with double exponential excitation on the planar boundary

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between two dielectrics.

II. EXACT FIELD COMPONENTS OF HORIZONTAL ELECTRIC DIPOLE WITH DOUBLE EXPONENTIAL EXCITATION ON THE BOUNDARY BETWEEN TWO DELECTRICS

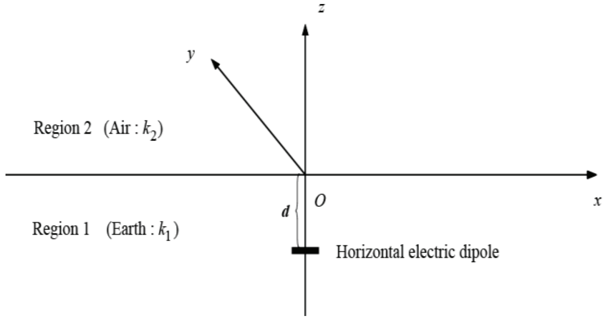

Fig. 1. Geometry of a horizontal electric dipole on the boundary between two dielectrics.

When a horizontal electric dipole is excited by double exponential currents, the problem of the exact solution on the transient field components will be in general more complicated. The relevant geometry and Cartesian coordinate system are shown in Fig. 1, where a horizontal electric dipole in the $\hat{x}$ direction is located at $(0,0,-d)$. The lower half-space $z \le 0$ (Region 1) is with the earth characterized by the permeability μ_0 and relative permittivity ε filled by with the air, and the upper half-space $z \ge 0$ (Region 2) is with the air characterized by the permeability μ_0 and uniform permittivity ε_0. The wave numbers of the two regions are $k_1 = \sqrt{\varepsilon}\omega/c$ and $k_2 = \omega/c$, respectively. When both the dipole and the observation point approach the boundary from above $(d \to 0^+)$ and $(z \to 0^+)$, with the time dependence of $e^{-i\omega t}$, the exact transient field of a horizontal electric dipole with a delta-function excitation on the boundary between two dielectrics can be obtained by using Fourier's transform technique [24]. From Chapter 8 in the book [24], the exact formulas for the transient field components with delta-function excitation can be written readily in the following forms:

$$
\begin{aligned}
\left[E_{2\rho}(\rho,t)\right]_\delta = &\frac{1}{2\pi\varepsilon_0 c\rho^2}\cdot\left[\delta\left(t-\frac{\rho}{c}\right)+\frac{1}{\sqrt{\varepsilon}}\delta\left(t-\frac{\sqrt{\varepsilon}\rho}{c}\right)\right] \\
&-\frac{1}{2\pi\varepsilon_0\rho^3(\varepsilon+1)} \\
&\times\begin{cases} 0 & ,\dfrac{ct}{\rho}<1 \\ 1-\dfrac{\varepsilon^2}{(\varepsilon-1)(\varepsilon+1)^{\frac{3}{2}}}\left(\dfrac{c^2t^2}{\rho^2}+\dfrac{2\varepsilon}{\varepsilon+1}\right)\left(\dfrac{c^2t^2}{\rho^2}-\dfrac{\varepsilon}{\varepsilon+1}\right)^{-\frac{5}{2}} & ,1<\dfrac{ct}{\rho}<\sqrt{\varepsilon} \\ 2 & ,\dfrac{ct}{\rho}>\sqrt{\varepsilon} \end{cases},
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
\left[E_{2\phi}(\rho,t)\right]_\delta = &\frac{1}{2\pi\varepsilon_0 c(\varepsilon-1)\rho^2}\left[\delta\left(t-\frac{\rho}{c}\right)-\sqrt{\varepsilon}\delta\left(t-\frac{\sqrt{\varepsilon}\rho}{c}\right)\right] \\
&+\frac{1}{2\pi\varepsilon_0(\varepsilon-1)\rho^2} \\
&\times\begin{cases} 0 & ,\dfrac{ct}{\rho}<1 \\ 2-\dfrac{1}{\varepsilon+1}+\dfrac{\varepsilon^2}{(\varepsilon+1)^{5/2}}\left(\dfrac{c^2t^2}{\rho^2}-\dfrac{\varepsilon}{\varepsilon+1}\right)^{-3/2} & ,1<\dfrac{ct}{\rho}<\sqrt{\varepsilon} \\ \dfrac{\varepsilon-1}{\varepsilon+1} & ,\dfrac{ct}{\rho}>\sqrt{\varepsilon} \end{cases},
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
\left[B_{2z}(\rho,t)\right]_\delta = &\frac{\mu_0}{2\pi\rho^2(\varepsilon-1)}\left[\delta\left(t-\frac{\rho}{c}\right)-\varepsilon\delta\left(t-\frac{\sqrt{\varepsilon}\rho}{c}\right)\right] \\
&-\frac{\mu_0}{2\pi\rho^3(\varepsilon-1)}\begin{cases} 0 & ,\dfrac{ct}{\rho}<1 \\ \dfrac{3ct}{\rho} & ,1<\dfrac{ct}{\rho}<\sqrt{\varepsilon} \\ 0 & ,\dfrac{ct}{\rho}>\sqrt{\varepsilon} \end{cases}.
\end{aligned}
\tag{3}
$$

In above formulas, the double exponential pulse is defined by:

$$i(t) = a\left[e^{-\alpha t} - e^{-\beta t}\right]U(t), \tag{4}$$

where $U(t)$ is unit step function. Thus, the exact formulas of the transient field components of a horizontal electric dipole excited by double exponential currents can be represented as follows:

$$E_{2\rho}(\rho,t) = \int_{-\infty}^{\infty}\left[E_{2\rho}(\rho,0;t-\zeta)\right]_\delta \cdot a\left(e^{-\alpha\zeta}-e^{-\beta\zeta}\right)U(\zeta)d\zeta, \tag{5}$$

$$E_{2\phi}(\rho,t) = \int_{-\infty}^{\infty}\left[E_{2\phi}\left(\rho,\frac{\pi}{2};t-\zeta\right)\right]_\delta \cdot a\left(e^{-\alpha\zeta}-e^{-\beta\zeta}\right)U(\zeta)d\zeta, \tag{6}$$

$$B_{2z}(\rho,t) = \int_{-\infty}^{\infty}\left[B_{2z}\left(\rho,\frac{\pi}{2};t-\zeta\right)\right]_\delta \cdot a\left(e^{-\alpha\zeta}-e^{-\beta\zeta}\right)U(\zeta)d\zeta. \tag{7}$$

Obviously, the formulas of the transient field can be expressed in terms of several integrals,

$$\frac{1}{a}E_{2\rho}(\rho,t) = \frac{1}{2\pi\varepsilon_0 c\rho^2}\left(I_1+\varepsilon^{-\frac{1}{2}}I_2\right) + \frac{1}{2\pi\varepsilon_0(\varepsilon+1)\rho^3}\left[I_3-\frac{1}{\varepsilon-1}\frac{\varepsilon^2}{(\varepsilon+1)^{\frac{3}{2}}}I_7+2I_5\right] , \quad (8)$$

$$\frac{1}{a}E_{2\phi}(\rho,t) = \frac{1}{2\pi\varepsilon_0 c(\varepsilon-1)\rho^2}\left(I_1-\varepsilon^{\frac{1}{2}}I_2\right)+\frac{1}{2\pi\varepsilon_0(\varepsilon-1)\rho^3} \times\left[\left(2-\frac{1}{\varepsilon+1}\right)\cdot I_3+\frac{\varepsilon^2}{(\varepsilon+1)^{2/5}}\cdot I_8+\frac{\varepsilon-1}{\varepsilon+1}\cdot I_5\right] , \quad (9)$$

$$\frac{1}{a}B_{2z}(\rho,t) = \frac{\mu_0}{2\pi\rho^2}\frac{1}{\varepsilon-1}(I_1-\varepsilon I_2)-\frac{3\mu_0 c}{2\pi\rho^3(\varepsilon-1)}\cdot I_9 . \quad (10)$$

The integrals I_1, I_2, I_3, and I_5 can be found in Chapter 15 in the book by King, Owens, and Wu [11], and Chapter 8 in the book by Li [24]. In this paper, we will not rewrite the exact solutions of the above four integrals. In the next step, the main task is evaluating the integrals I_7, I_8, and I_9. We write:

$$I_7=\int_{-\infty}^{\infty}\left[\frac{c^2(t-\zeta)^2}{\rho^2}+\frac{2\varepsilon}{\varepsilon+1}\right]\left[\frac{c^2(t-\zeta)^2}{\rho^2}-\frac{\varepsilon}{\varepsilon+1}\right]^{-5/2} \times\left[U\left(t-\frac{\sqrt{\varepsilon}\rho}{c}\right)-U\left(t-\frac{\rho}{c}\right)\right]\left(e^{-\alpha\zeta}-e^{-\beta\zeta}\right)U(\zeta)d\zeta , \quad (11)$$

$$I_8=\int_{-\infty}^{\infty}\left[\frac{c^2(t-\zeta)^2}{\rho^2}-\frac{\varepsilon}{\varepsilon+1}\right]^{-3/2} \times\left[U\left(t-\frac{\sqrt{\varepsilon}\rho}{c}\right)-U(t-\rho')\right]\left(e^{-\alpha\zeta}-e^{-\beta\zeta}\right)U(\zeta)d\zeta , \quad (12)$$

$$I_9=\int_{-\infty}^{\infty}\frac{c(t-\zeta)}{\rho}\left[U\left(t-\frac{\sqrt{\varepsilon}\rho}{c}\right)-U(t-\rho')\right]\left(e^{-\alpha\zeta}-e^{-\beta\zeta}\right)U(\zeta)d\zeta . \quad (13)$$

With $x=(t-\zeta)/\rho'$, $dx=-d\zeta/\rho'$, and $a^2=\varepsilon/\varepsilon+1$, the results become:

$$I_7=\begin{cases}\rho'\int_1^{\sqrt{\varepsilon}}\frac{x^2+2a^2}{(x^2-a^2)^{5/2}}\left[e^{-\alpha(t-\rho'x)}-e^{-\beta(t-\rho'x)}\right]dx & ,\sqrt{\varepsilon}\rho'\le t\\ \rho'\int_1^{t/\rho'}\frac{x^2+2a^2}{(x^2-a^2)^{5/2}}\left[e^{-\alpha(t-\rho'x)}-e^{-\beta(t-\rho'x)}\right]dx & ,\rho'\le t\le\sqrt{\varepsilon}\rho' ,\\ 0 & ,t<\rho'\end{cases} \quad (14)$$

$$I_8=\begin{cases}\rho'\int_1^{\sqrt{\varepsilon}}(x^2-a^2)^{-3/2}\left[e^{-\alpha(t-\rho'x)}-e^{-\beta(t-\rho'x)}\right]dx & ,\sqrt{\varepsilon}\rho'\le t\\ \rho'\int_1^{t/\rho'}(x^2-a^2)^{-3/2}\left[e^{-\alpha(t-\rho'x)}-e^{-\beta(t-\rho'x)}\right]dx & ,\rho'\le t<\sqrt{\varepsilon}\rho' ,\\ 0 & ,t<\rho'\end{cases} \quad (15)$$

$$I_9=\begin{cases}\rho'\int_1^{\sqrt{\varepsilon}}x\left[e^{-\alpha(t-\rho'x)}-e^{-\beta(t-\rho'x)}\right]dx & ,\sqrt{\varepsilon}\rho'\le t\\ \rho'\int_1^{t/\rho'}x\left[e^{-\alpha(t-\rho'x)}-e^{-\beta(t-\rho'x)}\right]dx & ,\rho'\le t<\sqrt{\varepsilon}\rho' .\\ 0 & ,t<\rho'\end{cases} \quad (16)$$

The integrals in (14)-(16) can be evaluated readily by using numerical method. Then, the transient field components with double exponential pulse excitation can be derived readily. We write:

$$\begin{aligned}\frac{1}{a}E_{2\rho}(\rho,t)=&\frac{1}{2\pi\varepsilon_0 c\rho^2}\Big\{\left[e^{-\alpha(t-\rho')}-e^{-\beta(t-\rho')}\right]U(t-\rho')\\&+\varepsilon^{-1/2}\left[e^{-\alpha(t-\sqrt{\varepsilon}\rho')}-e^{-\beta(t-\sqrt{\varepsilon}\rho')}\right]U(t-\sqrt{\varepsilon}\rho')\Big\}\\&+\frac{1}{2\pi\varepsilon_0(\varepsilon+1)\rho^3}\cdot\Big\{-\frac{1}{\alpha}\left[e^{-\alpha(t-\rho')}-e^{-\alpha(t-\sqrt{\varepsilon}\rho')}\right]\\&+\frac{1}{\beta}\left[e^{-\beta(t-\rho')}-e^{-\beta(t-\sqrt{\varepsilon}\rho')}\right]\Big\}U(t-\sqrt{\varepsilon}\rho')+\frac{1}{2\pi\varepsilon_0(\varepsilon+1)\rho^3}\\&\times\left\{-\frac{1}{\alpha}\left[e^{-\alpha(t-\rho')}-1\right]+\frac{1}{\beta}\left[e^{-\beta(t-\rho')}-1\right]\right\}\\&\times\left[U(t-\rho')-U(t-\sqrt{\varepsilon}\rho')\right]-\frac{1}{2\pi\varepsilon_0(\varepsilon+1)^{5/2}\rho^3}\frac{\varepsilon^2}{(\varepsilon-1)}\\&\times\Big\{\rho'\int_1^{\sqrt{\varepsilon}}\frac{x^2+2a^2}{(x^2-a^2)^{5/2}}\cdot\left[e^{-\alpha(t-\rho'x)}-e^{-\beta(t-\rho'x)}\right]dxU(t-\sqrt{\varepsilon}\rho')\\&+\rho'\int_1^{t/\rho'}\frac{x^2+2a^2}{(x^2-a^2)^{5/2}}\cdot\left[e^{-\alpha(t-\rho'x)}-e^{-\beta(t-\rho'x)}\right]dx\\&\times\left[U(t-\rho')-U(t-\sqrt{\varepsilon}\rho')\right]\Big\}+\frac{1}{\pi\varepsilon_0(\varepsilon+1)\rho^3}\\&\times\left\{-\frac{1}{\alpha}\left[e^{-\alpha(t-\sqrt{\varepsilon}\rho')}-1\right]+\frac{1}{\beta}\left[e^{-\beta(t-\sqrt{\varepsilon}\rho')}-1\right]\right\}U(t-\sqrt{\varepsilon}\rho'),\end{aligned} \quad (17)$$

$$\begin{aligned}
\frac{1}{a}E_{2\phi}(\rho,t) &= \frac{1}{2\pi\varepsilon_0(\varepsilon-1)\rho^2}\Big\{\Big[e^{-\alpha(t-\rho')}-e^{-\beta(t-\rho')}\Big]U(t-\rho') \\
&\quad -\varepsilon^{1/2}\Big[e^{-\alpha(t-\sqrt{\varepsilon}\rho')}-e^{-\beta(t-\sqrt{\varepsilon}\rho')}\Big]U(t-\sqrt{\varepsilon}\rho')\Big\} \\
&\quad +\frac{1}{2\pi\varepsilon_0(\varepsilon-1)\rho^3}\left(2-\frac{1}{\varepsilon+1}\right)\cdot\Big\{-\frac{1}{\alpha}\Big[e^{-\alpha(t-\rho')}-e^{-\alpha(t-\sqrt{\varepsilon}\rho')}\Big] \\
&\quad +\frac{1}{\beta}\Big[e^{-\beta(t-\rho')}-e^{-\beta(t-\sqrt{\varepsilon}\rho')}\Big]\Big\}U(t-\sqrt{\varepsilon}\rho') \\
&\quad +\frac{1}{2\pi\varepsilon_0(\varepsilon-1)\rho^3}\left(2-\frac{1}{\varepsilon+1}\right)\cdot\Big\{-\frac{1}{\alpha}\Big[e^{-\alpha(t-\rho')}-1\Big] \\
&\quad +\frac{1}{\beta}\Big[e^{-\beta(t-\rho')}-1\Big]\Big\}\Big[U(t-\rho')-U(t-\sqrt{\varepsilon}\rho')\Big] \\
&\quad -\frac{1}{2\pi\varepsilon_0(\varepsilon+1)^{5/2}\rho^3}\frac{\varepsilon^2}{(\varepsilon-1)} \\
&\quad \times\Big\{\rho'\int_1^{\sqrt{\varepsilon}}\left(x^2-a^2\right)^{-3/2}\cdot\Big[e^{-\alpha(t-\rho'x)}-e^{-\beta(t-\rho'x)}\Big]dx\,U(t-\sqrt{\varepsilon}\rho') \\
&\quad +\rho'\int_1^{t/\rho'}\left(x^2-a^2\right)^{-3/2}\cdot\Big[e^{-\alpha(t-\rho'x)}-e^{-\beta(t-\rho'x)}\Big]dx \\
&\quad \times\Big[U(t-\rho')-U(t-\sqrt{\varepsilon}\rho')\Big]\Big\}+\frac{1}{2\pi\varepsilon_0(\varepsilon+1)\rho^3} \\
&\quad \times\Big\{-\frac{1}{a}\Big[e^{-\alpha(t-\sqrt{\varepsilon}\rho')}-1\Big]+\frac{1}{\beta}\Big[e^{-\beta(t-\sqrt{\varepsilon}\rho')}-1\Big]\Big\}U(t-\sqrt{\varepsilon}\rho'),
\end{aligned} \tag{18}$$

$$\begin{aligned}
\frac{1}{a}B_{2z}(\rho,t) &= \frac{\mu_0}{2\pi\rho^2}\Big\{\Big[e^{-\alpha(t-\rho')}-e^{-\beta(t-\rho')}\Big]U(t-\rho') \\
&\quad -\varepsilon\Big[e^{-\alpha(t-\sqrt{\varepsilon}\rho')}-e^{-\beta(t-\sqrt{\varepsilon}\rho')}\Big]U(t-\sqrt{\varepsilon}\rho')\Big\} \\
&\quad -\frac{3\mu_0 c}{2\pi\rho^3(\varepsilon-1)}\Big\{\rho'\int_1^{\sqrt{\varepsilon}}x\Big[e^{-\alpha(t-\rho'x)}-e^{-\beta(t-\rho'x)}\Big]dx\,U(t-\sqrt{\varepsilon}\rho') \\
&\quad +\rho'\int_1^{t/\rho'}x\Big[e^{-\alpha(t-\rho'x)}-e^{-\beta(t-\rho'x)}\Big]dx \\
&\quad \times\Big[U(t-\rho')-U(t-\sqrt{\varepsilon}\rho')\Big]\Big\}.
\end{aligned} \tag{19}$$

By now, the analytical formulas for the three components $E_{2\rho}(\rho,t)$, $E_{2\phi}(\rho,t)$, and $B_{2z}(\rho,t)$ have been derived readily.

III. COMPUTATIONS AND CONCLUSIONS

From the above derivations and analysis, it is seen that the final exact formulas for the three components $E_{2\rho}(\rho,t)$, $E_{2\phi}(\rho,t)$, and $B_{2z}(\rho,t)$, radiated by a horizontal electric dipole with the double exponential pulse excitation can be expressed in terms of several fundamental functions.

After examining the equations (17)-(19), it is found that the three time-domain components $E_{2\rho}(\rho,t)$, $E_{2\phi}(\rho,t)$, and $B_{2z}(\rho,t)$ consist of two lateral electromagnetic pulses with the amplitude factor ρ^{-2}. In particular, the first pulse arrives at ρ/c, which travels along the boundary in Region 2 (air) with the velocity c; while the second pulse arrives at $\rho\sqrt{\varepsilon}/c$, which travels along the boundary in Region 1 (earth) with the velocity $c/\sqrt{\varepsilon}$.

In the following computations, the parameters of the double exponential pulse are taken as $a = 30$ kA, $\alpha = 2\times10^4\ \mathrm{sec}^{-1}$, and $\beta = 2\times10^5\ \mathrm{sec}^{-1}$. Assuming that the relative permittivity of the earth is $\varepsilon = 8$, both the electric field components $E_{2\rho}(\rho,t)$ and $E_{2\phi}(\rho,t)$ which are generated by a horizontal electric dipole with double exponential pulse excitation on the plane boundary between the air and the earth, are computed and shown in Figs. 2 and 3, respectively.

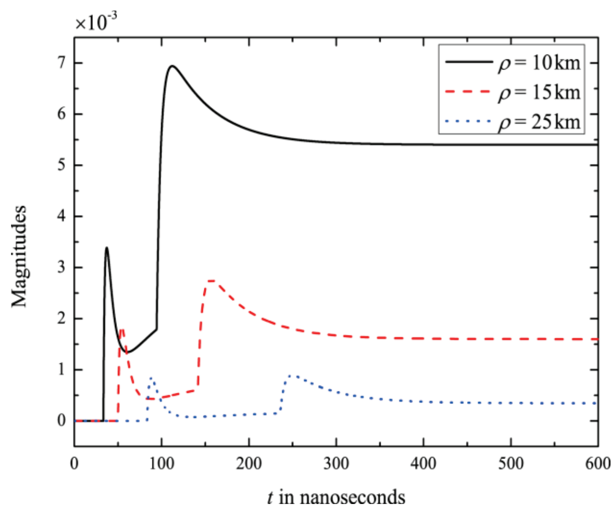

Fig. 2. Exact electric field $E_{2\rho}(\rho,t)$ on the boundary in air of a horizontal electric with the relative permittivity of the earth $\varepsilon = 8$.

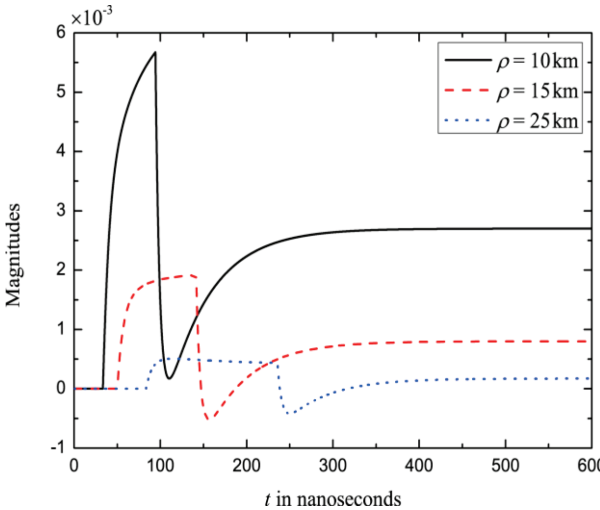

Fig. 3. Exact electric field $E_{2\phi}(\rho,t)$ on the boundary in air of a horizontal electric dipole with the relative permittivity of the earth $\varepsilon = 8$.

In these computations, the propagation distances are taken as $\rho = 10$ km, 15 km, and 25 km, respectively. Assuming that the propagation distance is $\rho =$ 15 km, both the electric components $E_{2\rho}(\rho,t)$ and $E_{2\phi}(\rho,t)$ are computed and plotted in Figs. 4 and 5, respectively.

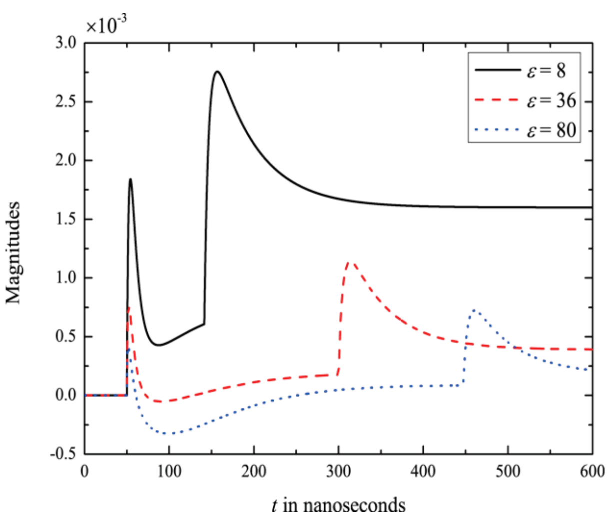

Fig. 4. Exact electric field $E_{2\rho}(\rho,t)$ on the boundary in air of a horizontal electric dipole at the propagation distance $\rho = 15$ km.

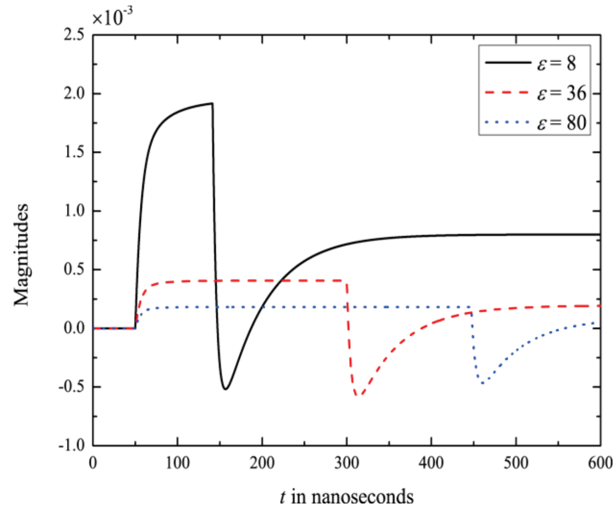

Fig. 5. Exact electric field $E_{2\phi}(\rho,t)$ on the boundary in air of a horizontal electric dipole at the propagation distance $\rho = 15$ km.

Actually, the amplitude of the second pulse for the component $E_{2\rho}(\rho,t)$ is smaller than that of the first pulse. It is noted that the term including the integral I_7 in $E_{2\rho}(\rho,t)$ has a large negative value near the first pulse, it is followed that the amplitude of the second pulse is larger than that of the first one. For the component $E_{2\phi}(\rho,t)$, there doesn't exists the term of the integral I_7. As a result, the amplitude of the first pulse is larger than that of the second pulse.

Similar to the case of the delta-function excitation, and that of the Gaussian excitation addressed in Chapter 8 in the book [24], the remaining three components $E_{2z}(\rho,t)$, $B_{2\rho}(\rho,t)$, and $B_{2\varphi}(\rho,t)$, cannot be expressed in terms of elementary functions and finite integrals.

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