Analytical Solution for Line Source Excitation of a PEMC Cylinder Coated with Multilayer Anisotropic Media

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Abstract **─** An analytical solution is presented for line source excitation of a cylinder with perfect electromagnetic conductor (PEMC) core which is coated with multilayer anisotropic materials. Exact solution is presented using analytical relations for scattering from cylindrical structures. For multilayer structures, we have to calculate the inverse of a matrix which is sparse, thus, we use recursive relations to calculate the fields. A recursive relation for solving the problem with PEMC boundary condition is presented. Finally, some examples are given using this method and the results are compared and validated with the simulation results and other works. The advantage of analytical relation proposed in this paper is much less run time compared to numerical simulation.

Index Terms **─** Line source, multilayer structures, perfect electromagnetic conductor, scattering RCS, special materials/anisotropic.

I. INTRODUCTION

Perfect electromagnetic conductor which was introduced by Lindell and Sihvola [3] is described using the following relations:

$$\vec{H} + M\vec{E} = 0, \tag{1}$$

$$\vec{D} - M\vec{B} = 0, \tag{2}$$

where M is the admittance of PEMC: a more general form of PEC or PMC, which can be simplified to show these boundary conditions.

PEMC is an ideal boundary, since it can be shown that pointing vector has only an imaginary part and no real part; thus, no real power flows into PEMC and it can be described as:

$$\hat{n} \times (\vec{H} + M\vec{E}) = 0. \tag{3}$$

PEMC boundary condition has been widely studied in several works from realization to a variety of scattering problems to applications. In references [6-8], the realization of PEMC boundary condition has been conducted. A grounded ferrite can be designed to have a Faraday rotation and thus show PEMC boundary condition. In [9-19], scattering problems containing PEMC boundary condition have been worked on. Reflection and transmission obliquely incident plane wave at the interface of a PEMC was considered in [9]. Analytical relations for scattering from a PEMC sphere and cylinder have been considered in [10] and [11], respectively. Scattering from a PEMC cylinder coated with single metamaterial layer was considered in [12]. Also, scattering from a PEMC cylinder buried in a dielectric half space was investigated in [13]. Scattering from two PEMC cylinders was investigated in [14] using iterative methods. In [15], a transformation method was introduced to solve the problems containing PEMC boundary condition and those involving PEMC objects in the air were treated. In [16], an extension to PEMC was introduced as "good electromagnetic conductor". Plane electromagnetic wave propagation in PEMC was considered in [17]. In [18] and [19] a PEMC cylindrical reflector has been studied and high frequency expressions were used.

Electromagnetic scattering from stratified media is the subject of many research and scientific articles [21]. By using stratified media, we can obtain some advantageous like wideband operation of our structure [22] and increased degree of freedom. Specifically, scattering analysis of anisotropic stratified medium is the subject of interest in many works [23-24]. For example, in [24], a stratified anisotropic medium was used to achieve invisibility. Therefore, it is interesting to investigate the problem of multilayer anisotropic coat with different cores including PEMC core. Solution of electromagnetic scattering from an anisotropic cylinder is studied in many resources. One of which is [20].

Most of the PEMC problems have concentrated on single PEMC core or single layer coated PEMC. For example, in [4], a single anisotropic layer coating a PEMC cylinder was considered and scattering of plane wave was investigates through this structure. Not many papers have investigated the scattering problem of a multilayer structure containing PEMC core. In this paper, this problem is considered for the cylindrical geometry with a simple method.

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Organization of the paper is as follows: In the first section, formulation of the problem is stated, general form of the field in each region is introduced, boundary conditions are presented and the problem of sparseness of a matrix which should be inverted to solve the problem is issued.

In the next section, recursive relations are introduced to solve the problem of sparseness of that matrix mentioned in the previous paragraph. The last section presents the results.

II. FORMULATION OF THE PROBLEM

In this section, we investigate the analysis of the above stated problem. Geometry of the problem is shown in Fig. 1. The permittivity and permeability tensors of the problem can be shown as:

$$\overline{\overline{\varepsilon}} = \begin{bmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix}_{\rho\phi z}, \quad \overline{\overline{\mu}} = \begin{bmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{bmatrix}_{\rho\phi z} \quad (4)$$

The form of incident electric and magnetic field for TM polarization is:

$$E_z^i = -\frac{\omega\mu_0}{4}\begin{cases} \sum_{n=-\infty}^{\infty} J_n(k_0\rho) H_n^2(k_0\rho') e^{jn(\phi-\phi')} & \rho<\rho', \\ \sum_{n=-\infty}^{\infty} J_n(k_0\rho') H_n^2(k_0\rho) e^{jn(\phi-\phi')} & \rho>\rho', \end{cases} \quad (5)$$

$$H_\phi^i = \frac{-k_0}{4j}\begin{cases} \sum_{n=-\infty}^{\infty} H_n^{(2)}(k_0\rho') J_n'(k_0\rho) e^{jn(\phi-\phi')} & \rho<\rho', \\ \sum_{n=-\infty}^{\infty} J_n(k_0\rho') H_n^{(2)\prime}(k_0\rho) e^{jn(\phi-\phi')} & \rho>\rho'. \end{cases} \quad (6)$$

And the form of scattered field would be:

$$E_z^s = -\frac{\omega\mu_0}{4}\sum_{n=-\infty}^{\infty} a_n H_n^{(2)}(k_0\rho) e^{jn(\phi-\phi')}, \quad (7)$$

$$\mathrm{H}_z^s = \frac{-k_0}{4j}\sum_{n=-\infty}^{\infty} a_n H_n^{(2)\prime}(k_0\rho) e^{jn(\phi-\phi')}. \quad (8)$$

Also, the form of field distribution in each anisotropic layer would be:

$$E_z^{(i)} = -\frac{\omega\mu_0}{4}\sum_{n=-\infty}^{\infty} [c_n^i H_{\upsilon ei}^{(1)}(u_i^\rho) + d_n^i H_{\upsilon ei}^{(2)}(u_i^\rho)]\; e^{jn(\phi-\phi')}, \quad (9)$$

$$\mathrm{H}_\phi^{(i)} = \frac{-k_0}{4j} Y_{ei}\sum_{n=-\infty}^{\infty} [c_n^i H_{\upsilon ei}^{(1)\prime}(u_i^\rho) + d_n^i H_{\upsilon ei}^{(2)\prime}(u_i^\rho)]\; e^{jn(\phi-\phi')}, \quad (10)$$

$$\upsilon ei = n\sqrt{\frac{\mu_{2i}}{\mu_{1i}}},\; \mathrm{u}_i^\rho = k_0\rho\sqrt{\varepsilon_{3i}\mu_{2i}},\; \mathrm{Y}_{ei} = \sqrt{\frac{\varepsilon_{3i}}{\mu_{2i}}}. \quad (11a\text{-}c)$$

And the field distribution for cross-pol components can be stated as:

$$H_z^s = \frac{-k_0}{4j}\sum_{n=-\infty}^{\infty} b_n H_n^{(2)}(k_0\rho) e^{jn(\phi-\phi')}, \quad (12)$$

$$E_\phi^s = \frac{-\omega\mu_0}{4}\sum_{n=-\infty}^{\infty} b_n H_n^{(2)\prime}(k_0\rho) e^{jn(\phi-\phi')}, \quad (\rho>\rho_N) \quad (13)$$

$$H_z^{(i)} = \frac{-k_0}{4j}\sum_{n=-\infty}^{\infty} [c_{n2}^i H_{\upsilon mi}^{(1)}(u_i^\rho) + d_{n2}^i H_{\upsilon mi}^{(2)}(u_i^\rho)]\ldots e^{jn(\phi-\phi')}. \quad (14)$$

$$E_\phi^{(i)} = \frac{-\omega\mu_0}{4} Z_{mi}\sum_{n=-\infty}^{\infty} [c_{n2}^i H_{\upsilon mi}^{(1)\prime}(u_i^\rho) + d_{n2}^i H_{\upsilon mi}^{(2)\prime}(u_i^\rho)]\; e^{jn(\phi-\phi')}, \quad (15)$$

$$\upsilon mi = n\sqrt{\frac{\varepsilon_{2i}}{\varepsilon_{1i}}},\; \mathrm{Z}_{mi} = \sqrt{\frac{\mu_{3i}}{\varepsilon_{2i}}},\; \mathrm{u}_i^\rho = k_0\rho\sqrt{\varepsilon_{2i}\mu_{3i}}. \quad (16)$$

Here, ρ_N is radius of the N[th] layer.

Now, we should apply boundary conditions to complete the formulation of the problem. Boundary conditions involve the continuity of tangential electric and magnetic fields at the boundary between two layers or between the last layer and free space. Since we have two polarizations, as one original polarization and one cross polarization, we should write boundary conditions for each polarization separately and apply PEMC boundary conditions between the field components of co and cross-polarized components of the first layer. In what follows, we have written boundary conditions in each region separately, the first two equations are for general polarization and the last two equations are for cross polarization. One of the two equations is for the electric field and the other is related to the magnetic field:

$$-\frac{\omega\mu_0}{4}[J_n(k_0\rho_N) H_n^2(k_0\rho') + a_n H_n^{(2)}(k_0\rho_N)] = -\frac{\omega\mu_0}{4}[c_n^N H_{\upsilon eN}^{(1)}(u_N^{\rho_N}) + d_n^N H_{\upsilon eN}^{(2)}(u_N^{\rho_N})], \quad (17)$$

$$\frac{-k_0}{4j}[H_n^{(2)}(k_0\rho') J_n'(k_0\rho_N) + a_n H_n^{(2)\prime}(k_0\rho_N)] = \frac{-k_0}{4j} Y_{eN}[c_n^N H_{\upsilon eN}^{(1)\prime}(u_N^{\rho_N}) + d_n^N H_{\upsilon eN}^{(2)\prime}(u_N^{\rho_N})], \quad (18)$$

$$\frac{-k_0}{4j}[c_{n2}^N H_{\upsilon mN}^{(1)}(u_N^{\rho_N}) + d_{n2}^N H_{\upsilon mN}^{(2)}(u_N^{\rho_N})] = \frac{-k_0}{4j} b_n H_n^{(2)}(k_0\rho_N), \quad (19)$$

$$\frac{-\omega\mu_0}{4} Z_{mN}[c_{n2}^N H_{\upsilon mN}^{(1)\prime}(u_N^{\rho_N}) + d_{n2}^N H_{\upsilon mN}^{(2)\prime}(u_N^{\rho_N})] = \frac{-\omega\mu_0}{4} b_n H_n^{(2)\prime}(k_0\rho_N), \quad (20)$$

$$-\frac{M\omega\mu_0}{4}[c_n^1 H_{\upsilon e1}^{(1)}(u_1^{\rho_1}) + d_n^1 H_{\upsilon e1}^{(2)}(u_1^{\rho_1})] - \frac{k_0}{4j}[c_{n2}^1\, H_{\upsilon m1}^{(1)}(u_1^{\rho_1}) + d_{n2}^1\, H_{\upsilon m1}^{(2)}(u_1^{\rho_1})] = 0, \tag{21}$$

$$\frac{-M\omega\mu_0}{4} Z_{m1}[c_{n2}^1\, H_{\upsilon m1}^{(1)}{}'(u_1^{\rho_1}) + d_{n2}^1\, H_{\upsilon m1}^{(2)}{}'(u_1^{\rho_1})] - \frac{k_0}{4j} Y_{e1}[c_n^1\, H_{\upsilon e1}^{(1)}{}'(u_1^{\rho_1}) + d_n^1\, H_{\upsilon e1}^{(2)}{}'(u_1^{\rho_1})] = 0. \tag{22}$$

Equations (17-20) are boundary conditions between last layer and free space. Equations (21-22) are PEMC boundary conditions.

Boundary conditions between the ith and (i+1)th layers are similar to Equations (17-20) with suitable changes.

After applying the boundary conditions (Equations (17-22)), the formulation of the problem is finished and we should find unknowns of the problem. Result of applying boundary conditions to the problem is a set of 4N linear equations with 4N unknowns. This system may be described as:

$$CX = D, \tag{23}$$

where X shows unknowns of the problem, C is a matrix containing coefficients of unknowns in the set of linear equations, and D is a vector of known values of the problem.

The matrix C becomes sparse. The reason is that each row has only three or four nonzero elements and others are zero. If we show the element of the matrix with A and B with subscripts ± and superscripts i, j then A shows coefficient of electric field boundary condition, B shows coefficient of magnetic field boundary condition, i is the layer in which fields exist, j is the layer boundary whose fields are evaluated, and + and – show direction of propagation of field. Consider a simple case where we do not have cross-polarized components in the problem. Thus, the coefficient matrix becomes as follows:

$$\begin{bmatrix} A_{(N+1)N} & A_{NN}^{+} & A_{NN}^{-} & 0 & 0 & \cdots & 0 \\ B_{(N+1)N} & B_{NN}^{+} & B_{NN}^{-} & 0 & 0 & \cdots & 0 \\ 0 & A_{N(N-1)}^{+} & A_{N(N-1)}^{-} & A_{(N-1)(N-1)}^{+} & A_{(N-1)(N-1)}^{-} & \cdots & 0 \\ 0 & B_{N(N-1)}^{+} & B_{N(N-1)}^{-} & B_{(N-1)(N-1)}^{+} & B_{(N-1)(N-1)}^{-} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 \\ 0 & \cdots & 0 & A_{21}^{+} & A_{21}^{-} & A_{11}^{+} & A_{11}^{-} \\ 0 & \cdots & 0 & B_{21}^{+} & B_{21}^{-} & B_{11}^{+} & B_{11}^{-} \end{bmatrix}. \tag{24}$$

As can be seen, there are so many zeroes in the matrix. The first and second rows have three nonzero elements, other rows have four nonzero elements, and the rest are all zero. For example, for a 3 layer problem, we have 7 unknowns; so, matrix is of order 7. There are 8 zeroes in the first and second rows and each other row has 3 zeroes. Totally, we have 23 zeroes in this matrix and 26 nonzero elements. It is obvious that this matrix is sparse. As the order of the matrix increases, this problem becomes more serious. The problem gets more difficult when we have PEMC boundary condition that causes the existence of cross-polarized components, which adds to the unknowns of the problem by the order of N.

Calculation of inverse of a sparse matrix is a tedious task and we want to prevent this difficulty so we use recursive relations instead which is introduced and formulated in the following section

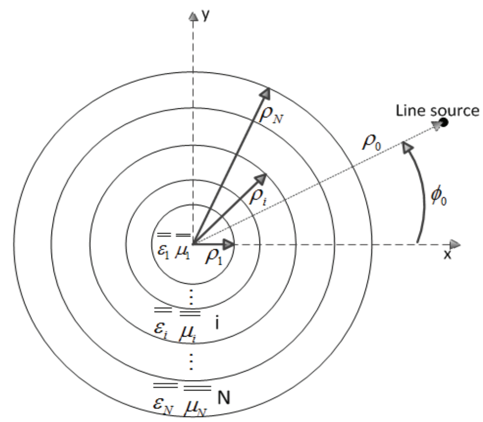


Fig. 1. Geometry of the problem.

III. RECURSIVE RELATIONS

In [2], a layered media was considered and solved using recursive relations. We get this idea and use it for our cylindrically layered media.

According to Equations (19) and (20), we can obtain recursive relations for cross-polarized components as follows:

$$\frac{d_{n2}^N}{c_{n2}^N} = \frac{F_A H_{\upsilon mN}^{(1)}{}'(u_N^{\rho_N}) - H_n^{(2)\prime}(k_0\rho_N) H_{\upsilon mN}^{(1)}(u_N^{\rho_N})}{H_n^{(2)\prime}(k_0\rho_N) H_{\upsilon mN}^{(2)}(u_N^{\rho_N}) - F_A H_{\upsilon mN}^{(2)}{}'(u_N^{\rho_N})}, \tag{25}$$

$$\frac{d_{n2}^i}{c_{n2}^i} = \frac{F_1(\frac{d_{n2}^{i+1}}{c_{n2}^{i+1}}) H_{\upsilon mi}^{(1)}{}'(u_i^{\rho_i}) - F_2(\frac{d_{n2}^{i+1}}{c_{n2}^{i+1}}) H_{\upsilon mi}^{(1)}(u_i^{\rho_i})}{F_2(\frac{d_{n2}^{i+1}}{c_{n2}^{i+1}}) H_{\upsilon mi}^{(2)}(u_i^{\rho_i}) - F_1(\frac{d_{n2}^{i+1}}{c_{n2}^{i+1}}) H_{\upsilon mi}^{(2)}{}'(u_i^{\rho_i})}, \tag{26}$$

$$F_A = Z_{mN} H_n^{(2)}(k_0\rho_N), \tag{27}$$

$$F_1(\frac{d_{n2}^{i+1}}{c_{n2}^{i+1}}) = Z_{mi}[H_{\upsilon m(i+1)}^{(1)}(u_{i+1}^{\rho_i}) + \frac{d_{n2}^{i+1}}{c_{n2}^{i+1}} H_{\upsilon m(i+1)}^{(2)}(u_{i+1}^{\rho_i})], \tag{28}$$

$$F_2(\frac{d_{n2}^{i+1}}{c_{n2}^{i+1}}) = Z_{m(i+1)}[H_{\upsilon m(i+1)}^{(1)}{}'(u_{i+1}^{\rho_i}) + \frac{d_{n2}^{i+1}}{c_{n2}^{i+1}} H_{\upsilon m(i+1)}^{(2)}{}'(u_{i+1}^{\rho_i})]. \tag{29}$$

It is observed that we have a certain known value for d_{n2}^N / c_{n2}^N; thus, we can express $d_{n2}^{N-1} / c_{n2}^{N-1}$ in terms of d_{n2}^N / c_{n2}^N according to 26 and continue it until getting to the first layer and obtain d_{n2}^1 / c_{n2}^1 which is known.

Afterwards, we make a relationship between co and cross-polarized components using the following relation (obtained from PEMC boundary condition):

$$\frac{d_n^1}{c_n^1}=\frac{F_3(\frac{d_{n2}^1}{c_{n2}^1})-(\frac{M\omega\mu_0}{4})^2 Z_{m1} H_{\upsilon e1}^{(1)}(u_1^{\rho_1})F_4(\frac{d_{n2}^1}{c_{n2}^1})}{(\frac{M\omega\mu_0}{4})^2 Z_{m1} H_{\upsilon e1}^{(2)}(u_1^{\rho_1})F_4(\frac{d_{n2}^1}{c_{n2}^1})-F_B F_3(\frac{d_{n2}^1}{c_{n2}^1})}, \quad (30)$$

$$F_B=(\frac{k_0}{4j})^2 Y_{e1} H_{\upsilon e1}^{(1)\prime}(u_2^{\rho_1}), \quad (31)$$

$$F_3(\frac{d_{n2}^1}{c_{n2}^1})=[H_{\upsilon m1}^{(1)}(u_1^{\rho_1})+\frac{d_{n2}^1}{c_{n2}^1}H_{\upsilon m1}^{(2)}(u_1^{\rho_1})], \quad (32)$$

$$F_4(\frac{d_{n2}^1}{c_{n2}^1})=[H_{\upsilon m1}^{(1)\prime}(u_1^{\rho_1})+\frac{d_{n2}^1}{c_{n2}^1}H_{\upsilon m1}^{(2)\prime}(u_1^{\rho_1})]. \quad (33)$$

Now, we move from the first layer to the last layer using recursive relations of co-polarized components of the field. We express d_n^2/c_n^2 in terms of d_n^1/c_n^1 which is known according to Equation (30) and continue to obtain d_n^N/c_n^N . We use the following recursive relation for this task:

$$\frac{d_n^{i+1}}{c_n^{i+1}}=\frac{F_5(\frac{d_n^i}{c_n^i})H_{\upsilon e(i+1)}^{(1)\prime}(u_{i+1}^{\rho_i})-F_6(\frac{d_n^i}{c_n^i})H_{\upsilon e(i+1)}^{(1)}(u_{i+1}^{\rho_i})}{F_6(\frac{d_n^i}{c_n^i})H_{\upsilon e(i+1)}^{(2)}(u_{i+1}^{\rho_i})-F_5(\frac{d_n^i}{c_n^i})H_{\upsilon e(i+1)}^{(2)\prime}(u_{i+1}^{\rho_i})}, \quad (34)$$

$$F_5(\frac{d_n^i}{c_n^i})=Y_{e(i+1)}[H_{\upsilon ei}^{(1)}(u_i^{\rho_i})+H_{\upsilon ei}^{(2)}(u_i^{\rho_i})\frac{d_n^i}{c_n^i}], \quad (35)$$

$$F_6(\frac{d_n^i}{c_n^i})=Y_{ei}[H_{\upsilon ei}^{(1)\prime}(u_i^{\rho_i})+H_{\upsilon ei}^{(2)\prime}(u_i^{\rho_i})\frac{d_n^i}{c_n^i}]. \quad (36)$$

When we obtain d_n^N/c_n^N , we can use Equations (17) and (18) to obtain a_n as follows:

$$a_n=\frac{F_7(\frac{d_n^N}{c_n^N})H_n^{(2)}(k_0\rho')J_n'(k_0\rho_N)-F_8(\frac{d_n^N}{c_n^N})J_n(k_0\rho)H_n^2(k_0\rho')}{F_8(\frac{d_n^N}{c_n^N})H_n^{(2)}(k_0\rho_N)-F_7(\frac{d_n^N}{c_n^N})H_n^{(2)\prime}(k_0\rho_N)}, \quad (37)$$

$$F_7(\frac{d_n^N}{c_n^N})=[H_{\upsilon eN}^{(1)}(u_N^{\rho_N})+H_{\upsilon eN}^{(2)}(u_N^{\rho_N})\frac{d_n^N}{c_n^N}], \quad (38)$$

$$F_8(\frac{d_n^N}{c_n^N})=Y_{eN}[H_{\upsilon eN}^{(1)\prime}(u_N^{\rho_N})+H_{\upsilon eN}^{(2)\prime}(u_N^{\rho_N})\frac{d_n^N}{c_n^N}]. \quad (39)$$

Thus, the problem is solved completely and we know the scattering coefficient and consequently scattering pattern and total field pattern.

Similar relations might be written for TE polarization. We can briefly describe what we did in this part:

We started from the last layer and used recursive relations of cross-polarized components (Equations (25) and (26)) to move through the layers to the first layer. In the first layer, we used PEMC boundary conditions (Equations (21) and (22)) and made a relationship between co- and cross-pol components (Equation (30)). Afterwards, we moved through the layers to the last layer using recursive relations of co-polarized components (Equation (34)) and finally obtained scattering coefficient. Thus, our movement was from the last to the first layers in terms of recursive relations of cross-polarized components and from the first to the last layers in terms of recursive relations of co-polarized components. This issue is schematically illustrated in Fig. 2.

Note that, according to Fig. 2, by starting with b_n, we mean that we use Equation (25) to start moving through the layers and there is no dependence on b_n.

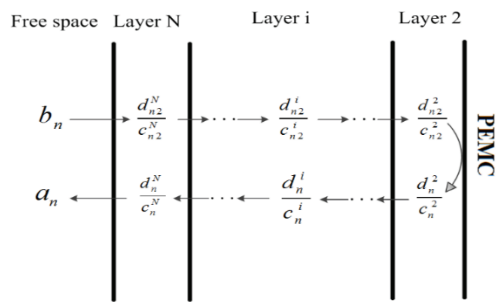


Fig. 2. Movements in layers in order to find scattering coefficient. Layers are cylindrical.

IV. RESULTS

In this section, we are going to illustrate the numerical results of the formulas presented earlier. Some methods are exerted to validate the results. One of them is to compare the results of a simple problem solved using our method with full-wave software like CST microwave studio. A simple problem refers to the one in which the core is PEC and the coat material is isotropic. Thus, we simulate a PEC cylinder coated with several dielectric layers illuminated by plane wave and then observe a near-field pattern. Afterwards, we are sure that our results are true for simple problems and recursive relations are written correctly. Finally, we compare the results with those of some of the previous works for PEMC core.

The result seen in CST is electric in TM polarization case and magnetic field in TE polarization case evaluated on a curve which is a circle concentric with a cylindrical structure.

Therefore, a PEC cylinder coated with 5 dielectric layers illuminated with TM and TE-polarized plane wave is simulated. Comparison is made between our code and CST simulation and then presented in Fig. 3. As can be seen from Fig. 3, comparisons show that our method has great accuracy and is thus reliable. The next step is to compare the results with other works.

In [4], Montaseri et al. solved the problem for one layer case. Here, we compare our results with those of their work. Two general cases are considered: Isotropic and Anisotropic. In each case, co and cross-pol results are compared.

The results in [4] are for the plane wave case. Normalized scattering cross-section with the following form is considered:

$$\frac{\sigma_{co}}{\lambda_0} = \frac{2}{\pi}\left|\sum_{n=-\infty}^{+\infty} a_n e^{jn(\phi-\phi_0)}\right|^2, \tag{40}$$

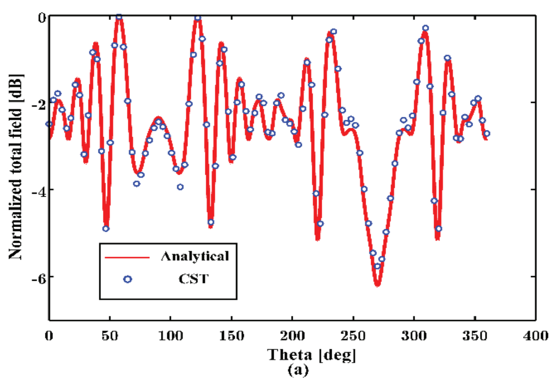
$$\frac{\sigma_{cross}}{\lambda_0} = \frac{2}{\pi}\left|\sum_{n=-\infty}^{+\infty} b_n e^{jn(\phi-\phi_0)}\right|^2, \tag{41}$$

where a_n is scattering coefficient for co-polarized component, b_n is scattering coefficient for cross-polarized coefficient, and φ_0 is angle of plane wave incidence.

In Figs. 4 and 5, isotropic dielectric cylinder coating a PEMC cylinder scattering is evaluated.

In Figs. 6 and 7, isotropic dielectric cylinder coating a PEMC cylinder scattering is evaluated.

Finally, we present the field pattern of line source from a 2-layer anisotropic structure with PEMC in the first layer. It is illuminated with electromagnetic fields of a line source and results are shown in Figs. 8 and 9 for co- and cross-pol, respectively.



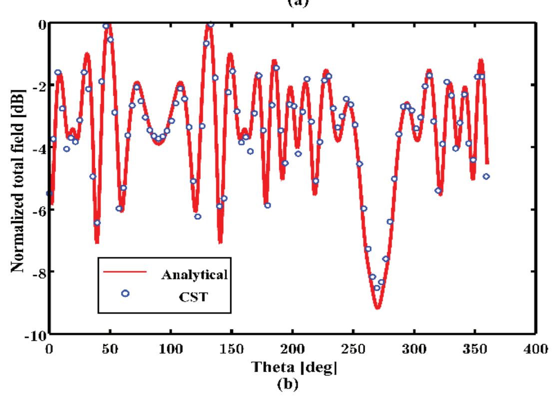


Fig. 3. Comparison between analytical relation stated in the paper and CST full-wave simulation for 5 layer dielectric cylinder with radiuses of layers described as 0.1λ, 0.2λ, 0.3λ, 0.4 λ, 0.5 λ with dielectric constants $\varepsilon_r = 2$, $\varepsilon_r = 3$, $\varepsilon_r = 4$, $\varepsilon_r = 5$, $\varepsilon_r = 6$ illuminated by a plane wave with angle of π/2 relative to x-axis. Near field is observed at: (a) TM polarization and (b) TE polarization.

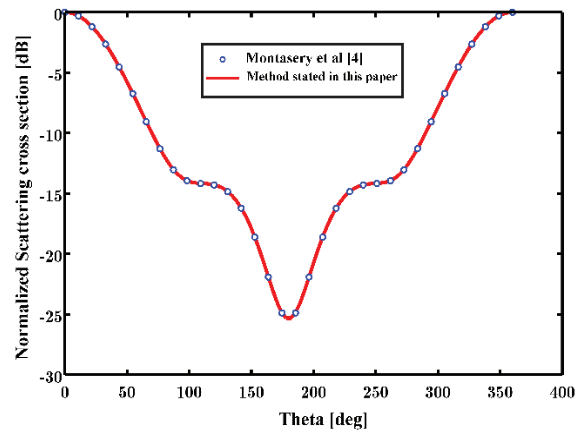


Fig. 4. Normalized co-polarization scattering cross section with parameters $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 9.8$, $\mu_1 = \mu_2 = \mu_3 = 1$, $\varphi_0 = \pi/2$, $M\eta_0 = \pm 1$, $a = \lambda/6$, $b = \lambda/3$.

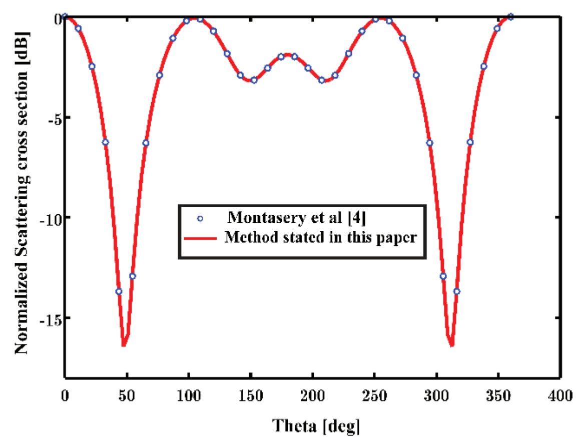


Fig. 5. Normalized cross-polarization scattering cross section with parameters $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 9.8$, $\mu_1 = \mu_2 = \mu_3 = 1$, $\varphi_0 = \pi/2$, $M\eta_0 = \pm 1$, $a = \lambda/6$, $b = \lambda/3$.

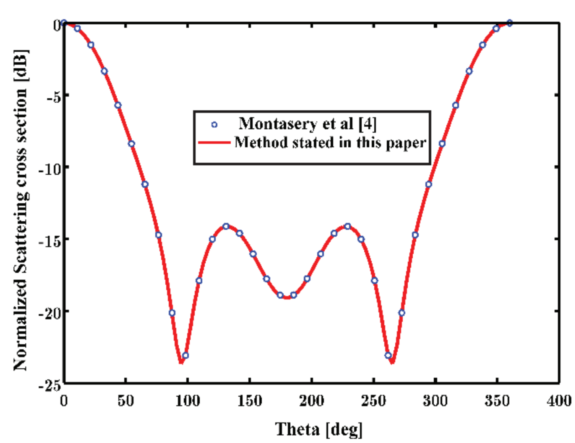


Fig. 6. Normalized co-polarization scattering cross-section with parameters $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 9.8$, $\mu_1 = 1$, $\mu_2 = 19$, $\mu_3 = 6$, $\varphi_0 = \pi/2$, $M\eta_0 = \pm 1$, $a = \lambda/6$, $b = \lambda/3$.

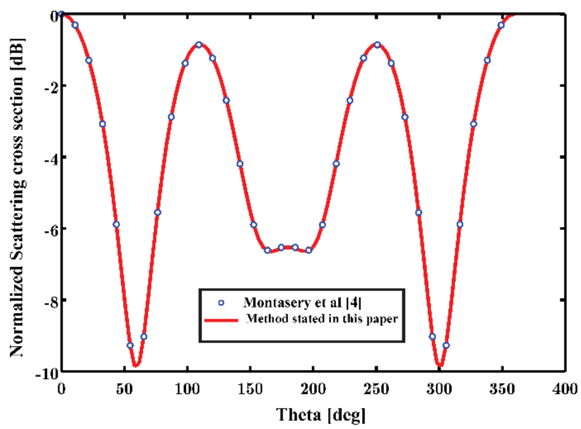


Fig. 7. Normalized cross-polarization scattering cross-section with parameters $\varepsilon_1 = 9.8$, $\varepsilon_2 = 16$, $\varepsilon_3 = 7$, $\mu_1 = 1$, $\mu_2 = 19$, $\mu_3 = 6$, $\varphi_0 = \pi/2$, $M\eta_0 = \pm 1$, $a = \lambda/6$, $b = \lambda/3$.

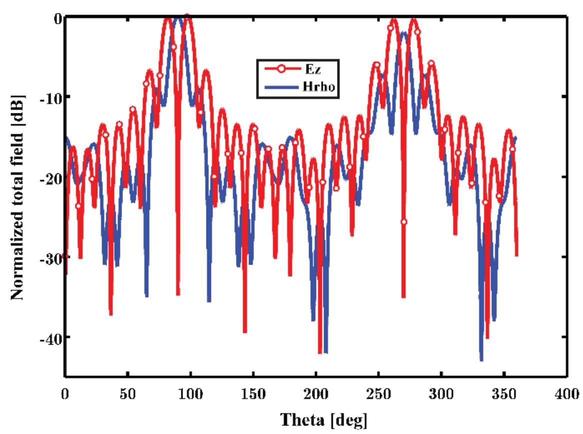


Fig. 8. Co-pol case with parameters described as $\varepsilon_{11} = 6.4$, $\varepsilon_{12} = 9.8$, $\varepsilon_{13} = 5.3$, $\varepsilon_{21} = 7$, $\varepsilon_{22} = 1.1$, $\varepsilon_{23} = 4$, $\mu_{ij} = 1$, $i = 1,2$, $j = 1,2$, $r_0 = \lambda/6$, $r_1 = \lambda/3$, $r_2 = \lambda/2$, $M\eta_0 = \pm 1$, $\rho_0 = 20\lambda$, $\varphi_0 = \pi/2$.

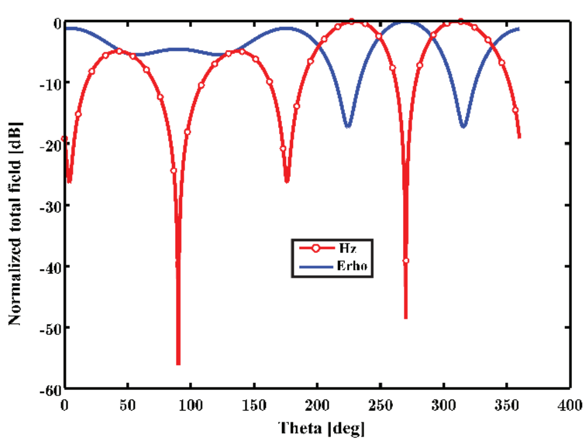


Fig. 9. Cross-pol case for the structure described in Fig 8.

V. CONCLUSION

In this paper, we analyzed line source scattering from a multilayer cylinder with a PEMC boundary condition at the first layer. Formulation of the problem was stated and boundary conditions were obtained. For the multilayer problems, the-inverse of a matrix that was sparse was calculated. Thus, we introduced a novel recursive method for the calculation of scattering coefficient for PEMC boundary condition. Finally, we illustrated the validation of the results using CST microwave studio and comparison with previous works and one case of multilayer problem was evaluated.

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