Incorporation of The Nihility Medium to Improve The Cylindrical Invisibility Cloak

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Abstract **—** Two dimensional cylindrical wave expansion method is used to analyze the cylindrical invisible nihility cloak. Ideal model of the cloak is set up to solve the problem and it is observed that a perfect equivalence with the ideal situation of the cloak can be achieved by using nihility medium at perturbed void region of the geometry. Also it is observed that the use of nihility medium at δ, the convergence rate is independent of the type of incident field. Both the transverse electric (TE) and transverse magnetic (TM) cases have been discussed.

Index Terms – Convergence, nihility, transverse electric, transverse magnetic, and wave expansion.

I. INTRODUCTION

Cloaks have attracted attentions of many researchers in the scientific community due to its amazing characteristics. Scientists have already developed such devices, which can render things invisible to electromagnetic waves. Metamaterials used for making such devices are required to have negative refraction index and are known as left-handed (LH) metamaterials. Metamaterials are a class of artificially engineered composite materials having extraordinary electromagnetic properties. Negative refraction index material has been discussed by many scientists [1-4]. Pendry is pioneer of the cloaking concept and Pendry et al., [5] first illustrated through theoretical simulations, that an object can be cloaked from electromagnetic fields by exploiting coordinate transformation in inhomogeneous and anisotropic metamaterials. Cummer et al., analyzed the full-wave simulations of electromagnetic cloaking structures [6]. They worked on the electromagnetic simulations of the cylindrical version of this cloaking structure using ideal and non-ideal electromagnetic parameters. They showed that the low reflections and power flow banding properties of electromagnetic cloaking structures are not especially sensible to modest permittivity and permeability variation. Schuring et al., described metamaterial electromagnetic cloak at microwave frequencies [7], which was the first practical realization of such a cloak. Ruan et al., [8] confirmed that a cloak with ideal material parameters in a perfect invisibility cloak by symmetrically studying the scattering coefficient from the near ideal case to the ideal one. Yan et al., explored important scattering characteristics of cylindrical invisibility cloak [9-12]. They described that cylindrical cloak having simplified material parameter inherently allowing the zeroth-order cylindrical wave to pass through the cloak as if the cloak is made of a homogenous isotropic medium and thus visible to all higher-order cylindrical waves. Their numerical simulation suggests that the simplified cloak inherits some properties of the ideal cloak, but some scattering exist. Greenleaf et al., worked on the improvement of cylindrical cloak with soft and hard surfaces (SHS) lining [13]. They showed that the cloak is significantly improved by the use of SHS lining with both the far field of the scattering wave significantly reduced and the blow up of electric field density and magnetic field density prevented. Leonhardt discussed optical conformal mapping in [14]. Shahzad et al., analyzed the cylindrical invisibility cloak incorporating PEMC

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at perturbed void region in [15]. A Green's function approach to calculate scattering width for cylindrical cloaks has been discussed by J. S. McGuirk [16].

The idea of nihility medium is given by Lakhtakia [17], and has received a lot of interest within the electromagnetic community. Nihility medium is a medium in which both relative permittivity and permeability are null-valued. This idea engaged many researchers in studying problem related to electromagnetics [18-24]. Due to this fact the medium does not allow the electromagnetic energy to propagate in it. For nihility condition Maxwell's equations have the form as,

$$\nabla \times \mathbf{E} = 0 \tag{1}$$

$$\nabla \times \mathbf{H} = 0. \tag{2}$$

Metamaterial with both negative permittivity and permeability, also known as backward (BW) medium or double negative (DNG) medium has been extensively studied. Scattering of electromagnetic plane waves by a nihility cylinder coated with different types of metamaterials is investigated by Ahmed et al., [23-24].

In the current work an invisibility cloak termed as cylindrical invisibility cloak incorporating nihility as core is discussed. Our main focus is to explore the scattering characteristics of the cylindrical invisible nihility cloak and to improve the convergence rate of the scattered fields in free space because in nihility case the zeroth-order contribution of Bessel functions changes to first-order for n = 0, thus decreasing the scattered fields.

The next sections deal with the formulation of the problem. Using the properties of the nihility medium, conclusions about the better cloak are drawn analytically. We have used e^{-jwt} time dependence, which is suppressed throughout the analysis.

II. ANALYTICAL FORMULATION

The geometry of 2-D cylindrical nihility cloak is shown in Figure 1. Invisibility cloak has been constructed to compress electromagnetic fields in a cylindrical region r' < b into concentric cylindrical shell a < r < b. The region outside radius b is termed as region 0, which is free space with k_0 as wave number and η_0 as impedance, which are given by

$$k_0 = w\sqrt{\mu_0 \epsilon_0} \;\; , \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \; .$$

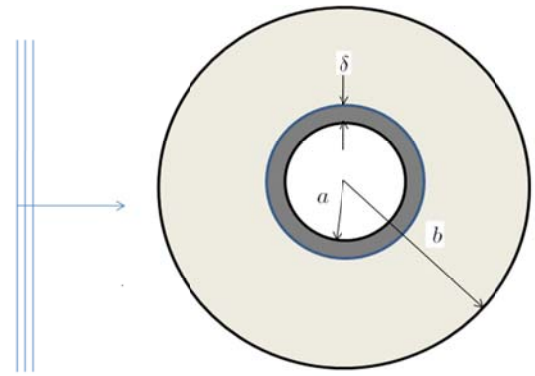

Figure1. Geometry of an invisible nihility cloak.

The region between a + δ < r < b is termed as region 1, which is composed of unknown medium. Radii of inner and outer cylinders are 'a' and 'b' respectively. 'δ' is the tiny perturbation. The region inside radius r < a + δ is region 2, which is assumed to be a dielectric medium at first instance with k_2 as wave number and η_2 as impedance, which are given as

$$k_2 = w\sqrt{\mu_2 \epsilon_2} \;\; , \eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} \; .$$

Consider the coordinate transformation in cylindrical coordinates [5] as,

$$f(r) = r' = a + \frac{r(b-a)}{b} \; ,$$

with the characteristics f (a) = 0 and f (b) =b, while θ and z are kept unchanged. Following the above coordinate transformation method [5], the permittivity and permeability of the cloak region may be obtained as,

$$\epsilon_r = \mu_r = \frac{f(r)}{rf'(r)}$$

$$\epsilon_z = \mu_z = \frac{f(r)f\prime(r)}{r} \, ,$$

$$\epsilon_\theta = \mu_\theta = \frac{f(r)f\prime(r)}{f(r)} \, ,$$

where the superscript ' denotes differentiation with respect to argument.

A. TE case

Consider the case when a TE wave is incident on the cloak from free space and it is assumed that the core of the cloak is composed of dielectric medium. The total electromagnetic field for the region r > b can be written as,

$$H_z = \sum_{n=-\infty}^{\infty} J_n (k_0 r) e^{jn(\theta)} + a_n H_n (k_0 r) e^{jn(\theta)} \tag{3}$$

$$E_\theta = \frac{k_0}{jw\epsilon_0} \sum_{n=-\infty}^{\infty} J_n' (k_0 r) e^{jn(\theta)} + a_n H_n' (k_0 r) e^{jn(\theta)} , \tag{4}$$

with $J_n(.)$, $H_n(.)$ are the Bessel and Hankel functions, respectively and a_n is the scattering coefficient in the region r>b. The scattering coefficients are required to be considered because region 1 is a dielectric medium at first instance and at the boundary of region 0 and region 1 some part of the fields should be reflected and some should be transmitted. The cloak having no scattering is termed as ideal one and the cloak having small scattering is nearest to ideal situation. That is the reason of considering these scattering coefficients. The total electromagnetic fields for region a<r< b are given as,

$$H_z = \sum_{n=-\infty}^{\infty} b_n J_n (k_1(r-a)) e^{jn(\theta)} + c_n H_n (k_1(r-a)) e^{jn(\theta)} \tag{5}$$

$$E_\theta = \frac{k_1}{jw\epsilon_\theta(r)} \sum_{n=-\infty}^{\infty} b_n J_n' (k_1(r-a)) e^{jn(\theta)} + c_n H_n'(k_1(r-a)) e^{jn(\theta)}, \tag{6}$$

with $k_1 = k_0 b/(b - a)$ and b_n, c_n as transmission and reflection coefficients, respectively.The fields in the region r<a are,

$$H_z = \sum_{n=-\infty}^{\infty} d_n J_n (k_2 r) e^{jn(\theta)} \tag{7}$$

$$E_\theta = \frac{k_2}{jw\epsilon_2} \sum_{n=-\infty}^{\infty} d_n J_n' (k_2 r) e^{jn(\theta)}, \tag{8}$$

where d_n is the transmission coefficient and k_2 is the wave number for region r<a.

The boundary conditions require that the tangential components of **E** and **H** fields to be continues across the interfaces at r = b and r = a + δ. After the application of the boundary conditions and utilizing the concept of impedance matching discussed in [11,15], we can find the unknown scattering coefficient a_n in free space and is given below,

$$a_n = \frac{k_2 J_n(k_1\delta) - \frac{k_1}{\epsilon_\theta(a+\delta)} L_n(k_0(a+\delta)) J_n'(k_0(a+\delta))}{\frac{k_1}{\epsilon_\theta(a+\delta)} L_n(k_0(a+\delta)) H_n'(k_0(a+\delta)) - k_2 H_n(k_1\delta)} \tag{9}$$

where

$$L_n(\chi) = \frac{J_n(\chi)}{J_n'(\chi)} . \tag{10}$$

B. TM case

Now considering the case where a TM wave is incident on the cloak from free space. The total electromagnetic field for the region r > b can be written as,

$$E_z = \sum_{n=-\infty}^{\infty} J_n (k_0 r) e^{jn(\theta)} + a_n' H_n (k_0 r) e^{jn(\theta)} \tag{11}$$

$$H_\theta = \frac{-k_0}{jw\mu_0} \sum_{n=-\infty}^{\infty} J_n' (k_0 r) e^{jn(\theta)} + a_n' H_n'(k_0 r) e^{jn(\theta)}. \tag{12}$$

The total electromagnetic field for the region a<r < b are given as,

$$E_z = \sum_{n=-\infty}^{\infty} b_n' J_n (k_1(r-a)) e^{jn(\theta)} + c_n' H_n (k_1(r-a)) e^{jn(\theta)} \tag{13}$$

$$H_\theta = \frac{-k_1}{jw\mu_\theta(r)} \sum_{n=-\infty}^{\infty} b_n' J_n' (k_1(r-a)) e^{jn(\theta)} + c_n' H_n'(k_1(r-a)) e^{jn(\theta)}. \tag{14}$$

The fields in the region r<a are,

$$E_z = \sum_{n=-\infty}^{\infty} d_n' J_n (k_2 r) e^{jn(\theta)} \tag{15}$$

$$H_\theta = \frac{-k_2}{jw\mu_2} \sum_{n=-\infty}^{\infty} d_n' (k_2 r) e^{jn(\theta)} , \tag{16}$$

Where a'_n, b'_n, c'_n, and d'_n are the scattering coefficients for TM case. These coefficients can be found by applying appropriate boundary conditions at r = b and r = a + δ. The scattering coefficients a'_n in free space can be written as,

$$a_n' = \frac{k_2 J_n(k_1\delta) - A * J_n'(k_0(a+\delta))}{A * H_n'(k_0(a+\delta)) - k_2 H_n(k_1\delta)} \tag{17}$$

Where $A = \frac{k_1}{\mu_\theta(a+\delta)} L_n(k_0(a+\delta))$.

C. Limiting procedure for nihility cloak

Now, the refractive index of nihility must benull-valuedbecause$\varepsilon_2 = \mu_2 = 0$as discussed by Lakhtahia in [17]. For the function $L_n(X)$ we have,

$$\lim_{\chi\to 0} \chi\, L_0(\chi) = -2, \quad n = 0 \tag{18}$$

$$\lim_{\chi\to 0} \chi^{-1} L_0(\chi) = n^{-1}, \quad n \neq 0 . \tag{19}$$

Therefore, after taking the limit $k_2 \to 0$, equations (9) and (17) for a nihility cloak simplify to,

$$a_0 = a_n' = \frac{J_1(k_0(a+\delta))}{H_1(k_0(a+\delta))}, \quad n = 0 \tag{20}$$

$$a_n = a_n' = \frac{J_{|n|}(k_0(a+\delta))}{H_{|n|}(k_0(a+\delta))}, \ n \neq 0. \tag{21}$$

From the results given in the above equations, it is clear that the nihility core ($\varepsilon_2 = \mu_2 \to 0$) is the void region because no fields exist in this region.

III. NUMERICAL RESULTS AND DISCUSSION

In this section some numerical results based on the proposed geometry are presented for cylindrical invisible cloak with nihility layer at δ. Wave number of the free space is taken as k_0=0.064/π. The radius of the inner boundary is taken as a =1.2π/k_0 and radius of outer boundary is b = 2π/k_0, while the range of tiny perturbation δ is taken as 10^{-8}a < δ < 10^{-2}a.

Figure 2 shows the scattering coefficient for n = 0. The result is compared with the published literature and found to be much better. In Fig. 2 the dotted red line is the nihility case while the solid blue line is the case without PEC [12]. The scattering coefficient approaches 0.1 at δ =10^{-8} while for the case of nihility approaches 10^{-19} at δ =10^{-8}. Hence better convergence rate is observed in case of nihility core. From the study of previous literature [9, 12, 15], it has been noted that better cloak may be achieved using PEC core for TM polarization and PMC core for TE polarization. In these combinations, for each case, zeroth scattering coefficient is function of first order cylindrical wave functions. This is due to the fact that nihility will have a better impedance matching with free space. Figure 3 shows the scattering coefficient for n = 0 and the result is compared with the case when PEC layer is incorporated at δ [12]. This result is found in good agreement with [12].

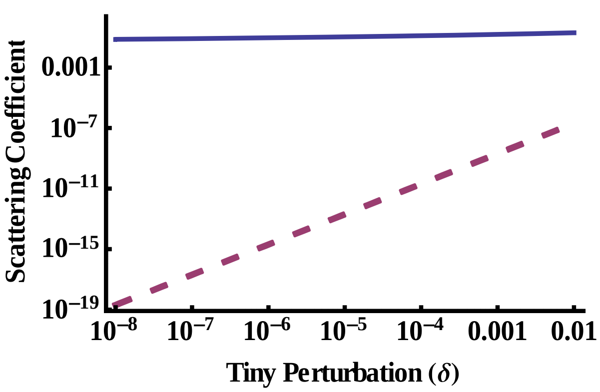

Fig. 2. Scattering coefficients for n = 0 with a=1.2π/k0 and b=2π/k0. Blue line is non PEC case [12] and dotted red line is nihility case.

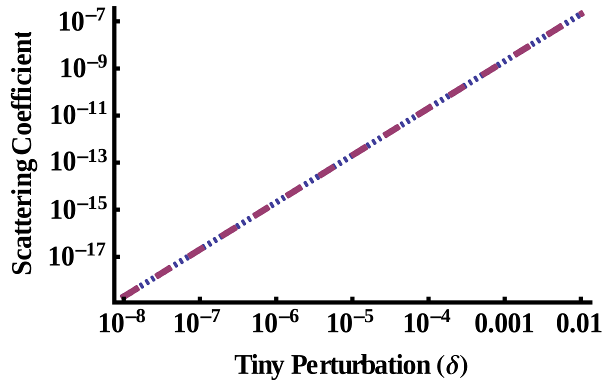

Fig. 3. Scattering coefficients for n=0 with a=1.2π/k0 and b=2π/k0. Dotted blue line is PEC case [12] and dashed red line is nihility case.

Figures 4 and 5 show the scattering coefficients for n = 1 and n = 2, respectively. These results are compared with the plots presented in [12] and found to be in good agreement.

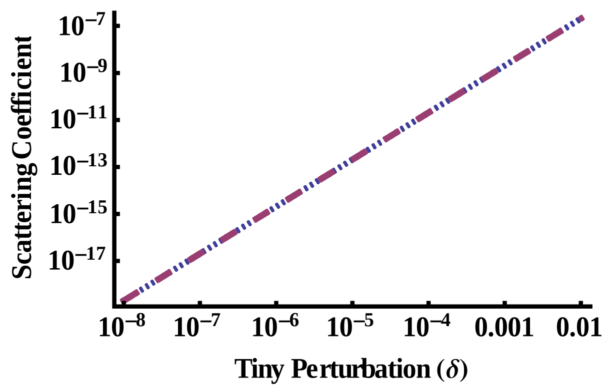

Fig. 4. Scattering coefficients for n=1 with a=1.2π/$k_0$and b=2π/k_0. Dottedblue line is PEC case [12] and dashed red line is nihility case.

For the case of the nihility core, same coefficients are obtained for TE and TM case as given in equation (21). This implies that these coefficients are independent of the polarization of the incident field, which is not possible for the case when PEC or PMC are used as core reference [12]. The main advantage of using nihility is nearest ideal situation, which is very powerful characteristic of the cloak because in nihility case the zeroth-order contribution of Bessel functions changes to first-order for n = 0 (equation (20)), thus decreasing the scattered fields.

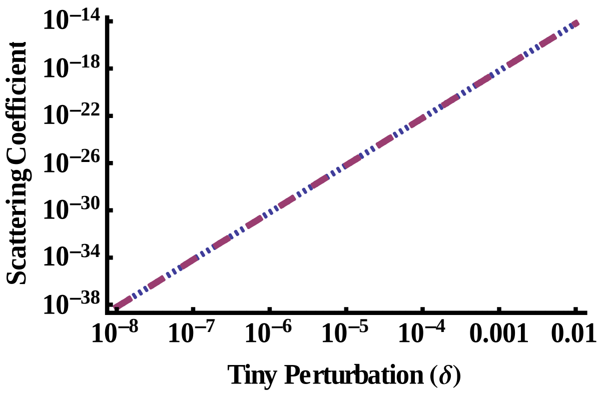

Fig. 5. Scattering coefficients for n=2 with a=1.2π/k0 and b=2π/k0. Dotted blue line is PEC case [12] and dashed red line is nihility case.

IV. CONCLUSIONS

From the analytical calculations done for both the TM and TE cases we can draw following conclusions:

i. It is shown that using nihility at δ layer o the geometry we can make cloak equal to the ideal situation.
ii. In case of nihility, the scattering coefficients are independent of the type of incident field.

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