Active Metamaterial Incorporating Gain Device/Medium: A Review

Qi Tang [1] and Hao Xin [1,2]

[1] Department of Electrical Engineering
[2] Department of Physics
University of Arizona, Tucson, AZ 85721, USA
tangqi@email.arizona.edu, hxin@ece.arizona.edu

Abstract **─** Metamaterials intrigue many exciting applications in the broad electromagnetic spectrum ranging from microwave to optics. However, many of the envisaged applications still remain in theory, largely because of the intrinsic loss and dispersion associated with passive metamaterials. Incorporating active devices or media into conventional passive metamaterial structures for loss compensation, as well as dispersion control, is very attractive and may finally enable many desired applications. In addition, because of the added design degree of freedom in active metamaterials, new and rich physical phenomena and insights can be discovered. In this paper, we review the recent progress in the realm of active, gain-assisted metamaterials. Physical limitations on loss and bandwidth of metamaterials are firstly discussed. Recent experimental efforts in transmission-line and volumetric metamaterials with net gain in the microwave and optical regime are then examined. The idea of utilizing non-Foster active devices to reduce the dispersion and achieve broad bandwidth is also presented. Finally, one of the important issues of active metamaterial design, stability, is briefly discussed.

Index Terms **─** Active metamaterials, gain, negative index.

I. BACKGROUND ON METAMATERIALS AND APPLICATIONS

Metamaterials are artificial composite materials that often involve periodic structures to achieve unconventional and advantageous material properties. The earliest study of artificial materials for manipulating electromagnetic waves dates back more than 100 years [1]. In 1967, Veselago theoretically studied materials with simultaneous negative permittivity (-ε) and permeability (-μ) [2], and predicted some unique properties such as the negative refractive index, opposite directions of the phase velocity and Poynting vector (the left-handedness of the wave propagation), the reversed Snell's law, and the reversed Doppler effects.

The first experimental demonstration of Negative Index Metamaterial (NIM) with simultaneous -ε and -μ (sometimes referred to as DNG, double negative) is conducted at microwave frequency (10.5 GHz). The designed NIM comprises a 2-D array of wires and Split Ring Resonators (SRRs) (see Fig. 1 (a)) [3]. Thereafter, various metamaterials including single negative (ε-negative, ENG and μ-negative, MNG) [4], near-zero index [5] and gradient index [6] materials have been studied. Another type of realization of metamaterials is based on the microwave transmission line theory [7-9], as shown in Fig. 1 (b). A simple analogy between the circuit model and the effective permittivity and permeability is described in [10,11].

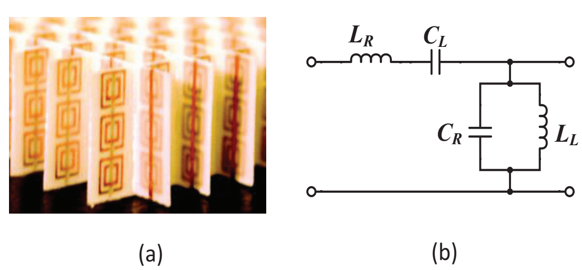


Fig. 1. Two most commonly studied metamaterial examples: (a) a wire (provides -ε) and SRR (provides -μ) array (figure adapted from [3]), and (b) a unit cell of composite Left-Hand/Right-Hand Transmission Line (CLRH-TL).

Submitted On: March 1, 2014
Accepted On: September 5, 2014

The unique properties of NIMs and other metamaterials lead to many exciting applications; for example, "invisible cloaking" [12-14], "perfect lens" [15,16], nanoplasmonics [17,18], electrically small antennas [19], highly directional antennas [20,21], leaky wave antennas [22], coupled-line couplers [10], phase shifting lines, broadband balun [11], etc. Despite of the large variety of intriguing applications, intrinsic loss and narrow bandwidth associated with existing metamaterials significantly hamper the wide implementation of metamaterials in practice.

Many theoretical studies assume lossless or low-loss scenario, which is hardly to achieve in reality. For example, at optical frequencies, all the demonstrated passive NIMs have large loss accompanying the negative refractive index. Even at microwave frequencies, especially for some typical resonant-type metamaterials, loss is still a problem and severely degrades or even completely eliminates the advantages of metamaterial based components and applications such as "cloaking" devices and electrically small antennas. Another example is the "perfect lens" [15], a slab with a refractive index $n=-1$ which overcomes the diffraction limit; however, the predicted "perfect image" cannot be realized unless the NIM slab is completely lossless [23].

Another bottleneck is the limited bandwidth. Metamaterials are intrinsically dispersive because of the causality requirement, thus only present interesting effective medium properties within a narrow bandwidth. Take the "cloaking" for example. It would be much less useful that the object inside the metamaterial layers would only be invisible for a single frequency, compared to a true broadband "cloaking" device. Also, for data communication or sensor applications, wide bandwidth is also desirable for high data rate and resolution.

Loss and dispersion are inherent in the constitutive parameters of natural materials [24,25] and even more so for NIMs, due to causality requirement. Some theoretical studies attempt to prove whether a lossless and dispersionless NIM violates any fundamental physics, and if it did not, what the optimal bandwidth with low loss one can achieve given a target constant material property, such as $n=-1$ for the 'perfect lens' application [26]. The loss generation in NIM can be divided into the dissipative loss due to the constitutive material properties [27,28], and the radiation loss [29] due to the shape and geometry of the unit cell structure. Various practical approaches to "passively" reduce the loss, for example, advances in fabrication, tailoring the shape and geometry [30,31], and the use of a stack of alternating negative and positive index layers [32], have been investigated but complete loss elimination remains elusive.

II. ACTIVE METAMATERIALS

The idea of incorporating gain device or medium into metamaterial structure to compensate the loss is quite natural and attractive [33,34]. First, losses associated with conductors, dielectrics and radiation can be compensated by the availability of gain. Second, with the additional gain as a design tradeoff, more bandwidth may be realized [35]. With consistent efforts on theoretical studies, fabrications and experimental validations, significant progress in active metamaterials has been made in recent years. Active metamaterials, recognized as the next stage of the technological revolution in metamaterials, also enables some new applications to appear on the horizon of possibilities, such as optical data processing and quantum information applications [36].

The term "active metamaterial", from an engineering point of view, is used to distinguish it from conventional passive metamaterial, such as SRR, fishnet structure, and CLRH-TL constructed from only passive constituents; i.e., metals and dielectrics. The word "active" can be either noted as embedding an energy source in the structure to ameliorate some undesirable properties, such as loss and dispersion, or adding an external control to realize tunable or other special functionalities. Some examples of the latter category include semiconducting varactors controlled impedance surfaces for antenna applications [37,38], electronically reconfigured magnetic-resonant metamaterial for phase modulator [39-42] and transistor-based nonreciprocal metamaterial for isolating functionality [43]. In this review, we will focus on the first category of active metamaterials for loss compensation and dispersion management only.

In the microwave regime, negative resistance elements can be implemented into the passive unit cell structure of a metamaterial to achieve loss compensation and even amplification [44-48]. Such negative resistance elements can be realized by active devices, such as resonant tunneling diodes, Gunn diodes, lambda diodes, negatron and transistor-based circuits [49,50], which have been widely applied in oscillator, amplifier, and mixer designs. A detailed review on a resonant tunneling diode based active CLRH-TL metamaterials will be given in Section IV.

In the optical regime, active gain media are well-established in lasers and optical amplifiers [51]. Loss compensation, steady-state net amplification, and nanoscopic lasing in optical frequencies become possible via the incorporation of gain materials adjacent to or within NIM structures [52-56]. We will also present a brief review on state of the art in optical active metamaterials in Section IV, although the main focus of this paper is on microwave active metamaterials. A more detailed review on active optical metamaterials can be found in [57] and [58].

Recently there have been increasing interests in designing non-dispersive metamaterials using "non-Foster" elements [59-64]. A non-Foster element is a negative inductance or negative capacitance realized by active electronic circuits [65,66]. Such circuits have been widely investigated in the designs of voltage controlled oscillators, active filters, amplifiers, and more recently electrically small antennas [67-69].

In the following sections, we begin by concisely reviewing some theoretical constraints on the properties of passive metamaterials, explaining why a dispersionless negative index metamaterial with negligible loss is not achievable and how to estimate the bounds of loss and bandwidth for metamaterials. We then give an overview of proof-of-concept realizations of gain-assisted active metamaterials. In the interest of brevity, we focus on the microwave regime, and briefly introduce the development of active optical metamaterials. The novel direction of non-dispersive active metamaterials by implementing active negative impedance circuits is reviewed next. We then present a brief discussion on stability issues of active metamaterials as all active circuits and materials are prone to instability. Finally, we conclude by an outlook of the important challenges that remain to be addressed in the future.

III. LOSS AND DISPERSION OF METAMATERIALS

A. Inevitable loss or dispersion

A straightforward method to prove a linear passive NIM with negligible loss has to be dispersive is by using a simplified Poynting's theorem [70]. Assuming a low-loss system with ε and μ such that $\varepsilon''<<|\varepsilon'|$ and $\mu''<<|\mu'|$, and consider a wave of a relatively narrow bandwidth in comparison to the bandwidth over which $\varepsilon(\omega)$ and $\mu(\omega)$ changes appreciably, the time-averaged internal stored energy can be calculated by [24,25]:

$$\overline{U}=\frac{1}{2}\mathrm{Re}[\frac{d(\omega\varepsilon)}{d\omega}]\bigg|_{\omega=\omega_0}\left\langle E^2\right\rangle+\frac{1}{2}\mathrm{Re}[\frac{d(\omega\mu)}{d\omega}]\bigg|_{\omega=\omega_0}\left\langle H^2\right\rangle, \quad (1)$$

where ω_0 is the central frequency of the narrow band signal. If ε and μ are not frequency dependent, i.e., dispersionless, the above Eq. (1) becomes to:

$$\overline{U}=\frac{1}{2}\varepsilon\left\langle E^2\right\rangle+\frac{1}{2}\mu\left\langle H^2\right\rangle. \quad (2)$$

By the law of the increase of entropy, we must have $\overline{U}>0$ for a passive medium. Therefore, the values of ε and μ in Eq. (2) cannot be both negative, which seems to contradict to the existence of NIMs. This leads to an important conclusion that dispersion cannot be neglected for a passive NIM. In other words, for a passive NIM with negligible loss, dispersion is unavoidable.

The second important question is whether a lossless NIM is achievable in principle. Reference [71] is one attempt to study the lower bound on the loss of NIM by imposing Kramers-Kronig relations on $\varepsilon(\omega)\mu(\omega)$, in which it asserts that "any loss compensation or significant reduction at and near the observation frequency will lead to the disappearance of the negative refraction itself due to the dispersion relation dictated by the causality." This statement caused a number of objections and later has been proved to be inaccurate [72-78]. A consensus is reached that negative refractive index is achievable at a single frequency with arbitrarily low loss. This point has been confirmed by a number of experimental results [46-48,52-56], which we will review in detail in Section IV. Reference [73] has further pointed out that a passive low loss NIM is achievable only if there is large loss or large dispersion immediately below the observation frequency, or, there are singularities at real frequencies.

B. Bounds for loss and dispersion of passive metamaterials

The derivation of bounds on dispersion and loss of passive NIMs is a nontrivial problem. For an analytical function at the upper half plane, the Hilbert transform gives the relationship between the real part and imaginary part of the function; i.e., Kramers-Kronig relations, yielding:

$$\varepsilon''(\omega) = \frac{2\omega}{\pi} P\int_0^\infty \frac{\varepsilon'(\omega')-1}{\omega^2-\omega'^2} d\omega', \quad \varepsilon'(\omega)-1 = \frac{2\omega}{\pi} P\int_0^\infty \frac{\varepsilon''(\omega')\omega'}{\omega'^2-\omega^2} d\omega', \tag{3}$$

where P denotes the Cauchy principal value. Causality in a linear dispersive medium implies that, $\varepsilon(t)$ and $\mu(t)$ have to be zero for $t<0$ and real values for $t \geq 0$. Therefore, in the frequency domain, for permittivity, it yields:

$$\varepsilon(-\omega^*) = \varepsilon^*(\omega) . \tag{4}$$

Furthermore, at very high frequency, the electrons behave like they are free, yielding the constraint of high-frequency asymptote:

$$|\varepsilon(\omega)-1| \sim \frac{1}{\omega^2}, \omega \to \infty . \tag{5}$$

Permeability also follows the same causality arguments as the permittivity in Eqs. (3)-(5). In addition, assuming a passive system, we have:

$$\int_{-\infty}^{T} \mathbf{E}(t)\cdot\frac{\partial \mathbf{D}(t)}{\partial t} dt \geq 0, \quad \int_{-\infty}^{T} \mathbf{H}(t)\cdot\frac{\partial \mathbf{B}(t)}{\partial t} dt \geq 0. \tag{6}$$

Using the above physical constraints, [75] derives the limitation of the bandwidth of passive NIMs by constructing and bounding Herglotz functions. Equation (7) gives a lower bound of loss within a finite bandwidth or an upper bound of bandwidth with a fixed loss for a passive medium.

$$\max_{\omega_1<\omega<\omega_2} |\varepsilon(\omega)-\varepsilon_m| \geq \frac{B}{1+B/2}(\varepsilon_\infty-\varepsilon_m)\begin{cases}1/2, & \text{lossy case,}\\ 1, & \text{lossless case,}\end{cases} \tag{7}$$

where ε_m is the targeted permittivity, ε_∞ is the high-frequency asymptote which equals to 1, and $B=(\omega_2-\omega_1)/\omega_0$ is the relative bandwidth with ω_0 as the center frequency. Take a medium with targeted $\varepsilon_m=-1$ as an example. For a deviation between the realized and targeted permittivity of 1%, the calculated maximum relative bandwidth is 1% for the lossy case, and approximates 0.5% for the lossless case.

For the case of an insulating medium, i.e., without static conductivity, the constraint can be further restricted to be:

$$\max_{\omega_1<\omega<\omega_2} \frac{|\varepsilon(\omega)-\varepsilon_m|}{|\varepsilon(\omega)-\varepsilon_\infty|} \geq \frac{B}{1+B/2}\frac{|\varepsilon_m-\varepsilon_s|}{(\varepsilon_s-\varepsilon_\infty)}\begin{cases}1/2, & \text{lossy case,}\\ 1, & \text{lossless case,}\end{cases} \tag{8}$$

where ε_s is the static permittivity (the low-frequency asymptote).

C. Active metamaterial for loss compensation/ gain and dispersion control

The consideration and analysis in III.A and III.B have led to the conclusion that a passive metamaterial cannot be simultaneously lossless and non-dispersive, and lower bound of loss and upper bound of bandwidth exist. However, with the incorporation of gain device/medium (or effectively, external energy source) in a metamaterial structure, the limits derived previously for passive metamaterials are no longer valid. Although, there have been controversies on the causality of an effective medium simultaneously with a negative refractive index and a net gain [71-78], many theoretical and experimental studies have been reported with the goal to achieve complete or even over compensation (i.e., gain) of loss for NIM over a wide frequency spectrum from microwave to optics [46-48,52-56]. In fact, based on the causality inherently described by the energy conservation, the condition for achieving a NIM with zero loss or gain from Poynting's theorem can be derived [79,80]. In the following sections, some of the interesting loss compensated metamaterials and broadband metamaterials utilizing active device/medium are reviewed.

IV. GAIN-ASSISTED METAMATERIALS

Incorporating active constituents with gain into metamaterials has been recognized as a promising technique for compensating losses. There have been sustained theoretical and experimental efforts to reduce or eliminate the loss associated with metamaterials. In this section, we review a number of these works with a focus on experimental validations [45-47,56,81-83].

A. Active transmission line metamaterials loaded with resonant tunneling diodes

Conventional microwave transmission line can be regarded as a 1-D homogeneous medium and modeled by cascading a number of unit cells with dimension much smaller than the wavelength. The equivalent circuit of the unit cell of a conventional transmission line can be expressed as a set of a series distributed inductance and a shunt distributed capacitance, which always leads to a positive phase velocity in the direction of energy flow. This characteristic can be classified as the Right-Handed Transmission Line (RH-TL). On the contrary, a unit cell of a Left-Handed Transmission Line (LH-TL) consists of a series distributed capacitance and a shunt distributed inductance, leading to a negative phase velocity. A pure LH-TL does not exist due to the unavoidable parasitics at high frequencies. Instead, a more realistic design of CLRH-TL is usually studied that exhibits negative phase velocity (LH) at low frequency range and positive phase velocity (RH) at high frequency range [84-86]. A bandgap generally exists between the LH and RH frequency region which is named as unbalanced. The gap would disappear, if the cutoff frequencies of LH and RH coincide (balanced). Such unique properties of CLRH-TLs are utilized in a number of applications, including leaky-wave antennas, compact coupled-line coupler, phase shifters, sub-wavelength resonators, distributed mixer/ amplifiers [10,11,22,87-91], etc. Because of the non-resonant nature of both permittivity and permeability, similar to a Drude medium, a CLRH-TL metamaterial exhibits larger frequency range of effective negative refractive index and lower loss compared to the resonant volumetric metamaterial [89]. Nevertheless, when frequency increases, losses in conductor and dielectric and due to radiation will inevitably increase. Moreover, lumped elements are essential components for this kind of transmission lines. At higher frequencies (i.e., higher than a few GHz), high quality lumped elements (especially inductors) are rare and limited in their achievable values.

Consider a unit cell of a CLRH-TL as shown in Fig. 2 (a), in which the series resistance R and the shunt conductance G representing the distributed losses. If a negative resistance and/or a negative conductance can be incorporated in the unit cell (i.e., R<0 and/or G<0), a CLRH-TL with loss compensation and even amplification could be realized. Figures 2 (b) and 2 (c) plot the calculated real part (α) and imaginary part (β) of the propagation constant ($\gamma=\alpha+j\beta$) of a CLRH-TL design with the following parameters: L_R=1 *nH*, C_L=1 *pF*, L_L=1 *nH*, C_R=1 *pF*, G=0 S, and R from -100 to 100 Ω.

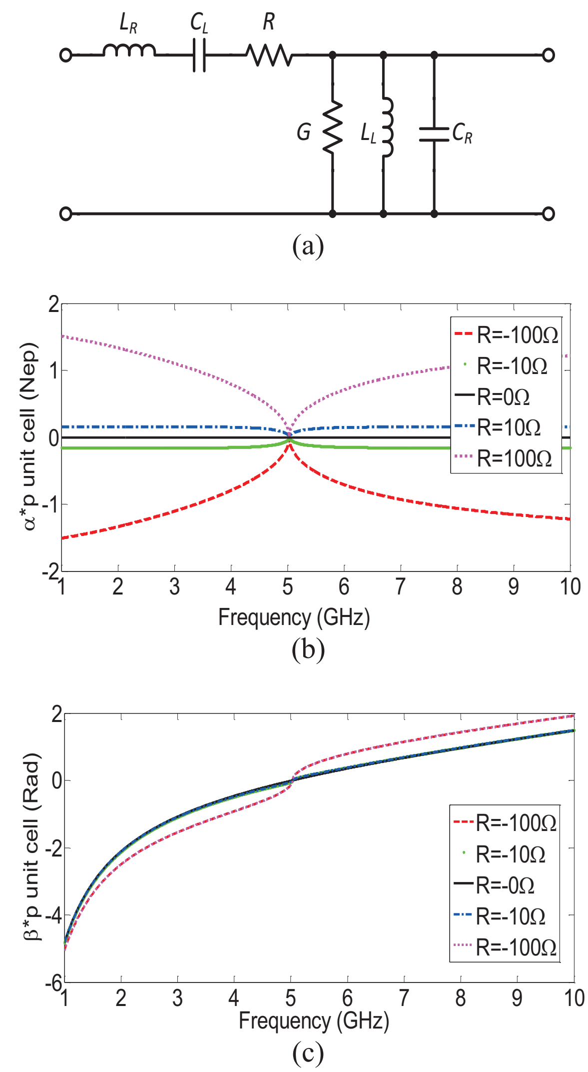


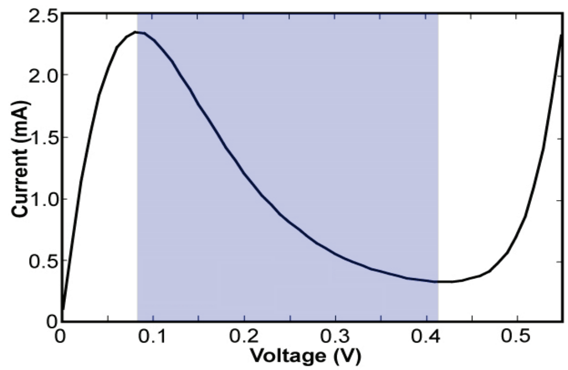
Fig. 2. (a) Equivalent circuit model of a unit cell of a CLRH-TL metamaterial, (b) calculated attenuation constant α (α>0-loss, α=0-lossless, α<0-gain), and (c) phase constant β (β>0-RH, β<0-LH) (figure adapted from [45]).

It can be observed in Figs. 2 (b) and 2 (c), that positive R leads to loss while negative R provides gain while the phase propagation constant β is not impacted much by the value of R. Thus, for this simple example, negative index of refraction and gain are achieved simultaneously by incorporating an ideal negative resistance in the unit cell.

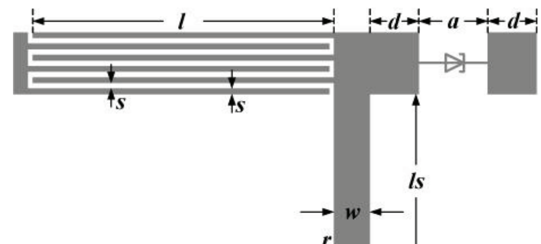
This above concept has been confirmed experimentally. In [46], a CLRH-TL at microwave frequency with net gain by incorporating a Germanium tunneling diode (General Electric TD261) at the unit cell level is implemented and tested. This tunneling diode has a pronounced Negative Differential Resistance (NDR) region, as can be observed in its I-V curve plotted in Fig. 3 (a). As the schematic of the unit cell in Fig. 3 (b) shows, a series interdigited capacitance (C_L), a shunt narrow line with a shorting via (L_L), and the tunneling diode device connected in series are used to implement the active CLRH-TL. It is demonstrated that the addition of the DC pumped diode not only maintains the left handedness, but also provides gain. In the design procedure, a passive CLRH-TL is first realized with appropriate equivalent circuit parameters to guarantee left-handedness in the interested frequency band. Then the tunneling diode with NDR behavior is added into the unit cell design to compensate for loss and provide gain while maintaining left-handedness of the transmission line. Three prototypes of one, two and three unit cells are studied experimentally.

A photo of the fabricated one unit-cell active CLRH-TL is shown in Fig. 3 (c). Two-port S-parameters of the fabricated samples are measured using a vector network analyzer. The complex propagation constant γ can be extracted from the measured S-parameters by Eq. (9), where p is the unit cell length and N is the number of unit cells of the CLRH-TL under consideration. The sign in Eq. (9) is determined according to [90]. An alternative method for extracting the propagation constant γ involving finding the Nth root of the ABCD matrix of the CLRH-TL [87] is also applied and the same propagation constant γ for all three cases (1, 2, and 3 unit cells) are obtained (note that the time dependence convention $e^{j\omega t-\gamma z}$ is used here).

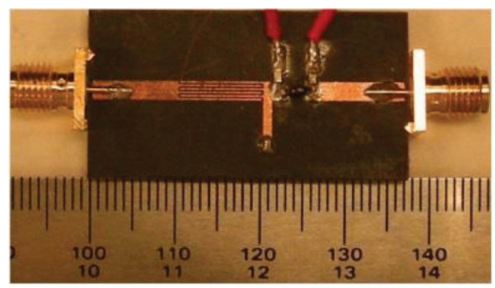
$$\gamma = \pm\frac{1}{Np}\cosh^{-1}\left(\frac{1+S_{12}S_{21}-S_{11}S_{22}}{2S_{21}}\right). \qquad (9)$$



(a)



(b)

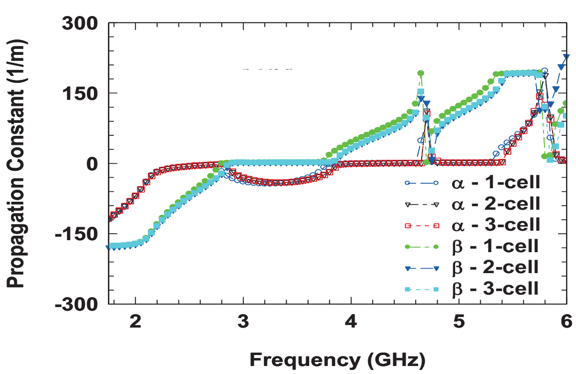


(c)

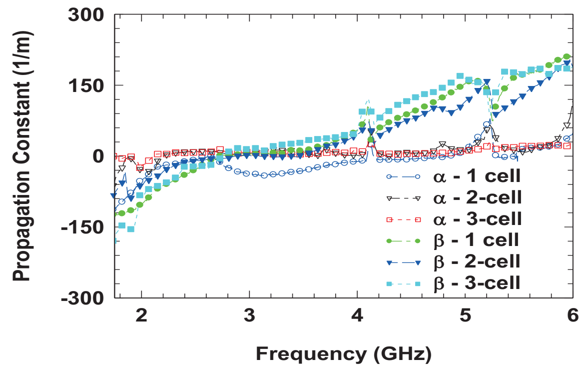
Fig. 3. (a) Current voltage relation (I-V curve) of the Germanium tunneling diode. The differential negative resistance appears at 0.15-0.3 V. (b) Schematic of a designed active CLRH-TL unit cell in microstrip configuration incorporating the resonant tunneling diode. The dimensions are: a=2.5 mm, d=1.5 mm, l=9.7 mm, w=1 mm, s=0.2 mm, l_s=5.9 mm, r=0.3 mm. (c) Photo of a fabricated single unit cell of the active CLRH-TL (figure adapted from [46]).

Figure 4 shows the simulated (Fig. 4 (a)) and measured (Fig. 4 (b)) propagation attenuation constant α and phase constant β, and the extracted material effective index of refraction (Fig. 4 (c)). The simulated results are obtained from the combination of full-wave electromagnetic models of the passive parts of the CLRH-TL and the equivalent circuit model of the tunneling diode. It can be observed from Fig. 4 (a) that for all three cases of one, two and three unit cells, the simulated propagation constants are identical with simultaneous negative β (negative index) and negative α (gain) from 1.75 GHz to 2.75 GHz.

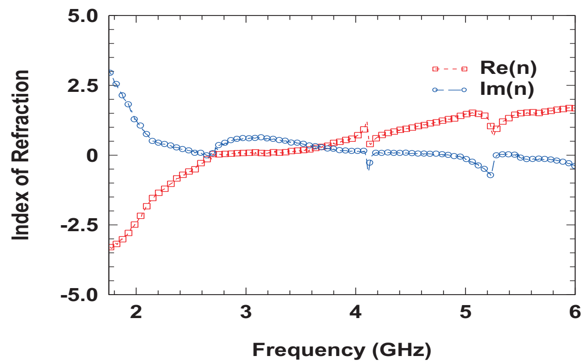
Compared to the passive CLRH-TL without the tunneling diode in [46], this active CLRH-TL is unbalanced with a lower transition frequency due to the extra parasitics of the tunneling diode. It is also observed from Fig. 4 (b), that for the single unit cell case, the measured β is negative from 1.75 GHz to 2.75 GHz while the measured α is also negative, indicating left-handedness with gain in that frequency range. The corresponding effective index of refraction n is calculated and plotted in Fig. 4 (c). From 1.75 GHz to 2.75 GHz, the real and imaginary part of n is negative and positive correspondingly, indicating negative index with gain. For the two- and three-unit cell cases, negative β are observed from 1.75 GHz to 2.75 GHz as well, quite similar to the single unit cell case. Most importantly, simultaneous negative β and negative α are also experimentally confirmed, although in narrower frequency ranges, from 1.75 GHz to 2.2 GHz and from 1.75 GHz to 2.1 GHz for the two- and three-unit cell cases, respectively. The similar behavior of the propagation constants for the one-, two- and three-unit cell cases confirm that this active CLRH-TL can be considered as an effective negative index material with gain. The differences in the measured propagation constants (especially for the attenuation constant α which is due to the negative resistance associated with the tunneling diode [45]) for the three cases are likely due to the non-uniformity of the diodes in each unit cell.



(a)



(b)



(c)

Fig. 4. (a) Simulated and (b) measured propagation constants of the active CLRH-TL incorporating tunneling diodes with one, two and three unit cells; (c) extracted refractive index from the measured one unit cell results with a time dependence convention $e^{j\omega t-\gamma z}$ (figure adapted from [46]).

Another interesting aspect of this active metamaterial transmission line is that the negative resistance value can be controlled by the bias voltage. Figure 5 plots the measured S_{21} of the one-unit cell structure under different bias voltages from 0.21 to 0.25 V. It is observed that with different bias voltage (thus different -R values), the level of loss compensation/gain can be controlled. Besides demonstrating the existence of NIM with gain in a finite bandwidth, the nonlinear power dependence and harmonics generation of the active CLRH-TL is also measured. It is observed that at low input power level (i.e., $P_{in}\leq$-35 dBm) the active CLRH-TL behaves linearly without significant harmonics generation. However, at higher input power, nonlinearity clearly sets in that leads to gain compression and harmonics generation. More detailed results can be found in [46].

It is very important to point out that since all active devices and gain media are inherently nonlinear, it is crucial to consider the nonlinearity and power handling capability in active metamaterial design and associated applications. Furthermore, similar to any other active components design, especially power amplifiers, stability issues of active metamaterials are of critical importance and can be challenging. References [92-94] study the wave propagation in nonlinear composite medium, and experimentally demonstrate the spatiotemporal dynamics of active metamaterials, including the generation and propagation of solitons for the backward-wave regime.

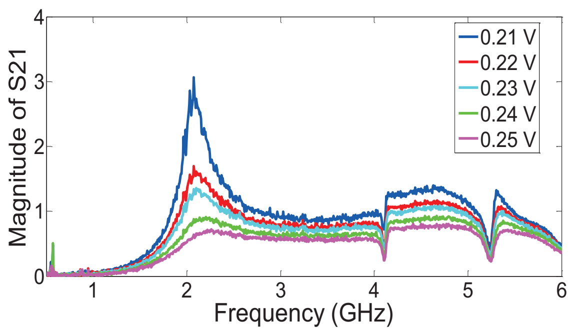


Fig. 5. Measured transmission S_{21} of the one-unit cell active CLRH-TL with the tunneling diode biased from 0.21 to 0.25 V. The level of loss compensation/gain can be controlled by the bias voltage.

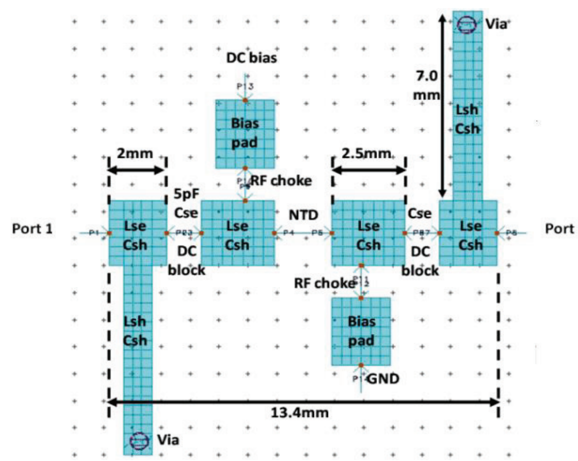
B. A balanced active CLRH-TL metamaterial

As discussed previously, the first active CLRH-TL incorporating tunneling diode is unbalanced due to the parasitics of the diode. In [47], an active CLRH-TL is designed to achieve the following goals which are of practical importance for many applications: balanced response with the diode parasitics taking into account; symmetric unit cell structure (i.e., S_{11}=S_{22}); and to evaluate the effect of number of unit cells on the CLRH-TL properties.

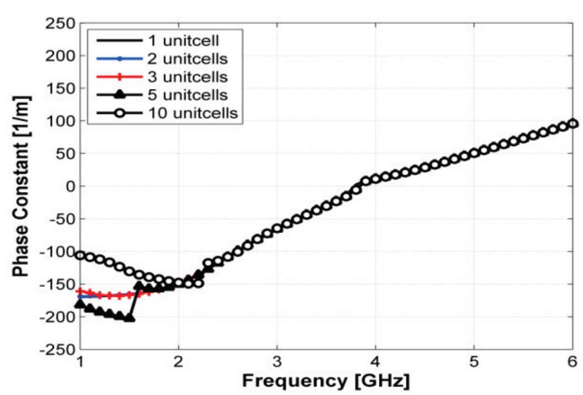
Figure 6 (a) shows the schematic layout of the improved design. Instead of using a series inter-digital capacitor as in [46], two lumped-element capacitors are placed symmetrically in the unit cell. The shunt inductor is also split into two symmetrically while the tunnel diode is placed at the center of the unit cell to maintain symmetry. The values of the equivalent circuit model elements and the associated dimensions are determined to obtain balanced response. According to the equivalent circuit model parameters, the estimated transition frequency is about 3.87 GHz.

Figures 6 (b) and 6 (c) plot the extracted phase constant β and attenuation constant α from simulated S-parameters for different number of unit cells, respectively. It is observed that left-handedness (or NRI) with gain is achieved from 2 to 3.83 GHz, with the transition frequency at 3.83 GHz, consistent with the equivalent circuit model estimation. The propagation properties of the CLRH-TL with different number of unit cells are mostly identical, demonstrating the validity of treating them as uniform transmission lines, or, effective media.

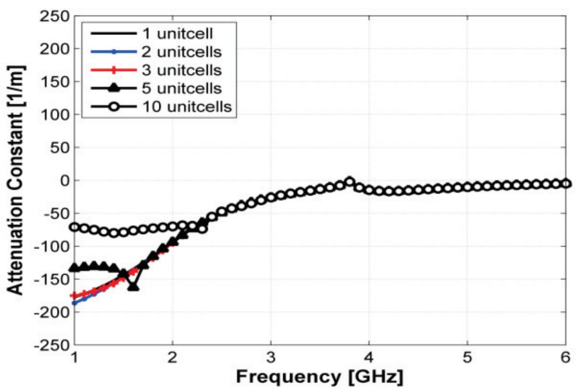
As a potential application, the performance of passive and active versions of a single unit-cell zeroth order resonator antenna based on the CLRH-TL is also compared. The benefit of the active CLRH-TL unit cell is demonstrated in terms of radiated power [47].



(a)



(b)



(c)

Fig. 6. (a) Schematic layout of a balanced and symmetric active CLRH-TL unit-cell design; simulated (b) phase constants β and (c) attenuation constants α of 1, 2, 3, 5 and 10 unit cells of the balanced active CLRH-TL (figure adapted from [47]).

C. Active volumetric metamaterials

The merit of a volumetric metamaterial is its convenience of frequency scaling. For example, SRR structures have been demonstrated from microwave to optical frequencies. In microwave frequency, loss compensation can still be achieved by embedding active devices such as tunneling diodes into the conventional sub-wavelength metamaterial unit cells. An active metamaterial with embedded microwave tunnel diodes exhibits a band-limited Lorentzian dispersion with an over-compensated loss (gain) and a negative refractive index is demonstrated in Fig. 7 [79]. It shows examples of sub-wavelength wire and SRR cells with embedded tunneling diode. This kind of active metamaterial design is very interesting because of its versatility. For example, by incorporating the NDR diode in the wire (permittivity)/SRR (permeability) part of the unit cell, activeness associated with the permittivity/permeability can be selected independently. Therefore, various model systems can be realized to investigate the rich physics related to active metamaterials.

Besides NIMs, other types of metamaterials incorporating active devices have also been studied. Reference [44] experimentally demonstrates an active SRR metamaterial unit cell with a control of different combinations of the real part and imaginary part of the permeability, which is achieved by embedding surface mount amplifiers, voltage controlled phase shifters, and a pair of sensing and driven loops. Despite of the "bulky" and complicated active loads, the unit length is still much smaller (~ $\lambda/10$) than the wavelength, taking advantages of the integrated circuit technology. The results show that $\mu' < 1$ is achieved near 590 MHz, and μ'' can be tuned from positive to negative values including zero by controlling the bias voltage of the phase shifter.

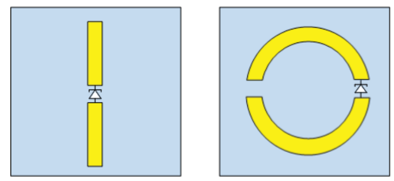


Fig. 7. A schematic example of a NDR diode-loaded metamaterial unit cell to compensate loss or provide gain while maintaining its negative permittivity (left) or negative permeability (right).

D. Optical gain-assisted metamaterials

At optical frequencies, NIMs are usually built from noble metal such as silver or gold. The material loss is severe, which plagues their potential applications in optics. Other known sources of loss in optical metamaterials stem from surface roughness, size effect, quantum effect and chemical interface effects. There are a number of reported theoretical studies with various gain models which predicts drastic improvement on loss compensation [95-97]. Reference [58] uses the Maxwell-Bloch methodology and transformed Poynting's theorem to predict that a steady-state net gain can be achieved when the pumped gain is bigger than the dissipation loss and smaller than the sum of dissipation and radiation loss. If there was no radiation loss, the two thresholds coincide, hence, suggesting only lasing light but not amplification would be possible. Techniques investigated experimentally include optically pumped gain media such as organic dyes, quantum wells and quantum dots, and nonlinear optical parametric amplification [52-58,81-83].

Reference [56] demonstrates an extremely low-loss optical NIM using organic dye molecules pumped by a laser pulse. A delicate "double-fishnet" structure is fabricated from two layers of silver with air or solvent as the spacer and alumina pillars as support. Epoxy doped with Rhodamine 800 dye material is coated on the structure afterwards. The sample is at first characterized by transmission and reflection measurements in the far field. The negative refractive index is obtained from 720 nm to 760 nm. A pump-probe experiment is then conducted to show a progressively increased transmission of the sample with pumped laser compared to the case without the pump, which proves that the loss compensation is provided by the dye material. The associated Figure-of-Merit (FOM = Re{n}/Im{n}) is increased from 1 to 26 with the pump wavelength of 737 nm, and to the order of 10^6 with the pump wavelength of 738 nm (Re{n}=-1.26 and Im{n}=1×10^{-6}), but the whole system still remains in lossy state.

Another useful technique to compensate the loss is to employ semiconductor gain materials, such as quantum dots and quantum wells, into the passive structures [80-82]. The advantage of using semiconductor gain materials is the long life time and stable performance compared to the dye materials. Reference [81] reports a gain-assisted magnetic metamaterial based on an array of SRRs fabricated on an InGaAs quantum well layer, pumped with 810 nm laser pulse. The transmission of the quantum well with SRRs changes substantially larger than the case of the quantum well alone, which indicates a strong local-field coupling between SRRs and the quantum well.

Although there have been many reports on optical active metamaterials for loss compensation, it is worth to point out that, to our knowledge, there has not been experimental demonstration of NIM with steady state net gain so far. This can probably be attributed to the challenges in fabrication requirements at optical wavelengths.

V. ACTIVE METAMATERIALS FOR DISPERSION CONTROL

Non-dispersive metamaterials (or more precisely, metamaterials with broadband response) have received increasing attentions in microwave regime in recent years [59-64]. Reference [35] firstly predicts the existence of non-dispersive metamaterials with active inclusions in 2001. The active inclusions involve the use of negative capacitance or inductance based on active feedback devices, which does not obey the Foster's reactance theorem (Fig. 8), therefore, named as non-Foster elements. Non-Foster elements can be realized by transistors, operational amplifiers, as well as negative resistance devices [65,66]. Although there are many different circuit configurations, the underlying mechanism is likely the same. That is all of them are using some positive feedback systems, therefore, the circuit is easily unstable. One common non-Foster configuration is based on Linvill's circuit, which is a pair of cross-coupled transistors, as shown in Fig. 9. The circuit has the same configuration as a typical oscillator; however, the only difference is non-Foster element is operating at its stable region.

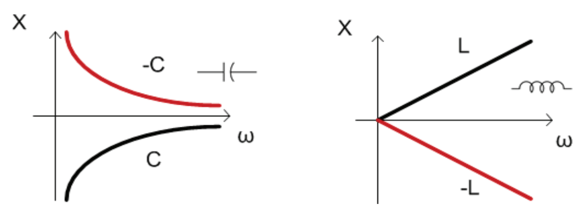


Fig. 8. The reactance behavior of negative capacitance and negative inductance compared with normal positive capacitance and inductance obeying Foster's reactance theorem.

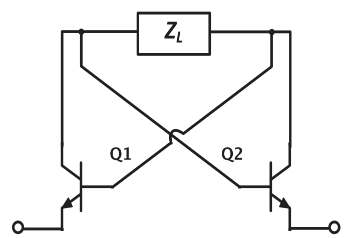


Fig. 9. A schematic of a negative impedance circuit example based on Linvill's configuration. The output impedance approximates to be Z_{in}=-Z_L.

Reference [60] experimentally demonstrates a transmission line type of a one-dimensional epsilon-near-zero metamaterial with negative capacitance, as shown in Fig. 10 (a). The measurement results verify the broadband behavior of the relative permittivity (from 0.27 to 0.37) within the frequency range of 2 to 40 MHz, spanning a bandwidth of 20:1.

A typical transmission line type of one-dimensional epsilon-near-zero metamaterial comprises a transmission line periodically loaded with shunt inductances, of which the effective permittivity can be expressed as:

$$\varepsilon_r(\omega) = \frac{1}{\varepsilon_0}(C_p - \frac{1}{\omega^2 L_p \Delta z}), \qquad (10)$$

where C_p is the distributed shunt capacitance per unit length and L_p is the shunt inductance per unit length. A broadband epsilon-near-zero material is designed by paralleling negative capacitance $-C_N$, so that:

$$\varepsilon_r(\omega) = \frac{1}{\varepsilon_0}(C_p - \frac{C_N}{\Delta z}) . \quad (11)$$

Therefore, the dispersion in the effective permittivity can be cancelled within a broad bandwidth.

Figure 10 (b) shows a schematic of another type of non-Foster metamaterials SRR with a negative inductance load. An approximated expression of effective permeability of the split-ring resonator without the negative inductance load is given as [90]:

$$\mu_{\text{eff}} = 1 - \frac{F}{1 - 1/\omega^2 LC + \mathrm{i}(R/\omega L)}, \quad (12)$$

where F is a factor related to the dimensions of the unit cell structure. If the gap reactance $-1/\omega C$ is replaced by a negative inductance $-\omega L_N$, the frequency dependency of the effective permeability would be cancelled (assuming negligible resistance and parasitics), yielding a broadband negative permeability medium,

$$\mu_{\text{eff}} = 1 - \frac{F}{1 - L_N/L + \mathrm{i}(R/\omega L)} \overset{R\to 0}{\approx} 1 - \frac{F}{1 - L_N/L} . \quad (13)$$

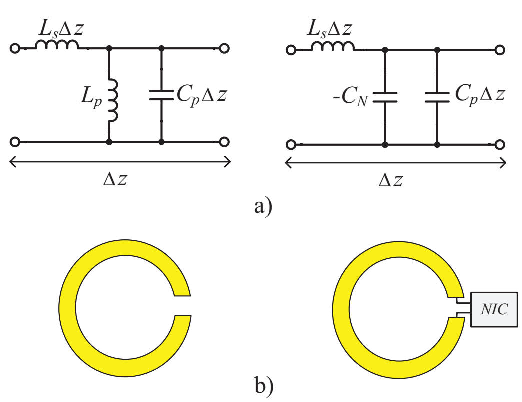


Fig. 10. (a) (left) An equivalent circuit model of a unit cell of a conventional epsilon-near-zero metamaterial transmission line, and (right) a broadband epsilon-near-zero metamaterial transmission line by replacing the periodic loaded inductance with a parallel negative capacitance [60]; (b) (left) a prototype of a SRR metamaterial with negative permeability, and (right) a scheme of a broadband SRR by embedding a negative impedance circuit at the gap.

VI. STABILITY ISSUE

Stability is one of the most challenging aspects in the design of active NIM incorporating devices or media with gain. For simplification, we consider an ideal infinite large system without boundary conditions. Two types of instability may exist in such an active system, namely absolute instability and convective instability. Absolute instability means that the EM fields blow up with time at each point of the space, while in the case of convective instability the fields at each point do not blow up with time but field grows along the propagation in space [98]. If the analytical function of the constitutive material properties are given, the medium is considered to be absolutely instable if $\varepsilon(\omega)\mu(\omega)$ contains poles or odd-order zeros in the upper half side of the complex plane. If an accurate equivalent circuit model of an active metamaterial was given, circuit stability analysis methods, for example, the normalized determinant function method [99] can be implemented. For "non-Foster" metamaterials, the internal stability of the active load (e.g., negative capacitance and inductance) as well as the overall stability of the whole system needs to be examined [100,101]. For example, a Linvill's negative impedance circuit may be internally unstable due to the feedback loop of the circuit configuration, and it highly depends on the load impedance. The parasitics of the non-Foster elements also have a severe impact on the stability and the performance. Therefore, a thoughtful design must include the consideration of the operating frequency, the cutoff frequency of the active devices, bias conditions and many others.

VII. SUMMARY AND OUTLOOK

There has been significant progress on the research of active metamaterial recently, including the understanding of fundamental limitations, fabrication and experiment techniques. Active metamaterials will not only pave the way for realizing low loss and broadband metamaterials and enabling many proposed electromagnetic applications such as "perfect" lens, "cloaking", and electrically small antennas, but also may lead to new and exciting physical insights and phenomena beyond the realm of passive metamaterials. Various challenges such as stability and nonlinearity are still currently being addressed. Further development of active metamaterials greatly depends on the advances in fabrication as well as the application demands.

REFERENCES

[1] R. W. Ziolkowski and N. Engheta, "Metamaterial special issue introduction," *IEEE Trans. Antennas and Propag*., 51, pp. 2546, October 2003.

[2] V. G. Veselago, "The electrodynamics of substances with simultaneously negative values of ε and μ," *Soviet Physics Uspekhi*, 10, 4, pp. 509-514, 1968.

[3] R. A. Shelby, D. R. Smith, and S. Schultz "Experimental verification of a negative index of refraction," *Science*, 292, pp. 77, April 2001.

[4] A. Alu, et al., "Single-negative, double-negative, and low-index metamaterials and their electromagnetic applications," *IEEE Antennas Propag. Mag*., 49, pp. 23, 2007.

[5] S. Enoch, "A metamaterial for direct emission," *Phys. Rev. Lett*., 76, pp. 213902, November 2002.

[6] D. Schurig, et al., "Metamaterial electromagnetic cloak at microwave frequencies," *Science*, 314, pp. 977, November 2006.

[7] A. K. Iyer and G. V. Eleftheriades, "Negative refractive index metamaterials supporting 2-D waves," *IEEE MTT-S IMS Digest*, 2, pp. 412, 2002.

[8] C. Caloz and T. Itoh, "Application of the transmission line theory of left-handed (LH) materials to the realization of a microstrip LH transmission line," *IEEE Antenna and Propagation Symp*., vol. 2, pp. 415, 2002.

[9] A. A. Oliner, "A periodic-structure negative-refractive-index medium without resonant elements," *IEEE Antenna and Propagation Symp*., pp. 41, 2002.

[10] C. Caloz and T. Itoh, "Metamaterials for high-frequency electronics," *Proceedings of IEEE*, 93, pp. 1744, October 2005.

[11] G. V. Eleftheriades, "Enabling RF/microwave devices using negative-refractive-index transmission line (NRI-TL) metamaterials," *IEEE Antennas Prop. Mag*., 49, pp. 34, April 2007.

[12] A. Alù and N. Engheta, "Achieving transparency with plasmonic and metamaterial coatings," *Phys. Rev. E*, vol. 72, no. 1, p. 016623, July 2005.

[13] J. B. Pendry, D. Schurig, and D. R. Smith, "Controlling electromagnetic fields," *Science*, vol. 312, no. 5781, pp. 1780-2, June 2006.

[14] D. Schurig, J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, "Metamaterial electromagnetic cloak at microwave frequencies," *Science*, vol. 314, no. 5801, pp. 977-80, November 2006.

[15] J. B. Pendry, "Negative refraction makes a perfect lens," *Phys. Rev. Lett*., vol. 85, no. 18, pp. 3966-3969, October 2000.

[16] N. Fang, H. Lee, C. Sun, and X. Zhang, "Sub-diffraction-limited optical imaging with a silver superlens," *Science*, vol. 308, no. 5721, pp. 534-7, April 2005.

[17] A. V Kabashin, P. Evans, S. Pastkovsky, W. Hendren, G. A. Wurtz, R. Atkinson, R. Pollard, V. A. Podolskiy, and A. V Zayats, "Plasmonic nanorod metamaterials for biosensing," *Nat. Mater*., vol. 8, no. 11, pp. 867-71, November 2009.

[18] N. Liu, T. Weiss, M. Mesch, L. Langguth, U. Eigenthaler, M. Hirscher, C. Sönnichsen, and H. Giessen, "Planar metamaterial analogue of electromagnetically induced transparency for plasmonic sensing," *Nano Lett*., vol. 10, no. 4, pp. 1103-7, April 2010.

[19] R. W. Ziolkowski and A. D. Kipple, "Application of double negative materials to increase the power radiated by electrically small antennas," *IEEE Trans. Antennas Propag*., vol. 51, no. 10, pp. 2626-2640, October 2003.

[20] S. Enoch, "A metamaterial for direct emission," *Phys. Rev. Lett*., 76, pp. 213902, November 2002.

[21] G. Lovat, et al., "Analysis of directive radiation from a line source in a metamaterial slab with low permittivity," *IEEE Trans. Antennas Propag*., 54, pp. 1017, 2006.

[22] A. Grbic and G. V. Eleftheriades, "Experimental verification of backward-wave radiation from a negative refractive index metamaterial," *J. Appl. Phys*., vol. 92, no. 10, p. 5930, October 2002.

[23] J. B. Pendry and S. A. Ramakrishna, "Refining the perfect lens," *Physica B: Condensed Matter*, vol. 338, pp. 329-332, 2003.

[24] L. D. Landau, L. P. Pitaevskii, and E. M. Lifshitz, "Electrodynamics of continuous media," *Butterworth-Heinemann*, 1979.

[25] J. D. Jackson, "*Classical electrodynamics third edition*," *Wiley*, 1998.

[26] J. Skaar and K. Seip, "Bounds for the refractive indices of metamaterials," *J. Phys. D. Appl. Phys*., vol. 39, no. 6, pp. 1226-1229, March 2006.

[27] V. P. Drachev, U. K. Chettiar, A. V. Kildishev, H. K. Yuan, W. Cai, and V. M. Shalaev, "The ag dielectric function in plasmonic metamaterials," *Opt. Express*, vol. 16, no. 2, p. 1186, 2008.

[28] P. Tassin, T. Koschny, M. Kafesaki, and C. M. Soukoulis, "A comparison of graphene, superconductors and metals as conductors for metamaterials and plasmonics," *Nat. Photonics*, vol. 6, no. 4, pp. 259-264, March 2012.

[29] K. L. Tsakmakidis, M. S. Wartak, J. J. H. Cook, J. M. Hamm, and O. Hess, "Negative-permeability electromagnetically induced transparent and magnetically active metamaterials," *Phys. Rev. B*, vol. 81, no. 19, p. 195128, May 2010.

[30] D. Güney, T. Koschny, and C. Soukoulis, "Reducing ohmic losses in metamaterials by geometric tailoring," *Phys. Rev. B*, vol. 80, no. 12, p. 125129, September 2009.

[31] L. Zhu, F. Y. Meng, L. Dong, J. H. Fu, and Q. Wu, "Low-loss magnetic metamaterial at THz frequencies by suppressing radiation losses," *IEEE Trans. Terahertz Sci. Technol*., vol. 3, no. 6, pp. 805-811, November 2013.

[32] J. A. Bossard, S. Yun, D. H. Werner, and T. S. Mayer, "Synthesizing low loss negative index metamaterial stacks for the mid-infrared using genetic algorithms," *Opt. Express*, vol. 17, no. 17, p. 14771, August 2009.

[33] D. J. Bergman and M. I. Stockman, "Surface plasmon amplification by stimulated emission of radiation: quantum generation of coherent surface plasmons in nanosystems," *Phys. Rev. Lett*., vol. 90, p. 027402, 2003.

[34] S. A. Ramakrishna and J. B. Pendry, "Removal of absorption and increase in resolution in a near-field lens via optical gain," *Phys. Rev. B*, vol. 67, no. 20, p. 201101, May 2003.

[35] S. A. Tretyakov, "Meta-materials with wideband negative permittivity and permeability," *Microw. Opt. Technol. Lett*., vol. 31, no. 3, pp. 163-165, November 2001.

[36] N. I. Zheludev, "Applied physics: the road ahead for metamaterials," *Science*, vol. 328, no. 5978, pp. 582-3, April 2010.

[37] D. F. Sievenpiper, J. H. Schaffner, H. J. Song, R. Y. Loo, and G. Tangonan, "Two-dimensional beam steering using an electrically tunable impedance surface," *IEEE Trans. Antennas Propag*., vol. 51, no. 10, pp. 2713-2722, October 2003.

[38] H. Xin, J. B. West, J. C. Mather, J. P. Doane, J. A. Higgins, H. Kazemi, and M. J. Rosker, "A two-dimensional millimeter wave phase scanned lens utilizing analog electromagnetic crystal (EMXT) waveguide phase shifters," *IEEE Trans. Antennas Propag*., vol. 53, no. 1, pp. 151-159, January 2005.

[39] H. T. Chen, W. J. Padilla, J. M. O. Zide, A. C. Gossard, A. J. Taylor, and R. D. Averitt, "Active terahertz metamaterial devices," *Nature*, vol. 444, no. 7119, pp. 597-600, November 2006.

[40] I. V. Shadrivov, S. K. Morrison, and Y. S. Kivshar, "Tunable split-ring resonators for nonlinear negative-index metamaterials," *Opt. Express*, vol. 14, no. 20, p. 9344, 2006.

[41] H. T. Chen, W. J. Padilla, M. J. Cich, A. K. Azad, R. D. Averitt, and A. J. Taylor, "A metamaterial solid-state terahertz phase modulator," *Nature Photonics*, vol. 3. pp. 148-151, 2009.

[42] Y. S. Kivshar, "Tunable and nonlinear metamaterials: toward functional metadevices," *Adv. Nat. Sci. Nanosci. Nanotechnol*., vol. 5, no. 1, p. 013001, November 2013.

[43] B. I. Popa and S. A. Cummer, "Nonreciprocal active metamaterials," *Phys. Rev. B*, vol. 85, no. 20, p. 205101, May 2012.

[44] Y. Yuan, B. I. Popa, and S. A. Cummer, "Zero loss magnetic metamaterials using powered active unit cells," *Opt. Express*, vol. 17, no. 18, pp. 16135-43, August 2009.

[45] L. M. Si, T. Jiang, K. Chang, T. C. Chen, X. Lv, L. Ran, and H. Xin, "Active microwave metamaterials incorporating ideal gain devices," *Materials*, vol. 4, no. 1, pp. 73-83, December. 2010.

[46] T. Jiang, K. Chang, L. M. Si, L. Ran, and H. Xin, "Active microwave negative-index metamaterial transmission line with gain," *Phys. Rev. Lett*., vol. 107, no. 20, p. 205503, November 2011.

[47] K. Chang, T. Jiang, L. Ran, and H. Xin, "Investigation of microwave negative refractive index (NRI) transmission lines incorporating tunnel diodes," *IEEE Antennas Wirel. Propag. Lett*., vol. 11, pp. 671-674, 2012.

[48] W. Xu, W. J. Padilla, and S. Sonkusale, "Loss compensation in metamaterials through embedding of active transistor based negative differential resistance circuits," *Opt. Express*, vol. 20, no. 20, pp. 22406-11, September 2012.

[49] E. W. Herold, "Negative resistance and devices for obtaining it," *Proc. IRE*, vol. 23, no. 10, pp. 1201-1223, October 1935.

[50] A. Cidronali, V. Nair, G. Collodi, J. H. Lewis, M. Camprini, G. Manes, and H. Goronkin, "MMIC applications of heterostructure interband tunnel devices," *IEEE Trans. Microw. Theory Tech*., vol. 51, no. 4, pp. 1351-1367, April 2003.

[51] M. Premaratne and G. P. Agrawal, "Light propagation in gain media," *Cambridge University Press New York*, 2011.

[52] A. K. Popov and V. M. Shalaev, "Compensating losses in negative-index metamaterials by optical parametric amplification," *Opt. Lett*., vol. 31, no. 14, p. 2169, 2006.

[53] M. A. Noginov, G. Zhu, M. Bahoura, J. Adegoke, C. E. Small, B. A. Ritzo, V. P. Drachev, and V. M. Shalaev, "Enhancement of surface plasmons in an ag aggregate by optical gain in a dielectric medium," *Opt. Lett*., vol. 31, no. 20, p. 3022, 2006.

[54] M. Noginov, G. Zhu, M. Mayy, B. Ritzo, N. Noginova, and V. Podolskiy, "Stimulated emission of surface plasmon polaritons," *Phys. Rev. Lett*., vol. 101, no. 22, p. 226806, November 2008.

[55] N. I. Zheludev, S. L. Prosvirnin, N. Papasimakis, and V. A. Fedotov, "Lasing spaser," *Nat. Photonics*, vol. 2, no. 6, pp. 351-354, May 2008.

[56] S. Xiao, V. P. Drachev, A V Kildishev, X. Ni, U. K. Chettiar, H. K. Yuan, and V. M. Shalaev, "Loss-free and active optical negative-index

metamaterials," *Nature*, vol. 466, no. 7307, pp. 735-8, August 2010.

[57] A. D. Boardman, V. V. Grimalsky, Y. S. Kivshar, S. V. Koshevaya, M. Lapine, N. M. Litchinitser, V. N. Malnev, M. Noginov, Y. G. Rapoport, and V. M. Shalaev, "Active and tunable metamaterials," *Laser Photon. Rev*., vol. 5, no. 2, pp. 287-307, March 2011.

[58] O. Hess, J. B. Pendry, S. A. Maier, R. F. Oulton, J. M. Hamm, and K. L. Tsakmakidis, "Active nanoplasmonic metamaterials," *Nat. Mater*., vol. 11, no. 7, pp. 573-84, July 2012.

[59] S. Hrabar, I. Krois, I. Bonic, and A. Kiricenko, "Negative capacitor paves the way to ultra-broadband metamaterials," *Appl. Phys. Lett*., vol. 99, no. 25, p. 254103, 2011.

[60] S. Hrabar, I. Krois, I. Bonic, and A. Kiricenko, "Ultra-broadband simultaneous superluminal phase and group velocities in non-foster epsilon-near-zero metamaterial," *Appl. Phys. Lett*., vol. 102, no. 5, p. 054108, 2013.

[61] S. Saadat, M. Adnan, H. Mosallaei, and E. Afshari, "Composite metamaterial and metasurface integrated with non-foster active circuit elements: a bandwidth-enhancement investigation," *IEEE Trans. Antennas Propag*., vol. 61, no. 3, pp. 1210-1218, March 2013.

[62] M. Barbuto, A. Monti, F. Bilotti, and A. Toscano, "Design of a non-foster actively loaded SRR and application in metamaterial-inspired components," *IEEE Trans. Antennas Propag*., vol. 61, no. 3, pp. 1219-1227, March 2013.

[63] Y. Fan, K. Z. Rajab, and Y. Hao, "Noise analysis of broadband active metamaterials with non-foster loads," *J. Appl. Phys*., vol. 113, no. 23, p. 233905, June 2013.

[64] P. Y. Chen, C. Argyropoulos, A. Alu, and A. Alù, "Broadening the cloaking bandwidth with non-foster metasurfaces," *Phys. Rev. Lett*., vol. 111, no. 23, p. 233001, December 2013.

[65] L. C. Verman, "Negative circuit constants," *Proc. IRE*, vol. 19, no. 4, pp. 676-681, April 1931.

[66] J. G. Linvill, "Transistor negative-impedance converters," *Proc. IRE*, vol. 41, no. 6, pp. 725-729, June 1953.

[67] S. E. Sussman-Fort and R. M. Rudish, "Non-foster impedance matching of electrically-small antennas," *IEEE Trans. Antennas Propag*., vol. 57, no. 8, pp. 2230-2241, August 2009.

[68] R. W. Ziolkowski, "Broad-bandwidth, electrically small antenna augmented with an internal non-foster element," *IEEE Antennas Wirel. Propag. Lett*., vol. 11, pp. 1116-1120, 2012.

[69] C. R. White, J. S. Colburn, and R. G. Nagele, "A non-foster VHF monopole antenna," *IEEE Antennas Wirel. Propag. Lett*., vol. 11, pp. 584-587, 2012.

[70] S. Tretyakov and S. Maslovski, "Veselago materials: what is possible and impossible about the dispersion of the constitutive parameters," *IEEE Antennas Propag. Mag*., vol. 49, no. 1, pp. 37-43, February 2007.

[71] M. Stockman, "Criterion for negative refraction with low optical losses from a fundamental principle of causality," *Phys. Rev. Lett*., vol. 98, no. 17, p. 177404, April 2007.

[72] T. Mackay and A. Lakhtakia, "Comment on 'criterion for negative refraction with low optical losses from a fundamental principle of causality'," *Phys. Rev. Lett.*, vol. 99, no. 18, p. 189701, November 2007.

[73] J. Skaar, "Causality and its implications for passive and active media," *URSI International Symp. Electromag. Theory, EMTS*, Berlin, 2010.

[74] P. Kinsler and M. McCall, "Causality-based criteria for a negative refractive index must be used with care," *Phys. Rev. Lett*., vol. 101, no. 16, p. 167401, October 2008.

[75] M. Gustafsson and D. Sjöberg, "Sum rules and physical bounds on passive metamaterials," *New J. Phys*., vol. 12, no. 4, p. 043046, April 2010.

[76] M. I. Stockman, "Spaser action, loss compensation, and stability in plasmonic systems with gain," *Phys. Rev. Lett*., vol. 106, no. 15, p. 156802, April 2011.

[77] S. Wuestner, A. Pusch, K. L. Tsakmakidis, J. M. Hamm, and O. Hess, "Comment on 'spaser action, loss compensation, and stability in plasmonic systems with gain'," *Phys. Rev. Lett*., vol. 107, no. 25, p. 259701, December 2011.

[78] J. B. Pendry and S. A. Maier, "Comment on 'spaser action, loss compensation, and stability in plasmonic systems with gain'," *Phys. Rev. Lett*., vol. 107, no. 25, p. 259703, December 2011.

[79] D. Ye, K. Chang, L. Ran, and H. Xin "Volumetric microwave gain medium with negative refractive index," *under review*.

[80] V. V. Varadan and R. Ro, "Analyticity, causality, energy conservation and the sign of the imaginary part of the permittivity and permeability," *In 2006 IEEE Antennas Propag. Society Intern. Symp*., pp. 499-502, 2006.

[81] N. Meinzer, M. Ruther, S. Linden, C. M. Soukoulis, G. Khitrova, J. Hendrickson, J. D. Olitzky, H. M. Gibbs, and M. Wegener, "Arrays of ag split-ring resonators coupled to InGaAs single-quantum-well gain," *Opt. Express*, vol. 18, no. 23, pp. 24140-51, November 2010.

[82] K. Tanaka, E. Plum, J. Y. Ou, T. Uchino, and N. I. Zheludev, "Multifold enhancement of quantum dot luminescence in plasmonic metamaterials," *Phys. Rev. Lett*., vol. 105, no. 22, p. 227403, November

2010.

[83] E. Plum, V. A. Fedotov, P. Kuo, D. P. Tsai, and N. I. Zheludev, "Towards the lasing spaser: controlling metamaterial optical response with semiconductor quantum dots," *Opt. Express*, vol. 17, no. 10, p. 8548, May 2009.

[84] G. V. Eleftheriades, A. K. Iyer, and P. C. Kremer, "Planar negative refractive index media using periodically L-C loaded transmission lines," *IEEE Trans. Microw. Theory Tech*., vol. 50, no. 12, pp. 2702-2712, December 2002.

[85] A. Lai, C. Caloz, and T. Itoh, "Composite right/left-handed transmission line metamaterials," *IEEE Microw. Mag*., vol. 5, no. 3, pp. 34-50, September 2004.

[86] N. Engheta, "Circuits with light at nanoscales: optical nanocircuits inspired by metamaterials," *Science*, vol. 317, no. 5845, pp. 1698-702, September 2007.

[87] S. Paulotto, P. Baccarelli, F. Frezza, and D. R. Jackson, "Full-wave modal dispersion analysis and broadside optimization for a class of microstrip CLRH leaky-wave antennas," *IEEE Trans. Microw. Theory Tech.*, vol. 56, no. 12, pp. 2826-2837, December 2008.

[88] Q. Tang, F. Y. Meng, Q. Wu, and J. C. Lee, "A balanced composite backward and forward compact waveguide based on resonant metamaterials," *J. Appl. Phys*., vol. 109, no. 7, p. 07A319, April 2011.

[89] C. Caloz, "Perspectives on EM metamaterials," *Mater. Today*, vol. 12, no. 3, pp. 12-20, 2009.

[90] H. Chen, L. Ran, J. Huangfu, T. M. Grzegorczyk, and J. A. Kong, "Equivalent circuit model for left-handed metamaterials," *J. Appl. Phys*., vol. 100, no. 2, p. 024915, July 2006.

[91] F. P. Casares-Miranda, C. Camacho-Penalosa, and C. Caloz, "High-gain active composite right/left-handed leaky-wave antenna," *IEEE Trans. Antennas Propag*., vol. 54, no. 8, pp. 2292-2300, August 2006.

[92] A. B. Kozyrev, I. V. Shadrivov, and Y. S. Kivshar, "Soliton generation in active nonlinear metamaterials," *Appl. Phys. Lett*., vol. 104, 084105, 2014.

[93] N. L. Tsitsas, A. Lakhtakia, and D. J. Frantzeskakis, "Solitons in a homogenized two-phase, isotropic, nonlinear, particulate composite medium," *J. Opt. Soc. Am. B*, vol. 29, no. 9, pp. 2610-2616, September 2012.

[94] N. L. Tsitsas, "Analysis of electromagnetic wave propagation in frequency bands of nonlinear metamaterials," *Applied Computational Electromagnetics Society Journal*, vol. 27, no. 2, pp. 169-180, February 2012.

[95] M. Wegener, J. L. García-Pomar, C. M. Soukoulis, N. Meinzer, M. Ruther, and S. Linden, "Toy model for plasmonic metamaterial resonances coupled to two-level system gain," *Opt. Express*, vol. 16, pp. 19785-19798, 2008.

[96] A. Fang, T. Koschny, and C. M. Soukoulis, "Self-consistent calculations of loss-compensated fishnet metamaterials," *Phys. Rev. B*, vol. 82, no. 12, p. 121102, September 2010.

[97] Y. Sivan, S. Xiao, U. K. Chettiar, A. V Kildishev, and V. M. Shalaev, "Frequency-domain simulations of a negative-index material with embedded gain," *Opt. Express*, vol. 17, pp. 24060-24074, 2009.

[98] A. D. Boardman, Y. G. Rapoport, N. King, and V. N. Malnev, "Creating stable gain in active metamaterials," *J. Opt. Soc. Am. B*, vol. 24, no. 10, p. A53, 2007.

[99] A. P. A. Platzker, W. S. W. Struble, and K. T. H. K. T. Hetzler, "Instabilities diagnosis and the role of K in microwave circuits," *1993 IEEE MTT-S Int. Microw. Symp. Dig*., 1993.

[100] K. Z. Rajab, Y. Hao, D. Bao, C. G. Parini, J. Vazquez, and M. Philippakis, "Stability of active magnetoinductive metamaterials," *J. Appl. Phys*., vol. 108, no. 5, p. 054904, 2010.

[101] K. Z. Rajab, Y. F. Fan, and Y. Hao, "Characterization of active metamaterials based on negative impedance converters," *J. Opt*., vol. 14, no. 11, p. 114004, November 2012.

Qi Tang received his B.S. degree in Communication Engineering, in 2009, and his M.S. degree in Microwave Engineering, in 2011, from Harbin Institute of Technology, Harbin, China. He is currently working towards his Ph.D. degree with the Electrical and Computer Engineering department, University of Arizona, Tucson, AZ, USA. His doctoral research focuses on metamaterials, microwave circuits, antennas, THz technologies, and millimeter wave wireless communication systems.

Hao Xin Professor of Electrical and Computer Engineering at the University of Arizona. He was named an Arizona Engineering fellow in Aug. 2013. He received his Ph.D in Physics from Massachusetts Institute of Technology in February 2001.

From 2000 to 2003, he was a Research Scientist with the Rockwell Scientific Company. He was a Sr. Principal Multidisciplinary Engineer with Raytheon Company from 2003 to 2005.

His primary research interests are in the area of microwave/millimeter wave/THz devices, circuits, antennas and their applications in wireless communication and sensing systems. His recent research activities have covered a broad range of high frequency technologies, including applications of new technologies and materials in microwave and millimeter wave circuits such as electromagnetic band gap crystals and other meta-materials, carbon nano-tubes devices, solid state devices and circuits, active or semi-active antennas, and passive circuits. He has authored over 200 referred publications and 14 patents (12 issued and 2 pending) in the areas of microwave and millimeter-wave technologies, random power harvesting based on ferro-fluidic nano-particles and carbon nanotube based devices. He is a senior member of IEEE and Chair of the joint chapter of IEEE AP/MTT/EMC/COMM in Tucson AZ. He is a General Co-Chair of the 8th International Workshop on Antenna Technology. He also serves as an Associate Editor for IEEE Antennas and Wireless Propagation Letters.