# Improvement of the Broadside Radiation Pattern of a Conformal Antenna Array using Amplitude Tapering 

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#### Abstract

In this paper, an algorithm for evaluation of near optimal amplitude excitation coefficients for a conformal array on a wedge-shaped surface has been developed in order to maximize the broadside radiation pattern. The radiation pattern for an eight-element conformal antenna has been analyzed and the derived analytical expressions are used to recover the distorted broadside radiation pattern. A heuristic amplitude tapering algorithm is developed to maximize the broadside radiation pattern through the control of individual voltage variable attenuator of each radiating element in the array. It is shown that with appropriate amplitude tapering, the broadside radiation pattern of the array can be improved without phase correction. The pattern recovery information is used to develop a new $1 \times 8$ microstrip patch antenna array bent at certain angles on a single curved non-conducting structure and the measured results are shown to agree well with the analytical solutions.


Index Terms - Adaptive arrays, antenna radiation patterns, conformal antennas, microstrip antenna arrays, phased arrays.

## I. INTRODUCTION

Conformal antennas have attracted attention in many wireless applications that require antennas to be placed on non-planar surfaces [1] and well defined techniques are available to use linear and planar array configurations to achieve a desired radiation pattern [2, 3]. However, these techniques assume a fixed position of the antenna elements. In practice though, the position of the antenna elements may be changed (e.g., bending the antenna array), which results in an unwanted change of the original radiation pattern. Several phase- and amplitude- compensation techniques have been investigated and reported to autonomously recover the distorted radiation pattern of a conformal antenna [4-

7,12] and more recently genetic algorithms for phase compensation [13,16,19,20]. In [14,15], photo-conductive attenuators have been designed for adaptive nulling antenna array. Traditionally for conformal antenna arrays, the designers use both amplitude and phase correction to correct the broadside radiation pattern and to control the sidelobe characteristics to a desired level, see for example [5-7]. Since additional phase shifters and attenuators are required to implement these compensation techniques, the cost of the system and complexity of the feed network is considerably increased. This work investigates the possibilities of simplifying these techniques and studies the effects of amplitude tapering (instead of altering the phase of each antenna element) on broadside and side lobe level (SLL) radiation patterns. As a result, a low-cost solution can be utilized to improve the broadside pattern recovery and sidelobe level control on a conformal surface. The amplitude tapering for conformal antennas has been investigated in [9] where the amplitude weights have been determined using empirical techniques. No automated algorithm has been proposed to deal with different scenarios.

In this work, an amplitude tapering algorithm is developed using the array factor expression, which nearly gives an optimal amplitude coefficients to improve the radiation pattern in the target (broadside) direction and reduces the energy level in the first side lobes, while keeping the transmit power the same as that of uniform illumination. The amplitude tapering is created by redistributing the power among the antenna elements, so that the total transmit power remains the same, i.e., the sum of the squares of amplitude tapering voltages is the same as the sum of squares of a uniform amplitude excitation voltages of the same number of antenna elements.

This work is organized in the following manner. In Section 2, the radiation pattern correction is presented using analytical expressions and an optimization
algorithm. Section 3 presents simulation and measurement validation of the pattern correction for a 45 degree conformal wedge. Finally Section 4 concludes the paper.

## II. ARRAY FACTOR (AF) AND OPTIMIZATION ALGORITHM

Consider the eight-element array attached to a conformal wedge-shaped surface shown in Fig. 1. The location of the $n^{t h}$ element in the array is represented as a function of distance $a_{n}$ from the origin, and angle $\phi_{n}$ from the $x$-axis, where $n=1,2, \ldots, 8 . \phi_{b}$ is the bend angle of the wedge and the broadside main beam direction is in the $+y$-direction.


Fig. 1. A drawing of the eight-element amplitude-tapered array antenna on a conformal wedge-shaped surface.

## A. Array Factor (AF)

The radiation pattern produced by the array configuration in Fig. 1 can be written as (chapter 5 in [1]):

$$
A F=\sum_{n=1}^{N} F_{n}(\theta, \phi) w_{n} e^{j k\left[x_{n} \cos \phi+y_{n} \sin \phi\right]}
$$

where $k$ is the free space wave number, $N$ is number of antenna elements, $\left(x_{n}, y_{n}\right)$ is the location of $n^{\text {th }}$ antenna element on the conformal surface in Fig. 1, and $F_{n}(\theta, \phi)=\cos \left(\phi-\phi_{b}\right)$ is the pattern for right fourelements and $F_{n}(\theta, \phi)=\cos \left(\phi+\phi_{b}\right)$ is the pattern of left four-elements in Fig. 1. $w_{n}=I_{n} e^{j \Delta \phi}$ is the complex weighting function required to drive the $n^{\text {th }}$ antenna element. For this work, the phase difference $\Delta \phi$ between adjacent antenna elements was made zero, and the amplitude tapering coefficient $I_{n}$ was computed for various bend angles to correct the main beam direction towards the broadside. Putting $w_{n}=I_{n}$ for amplitude tapering, (1) can be written as:

$$
\begin{equation*}
A F=\sum_{n=1}^{N} F_{n}(\theta, \phi) I_{n} e^{j k\left[x_{n} \cos \phi+y_{n} \sin \phi\right]} . \tag{2}
\end{equation*}
$$

To determine the amplitude tapering coefficients $I_{n}$ in (2), the following optimization algorithm was
developed to correct the broadside main beam target direction.

## B. Algorithm development to compute amplitude coefficients

A classical antenna array synthesis problem assigns complex weights to each array element so as to maximize the radiation pattern in the target directions while minimizing the side-lobes below a certain level. This can be easily formulated as a convex optimization problem which minimizes the total radiated power subject to the constraints on the radiation in the target direction while suppressing the side-lobes radiation to a value below $\gamma_{S L L}$ [8]:

$$
\begin{array}{ll} 
& \min |w|_{2}^{2} \\
\text { subject to } & \left|A F_{t a r}(w)\right|=1  \tag{3}\\
\text { and } & \left|A F_{S L L}(w)\right| \leq \gamma_{S L L},
\end{array}
$$

where $w=\left[w_{1}, w_{2}, \ldots, w_{N}\right]^{T}$ represents the complex antenna element weights. $A F_{\text {tar }}$ is the radiation in the target direction and $A F_{S L L}$ is the radiation pattern of the side lobes. The objective function in (3) ensures that the total radiated power of the antennas is constrained. Its implementation, however, necessitates both attenuators and phase shifters which leads to an expensive solution. A less expensive but suboptimal approach is to use only the amplitude tapering, i.e.,

$$
\begin{align*}
& \arg \max _{I}\left|A F_{t a r}(I)\right|, \\
& \text { subject to }|I|_{2}^{2} \leq 1,  \tag{4}\\
& \text { and }\left|A F_{S L L}(I)\right| \leq \gamma_{S L L},
\end{align*}
$$

where the weights $I=\left[I_{1}, I_{2}, \ldots, I_{N}\right]^{T}$ represents the amplitude coefficients $I_{n}$ in (2). Although this problem can be solved through the classical descent algorithms, a simpler approach involves growing $I$ incrementally one antenna element at a time by making a locally optimal choices at each stage. During each incremental increase in $I$, a multi-criteria objective function is formed using the change in the radiated power in the target direction $\Delta A F_{\text {tar }}^{2}(I)$ and the maximum change encountered in the first sidelobe $\max \left(\Delta A F_{S L L}^{2}(I)\right)$. A scalarization parameter $\beta$ is used to controlthe emphasis to either of these two parameters, i.e., $\Delta A F_{\text {tar }}^{2}$ and $\Delta A F_{S L L}^{2}$. This multi-criteria objective function is then normalized with respect to the change in the total transmit power given by the increase in the square of Euclidean modulus of the weight vector $\Delta|I|_{2}^{2}$. The process of adding weights continues for as long as the weight constraint in (4) is not violated. The objective function over each incremental change is now written as:

$$
\begin{align*}
& \arg \max _{I_{n}^{\prime}}^{\prime} \frac{\Delta A F_{\mathrm{tar}^{2}}-\beta \max \left(\Delta A F_{\mathrm{SLL}}{ }^{2}\right)}{\Delta \mid I_{2}^{2}}  \tag{5}\\
& =\underset{I_{n}^{\prime}}{\operatorname{argmax}} \frac{\left\{A F_{\operatorname{tar}}^{2}\left(I^{\prime}\right)-A F_{\text {tar }}^{2}(I)\right\}-\beta \max \left\{A F_{S L L}^{2}\left(I^{\prime}\right)-A F_{S L L}^{2}(I)\right\}}{\left|I^{\prime}\right|_{2}^{2}-|I|_{2}^{2}},
\end{align*}
$$

where $I^{\prime}=\Delta I_{n}+I, \Delta I_{n}$ is the incremental change in the $n^{t h}$ antenna element weight that maximizes the above
ratio and $I^{\prime}$ is the corresponding new weight vector. Both the half beamwidth $\zeta$ and the scalarization parameter $\beta$ are also varied in order to obtain a solution that gives the best tradeoff between the antenna gain, the beam-width, and the energy gap to the first sidelobe.

Table 1: Optimization algorithm
Algorithm: Computation of Amplitude
Excitation Coefficients

```
Inputs: \(N, a_{n}, \phi_{n}, \theta_{\text {tar }}, \phi_{b}, F_{n}(\theta, \phi)\)
Outputs: I, \(\beta_{o p t}, \zeta_{o p t}\)
For \(\zeta=\theta_{1}\) to \(\theta_{2}\) beam-width
        For \(\beta=\beta_{1}\) to \(\beta_{2}\) scalarization parameter
            Initialization: \(I_{N \times 1}=[0,0, \cdots 0]^{T}\),
\(A F=0\);
        while \(|I|_{2}^{2}<1\) do
            for \(n=1 \rightarrow N\) do
                \(I_{n}{ }^{\prime}=I_{n}+\Delta I\)
                \(\Omega(n)=\frac{\Delta A F^{2}}{\Delta|I|_{2}^{2}}\)
    \(=\frac{\Delta A F_{\text {tar }}^{2}-\beta \max \left(\Delta A F_{S L L}^{2}\right)}{\Delta|I|_{2}^{2}}\)
        end for
            \(I^{*} \leftarrow \arg \max _{n} \Omega(n)\)
                \(I_{n^{*}}{ }^{\prime}=I_{n^{*}}+\Delta I\)
                    \(I_{m} \leftarrow I_{n^{*}}\)
        end while
    end For
    end For
    \(\arg \max _{I, \zeta, \beta}\left(\frac{A F_{t a r}}{A F_{S L L}}\right)\)
```

Table 1 gives the algorithm for the weights selection. It uses a greedy approach, using the locally optimal choice of increasing the power in only one antenna element at a time that maximizes the ratio in (4). The algorithm is optimized in a manner, so that the total transmit power for various bend angles is always normalized to one, i.e., $\operatorname{sum}\left(I_{n}{ }^{2}\right)=1$.

In Fig. 2, the radiation pattern of the proposed amplitude tapering algorithm (labeled as 'Amplitude tapering') is compared with the optimal phase and amplitude correction algorithm (2) (labeled as 'Optimal') and the uncorrected radiation pattern (labeled as 'Uncorrected'). All three algorithms are simulated for eight-element antenna array for various bend angles of 0 to $45^{\circ}$ of a wedge-shaped conformal surface. The value of $\beta$ is varied from 0 to 8 , the half beamwidth is varied between $10^{\circ}$ to $45^{\circ}$ and the value of $\gamma_{S L L}$ in (2) is fixed to -10 dB .

For bend angles less than $10^{\circ}$, the difference between the gains of corrected and uncorrected radiation patterns is very small. The effect of bend angles on the uncorrected pattern becomes more pronounced at higher
angles ( $\phi_{b} \geq 10^{\circ}$ ) with the sidelobe levels becoming comparable or even bigger than the main beam in the target direction. The proposed algorithm not only improves the main lobe gain but also reduces the SLL considerably as shown in Fig. 2. The difference in the gains between uncorrected and corrected patterns is highlighted for the bend angles shown. For the simulated bend angles 0 to $45^{\circ}$, the proposed amplitudetapering algorithm gain stays within 6 dB of the optimal scheme, which is within the acceptable limit $[5,6,17,18]$.

The aperture efficiency $\eta_{\text {aperture }}$ can be calculated using the expression in [10]:

$$
\begin{equation*}
\eta_{\text {aperture }}=\frac{\left(\sum_{n=1}^{N} I_{n}\right)^{2}}{N \sum_{n=1}^{N}\left(I_{n}\right)^{2}} . \tag{6}
\end{equation*}
$$

The aperture efficiencies for various bend angles were computed using (6) and are given in Table 2. The results in Table 2 show that conformal surfaces with different bending angles require various amplitude weighting distributions, aperture efficiencies, and hence various deviations of pattern recovery patterns from optimal patterns as illustrated in Fig. 2.

Table 2: Aperture efficiency for various bend angles of the eight-element conformal array

| $\phi_{b}$ | $I_{n}$ | $\eta_{\text {aperture }}$ <br> $(\%)$ |
| :---: | :---: | :---: |
| 0 | $[0.23,0.24,0.41,0.43,0.43,0.42,0.24,0.23]$ | 100 |
| 5 | $[0.23,0.34,0.36,0.43,0.43,0.36,0.34,0.23]$ | 97.5 |
| 10 | $[0.14,0.4,0.46,0.32,0.32,0.46,0.4,0.14]$ | 88.4 |
| 15 | $[0,0.29,0.53,0.36,0.36,0.53,0.29,0]$ | 70.7 |
| 20 | $[0,0,0.46,0.53,0.53,0.46,0,0]$ | 50 |
| 25 | $[0,0,0.46,0.53,0.53,0.46,0,0]$ | 50 |
| 30 | $[0,0,0,0.71,0.71,0,0,0]$ | 25 |
| 35 | $[0.5,0,0,0.5,0.5,0,0,0.5]$ | 50 |
| 40 | $[0.5,0,0,0.5,0.5,0,0,0.5]$ | 50 |
| 45 | $[0.46,0,0,0.53,0.53,0,0,0.46]$ | 50 |


(a)

(b)

(c)

(d)


Fig. 2. Simulated radiation pattern of the eight-element conformal array for:(a) $\phi_{b}=0^{\circ}$,(b) $\phi_{b}=15^{\circ}$,(c) $\phi_{b}=20^{\circ}$, (d) $\phi_{b}=25^{\circ}$, (e) $\phi_{b}=35^{\circ}$, and (f) $\phi_{b}=45^{\circ}$.

## III. MEASUREMENT VALIDATION

For measurement purposes, the eight-element testing array in Fig. 3 (a) was fabricated, and used to validate (1) on a conformal wedge-shaped surface with bend angle $\phi_{b}=45^{\circ}$ (shown in Fig. 3 (b)). The array consisted of voltage variable attenuators, a 8-way power splitter, amplifier and eightmicrostrip patches designed to operate at 2.45 GHz with an inter-element spacing of $0.5 \lambda_{0}$. The power splitter, attenuators and amplifiers were manufactured by Mini-Circuits [11] (PNs: ZN8PD1-53-S+, ZX73-2500-S+ and ZX60-33LN-S+, respectively). Identical SMA cables were used to
connect each patch to a port on the power splitter through an attenuator.


Fig. 3. (a). Topology of the eight-element array used for attachment to conformal surfaces, and (b) picture of the eight-element array attached to a wedge in the anechoic chamber for bend angle $\phi_{b}=45^{\circ}$.

The algorithm in Table 1 was used to calculate the amplitude distribution coefficients to maximize the broadside main beam radiation pattern and -10 dB first sidelobe constraints was defined. The computed amplitude coefficients for $\emptyset_{b}=45^{\circ}$ are given in Table 2. The uniform amplitude tapering coefficients for the uncorrected case are given by $[1,1,1,1,1,1,1,1] / \sqrt{8}$.

Next, the eight-element wedge antenna array of Fig. 3 (b) was driven with uncorrected and corrected amplitude distribution coefficients, and the results for bend angle $\phi_{b}=45^{\circ}$ are shown in Fig. 4 (amplitude tapering correction results were normalized with respect to optimal scheme). With amplitude tapering, the broadside main beam direction has been corrected by 8.4 dB over the uncorrected pattern.


Fig. 4. Measurement and analytical results of the eightelement conformal array for bend angle $\phi_{b}=45^{\circ}$.

## IV. CONCLUSION

Amplitude tapering algorithm was developed to improve the radiation pattern of eight-element conformal array towards the broadside target direction. The proposed algorithm was compared with the optimal scheme (that utilizes both phase and amplitude correction) for various bend angles of conformal wedge. It was shown that the broadside main beam pattern can be corrected using amplitude excitation coefficients only (instead of both phase and amplitude correction). Therefore a low-cost solution can be utilized to improve the broadside pattern recovery on a conformal surface. A measurement case of $45^{\circ}$ conformal wedge showed that 8.4 dB correction of main beam direction towards the broadside over uncorrected case can be achieved using the proposed amplitude tapering algorithm.

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