A General Equivalent Model for Multi-Coil Wireless Power Transfer System Analysis and its Application on Compensation Network Design

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***Abstract*─** This paper presents a novel general equivalent model of multi-coil coupled wireless power transfer (WPT) system and its application on compensation network design. The proposed equivalent model has the advantages of concise expressions, good accuracy, and fast calculation speed. Firstly, the general equivalent model is established to get the concise expressions of system efficiency and output power. Then, based on the proposed model, compensation network design method is discussed, considering several system performance indicators. Furthermore, the proposed model and method are verified by a developed WPT prototype. Meanwhile, the equivalent characteristics and the mutual-resistance effect are analyzed. Also, numerical simulations are conducted to study the magnetic flux distribution, the magnetic field exposure issue, and the current distribution in coil Litz wire. Finally, a varied capacitor compensation method is presented to improve system efficiency on the conditions of coil misalignments.

***Index Terms*─** Compensation network design, efficiency improvement, general equivalent model, Wireless power transfer.

I. INTRODUCTION

Wireless power transfer (WPT) can make people free from connecting wires and bring convenience to consumers. So, it has attracted much attention and has been used in implanted devices, sensors, mobile phone and electric vehicle (EV) charging, etc. [1,2].

Coupling coils are a key part of WPT system. They can be described by coupled-mode theory [3], circuit model [4], or some electromagnetic descriptions [5]. WPT system works through the electromagnetic coupling among the coils. So, it is an electromagnetic problem essentially, affected by coil sharp, size, position, and so on [5,6]. However, the coils can be considered to be electrically small, because their sizes and lengths are much smaller than the wavelength of WPT operating frequency [5]. Moreover, distance between the coils is much smaller than the wavelength of WPT operating frequency. So, we can think the energy is stored in the near-field and not radiated into space. Hence, the radiation loss can be neglected. Based on the above two points, WPT system can be approximately equivalent to a lumped parameter circuit.

Lumped parameter circuit model of WPT system can be obtained from voltage-current equations based on impedance matrix [7,8]. But for the multi-coil WPT system, the circuit models usually contain too many parameters, and cannot get a concise expression of system efficiency or output power [4,7]. So, several methods are proposed to solve this problem, including making the coils and compensation networks working in the resonance states [9], assuming the coils are identical [10], adopting S-parameters to describe the WPT system [1], and reducing the order of the multi-coil WPT model [11]. The most common way is considering only the mutual-inductances between adjacent coils to simplify the model [12,13]. This simplified model could give concise expressions, but will lead to inaccurate results in some applications.

The other key part of WPT system is compensation networks, which include series capacitor, paralleled capacitor, LCC compensation network and so on [14,15]. They can be designed in the resonance state, which has the advantages that the resonant frequency is irrelevant with the coupling coefficient and is also independent of the load condition [15]. But in some close range applications, the optimal system does not work in the completely resonance state [4]. So, the compensation networks need to be optimized considering features of both the coils and the load [16], in order to adjust system output power and keep system efficiency in a high value.

Based on the research above, a novel general equivalent model of multi-coil coupled WPT system is proposed in this paper. This model contains complete information of the multi-coil WPT system, so it has good accuracy. At the same time, concise efficiency and output power expressions can be obtained, which are

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more convenient for system analysis and design. Also, the proposed model can achieve faster calculation speed, because the n-order impedance matrix has been transferred to a two-order one, so the calculation time of the reverse operation will be significantly reduced. This paper is organized as follows. Section II presents the general equivalent model. Section III shows the compensation network design method. Section IV gives an example of a four-coil system, corresponding experimental verifications and analysis.

II. GENERAL EQUIVALENT MODEL

The electromagnetic coupling problem of WPT system can be analyzed through the lumped parameter circuit model, since it meets the requirements of electrically small. Circuit model of a typical multi-coil WPT system is shown in Fig. 1. Where, compensation networks with series capacitors are adopted, since they have simple structures and been widely used in multi-coil system; U_S is equivalent voltage source; R_L is equivalent load resistance, I_1, I_2, I_3, … I_{n-1}, I_n are currents in the coils; L_1, L_2, L_3, … L_{n-1}, L_n are self-inductances of the coils; R_1, R_2, R_3, … R_{n-1}, R_n are stray resistances of the coils' loops; C_1, C_2, C_2, … C_{n-1}, C_n are series compensation capacitances; M_{12}, M_{13}, M_{23}, … $M_{2(n-1)}$, $M_{3(n-1)}$, M_{3n} , … $M_{n(n-1)}$ are mutual-inductances among the coils.

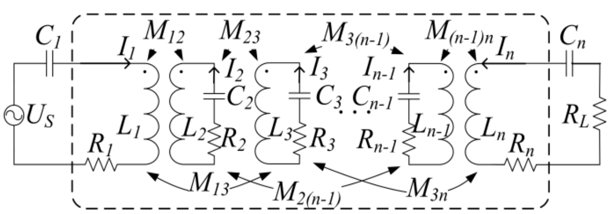

Fig. 1. Circuit model of the multi-coil coupled WPT system with series compensation capacitors.

According to Kirchhoff's law and mesh current analysis method, the voltage-current equation of the circuit model in Fig. 1 is given by Equation (1a):

$$\begin{bmatrix} Z_{11}' & Z_{12} & Z_{13} & \cdots & Z_{1(n-1)} & Z_{1n} \\ Z_{12} & Z_{22} & Z_{23} & \cdots & Z_{2(n-1)} & Z_{2n} \\ Z_{13} & Z_{23} & Z_{33} & \cdots & Z_{3(n-1)} & Z_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ Z_{1(n-1)} & Z_{2(n-1)} & Z_{3(n-1)} & \cdots & Z_{(n-1)(n-1)} & Z_{(n-1)n} \\ Z_{1n} & Z_{2n} & Z_{3n} & \cdots & Z_{(n-1)n} & Z_{nn}' \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_{n-1} \\ I_n \end{bmatrix} = \begin{bmatrix} U_S \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}. \tag{1a}$$

Where, system impedances are defined by Equation (1b), and ω is the angle frequency:

$$\begin{aligned} &Z_{11}' = Z_{11} + 1/\,j\omega C_1 = R_1 + j\omega L_1 + 1/\,j\omega C_1; \\ &Z_{nn}' = Z_{nn} + R_L + 1/\,j\omega C_n = R_n + j\omega L_n + R_L + 1/\,j\omega C_n; \\ &Z_{pp} = R_p + j(\omega L_p - 1/\,\omega C_p);\ \ p = 2,3\ldots n-1; \\ &Z_{pq} = j\omega M_{pq};\ \ p,q = 1,2,3\ldots n-1, n,\ and\ \ p \neq q. \end{aligned} \tag{1b}$$

Equations (1a) and (1b) indicate that the inverse matrix of the n-order impedance matrix has to be solved to get the results. This inverse operation will lead to lots of calculations and complex final expressions. For the sake of solving this problem, we consider to reduce the order of the impedance matrix by making an equivalent with a two-coil system like shown in Fig. 2. In the equivalent process, U_S, R_L, I_1, I_n, C_1 and C_n are kept at the same. We build an equivalent relationship between the part in dashed box in Fig. 1 and the one in dashed box in Fig. 2. This conversion will change the n-order impedance matrix to a two-order one, so the corresponding calculations of the inverse operation will be significantly reduced, and it is possible to obtain concise equation solutions.

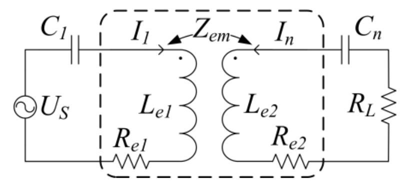

Fig. 2. Equivalent two-coil model used to express the multi-coil WPT system.

In Fig. 2, L_{e1} and L_{e2} are the self-inductances, and R_{e1} and R_{e2} are the resistances of the two equivalent coils' loops, respectively. Also, primary side impedance Z_{e1}, secondary side impedance Z_{e2}, and mutual-impedance Z_{em} are defined as follows: $Z_{e1}=R_{e1}+j\omega L_{e1}$, $Z_{e2}=R_{e2}+j\omega L_{e2}$, $Z_{em}=R_{em}+j\omega M_e$. Where, R_{em} and M_e are mutual-resistance and mutual-inductance between the two equivalent coils. It should be noticed that mutual-impedance between the two equivalent coils contains resistance component R_{em}, which is different from the expressions of the traditional two-coil WPT system.

Using the general equivalent model, expressions of system efficiency and output power can be got according to the following steps. Firstly, the n-coil coupled circuit in Fig. 1 is transferred to the equivalent two-coil model in Fig. 2, to the aim of reducing the impedance matrix order. Specifically, we find the relationships between impedances of the n-coil coupled circuit model (Z_{pp} and Z_{pq}) and impedances of the equivalent two-coil model (Z_{e1}, Z_{e2}, and Z_{em}), as the functions shown in Equation (2):

$$\begin{aligned} &Z_{e1} = f_{e1}(Z_{pp}, Z_{pq});\ \ Z_{e2} = f_{e2}(Z_{pp}, Z_{pq}), \\ &Z_{em} = f_{\mathrm{em}}(Z_{pp}, Z_{pq});\ \ p,q = 1,2,3\ldots n-1, n,\ and\ \ p \neq q. \end{aligned} \tag{2}$$

Then, based on the transfer relationships in Equation (2), the voltage-current equation of the equivalent two-coil model in Fig. 2 is given by Equation (3):

$$\begin{bmatrix} Z_{e1}+1/j\omega C_1 & Z_{em} \\ Z_{em} & Z_{e2}+R_L+1/j\omega C_n \end{bmatrix}\begin{bmatrix} I_1 \\ I_n \end{bmatrix}=\begin{bmatrix} U_S \\ 0 \end{bmatrix}. \quad (3)$$

At last, system efficiency and output power expressions can be calculated and given by Equations (4) and (5). Where, $R_{sec}=R_{e2}+R_L$, $X_{e1}=\omega L_{e1}$-$1/\omega C_1$, $X_{e2}=\omega L_{e2}$-$1/\omega C_n$, $Z_{eq}=R_{e1}\,R_{sec}$-$R_{em}^2+\omega^2 M_e^2$.

$$\eta=\frac{(\omega^2 M_e{}^2+R_{em}{}^2)R_L}{R_{\sec}(\omega^2 M_e{}^2-R_{em}{}^2)+R_{e1}(R_{\sec}{}^2+X_{e2}{}^2)-2\omega M_e R_{em}X_{e2}}. \quad (4)$$

$$P_o=\frac{(\omega^2 M_e{}^2+R_{em}{}^2)R_L U^2}{(Z_{eq}-X_{e1}X_{e2})^2+(R_{e1}X_{e2}+R_{\sec}X_{e1}-2\omega M_e R_{em})^2}. \quad (5)$$

Equations (4) and (5) suggest that concise solutions can be obtained from the proposed equivalent model. They will be helpful for system analysis and design.

Furthermore, Equations (4) and (5) can be used to get optimal system states, such as maximum efficiency and so on. According to the traditional two-coil WPT model, system can achieve maximum efficiency, when the secondary side is in resonance state, which means X_{e2}=0. However, the situation in the proposed equivalent two-coil model is different, because of the introduction of R_{em}. The optimal X_{e2} to achieve maximum efficiency is calculated and given by Equation (6):

$$X_{e2\_opt}=\omega M_e R_{em}/R_{e1}. \quad (6)$$

Through Equation (6), the maximum system efficiency η_{max} can be solved, as well as system output power P_{om} when having maximum efficiency. Their expressions are given in Equations (7) and (8). Based on these equations, compensation capacitors can be designed to achieve high system performance:

$$\eta_{\max}=\frac{(\omega^2 M_e{}^2+R_{em}{}^2)R_L R_{e1}}{R_{e1}R_{\sec}(\omega^2 M_e{}^2-R_{em}{}^2)+R_{e1}{}^2 R_{\sec}{}^2-\omega^2 M_e{}^2 R_{em}{}^2}. \quad (7)$$

$$P_{om}=\frac{(\omega^2 M_e{}^2+R_{em}{}^2)R_L U^2}{(Z_{eq}-X_{e1}\omega M_e R_{em}/R_{e1})^2+(R_{\sec}X_{e1}-\omega M_e R_{em})^2}. \quad (8)$$

Besides, the proposed equivalent model can also be achieved and investigated through electromagnetic theory, but the circuit model is more suitable and has higher accuracy on the specific condition in this paper.

III. COMPENSATION NETWORK DESIGN

Compensation networks are designed to make the WPT system achieve high performance, such as maximum efficiency, rated output power, high coil misalignment tolerance, etc. Since every coil has a corresponding compensation capacitor as shown in Fig. 1, the system will have n degrees of freedom, which can be used for design. This means n equations can be simultaneously solved to reach several design targets at the same time.

Firstly, efficiency and output power targets are most important. So, Equation (6) should be included in the target equations to reach the maximum system efficiency. Also, $P_{om}=P_{ref}$ should be solved based on Equation (8) to make the system output rated power. Where, P_{ref} is rated system output power.

Then, system input impedance characteristics need to be adjusted to match the power source. For example, sometimes system input resistance should be 50Ω to match the radio frequency (RF) source. Moreover, if the source is an inverter, system input reactance should be adjusted to control the inverter load reactive power, in order to reduce electric stress of power electrics devices. Equation (9) shows the expression of system input reactance. In order to adjust the reactive power, $X_{inv}=X_{req}$ needs to be added in the target equations. Where, X_{req} is the reference reactance:

$$X_{inv}=\frac{R_{\sec}(R_{\sec}X_{e1}-2\omega M_e R_{em})+X_{e2}(R_{em}{}^2-\omega^2 M_e{}^2+X_{e1}X_{e2})}{R_{\sec}{}^2+X_{e2}{}^2}. \quad (9)$$

Furthermore, coil misalignment tolerance could be considered, if there still have design degrees of freedom. For example, making the WPT system reach maximum efficiency or achieve reference efficiency, when the coils have a certain misalignment distance. Equation (10) gives the target equation which make the system reach maximum efficiency when misalignment. Where, subscripts '_d' represent the corresponding equivalent model parameters in the coil misalignment distance d:

$$X_{e2\_opt\_d}=\omega L_{e2\_d}-1/\omega C_n=\omega M_{e\_d}R_{em\_d}/R_{e1\_d}. \quad (10)$$

Solving process of the target equation is a little complex, if we want WPT system to achieve reference efficiency η_d in the coil misalignment distance d. This is caused by the efficiency expression is a two-order function of X_{e2}, as shown in Equation (4). So, firstly, Equation (11a) is used to determine whether the efficiency equation is solvable. If there is a solution, the target equation can be given by Equation (11b):

$$\Delta=4\omega^2 M_{e\_d}{}^2 R_{em\_d}{}^2-4R_{e1\_d}(R_{e1\_d}R_{\sec\_d}{}^2+R_{\sec\_d}{}^2 \times(\omega^2 M_{e\_d}{}^2-R_{em\_d}{}^2)-\frac{(\omega^2 M_{e\_d}{}^2+R_{em\_d}{}^2)R_L}{\eta_d})\geq 0. \quad (11a)$$

$$X_{e2\_opt\_d}=\frac{2\omega M_{e\_d}R_{em\_d}+\sqrt{\Delta}}{2R_{e1\_d}}. \quad (11b)$$

Through simultaneously solving the above target equations, values of the compensation capacitors can be obtained by considering several system performance indicators. It should be noticed that sometimes there is no analytical solution for these equations. Numerical solution methods need to be used on this condition. Also, calculated values of the compensation capacitors require fine tuning in practice to get better results.

IV. EXAMPLE OF A FOUR-COIL SYSTEM

A. Equivalent model of four-coil system

Circuit model of a typical four-coil WPT system is shown in Fig. 3. Where, the four coupled coils are named as drive coil, transmit coil, receive coil, and load coil, in the order from source to load; the variables have the same meaning with the ones shown in Fig. 1.

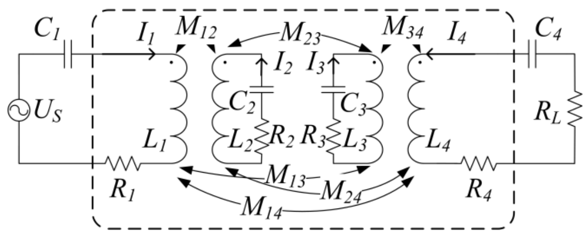

Fig. 3. Circuit model of the four-coil coupled WPT system with series compensation capacitors.

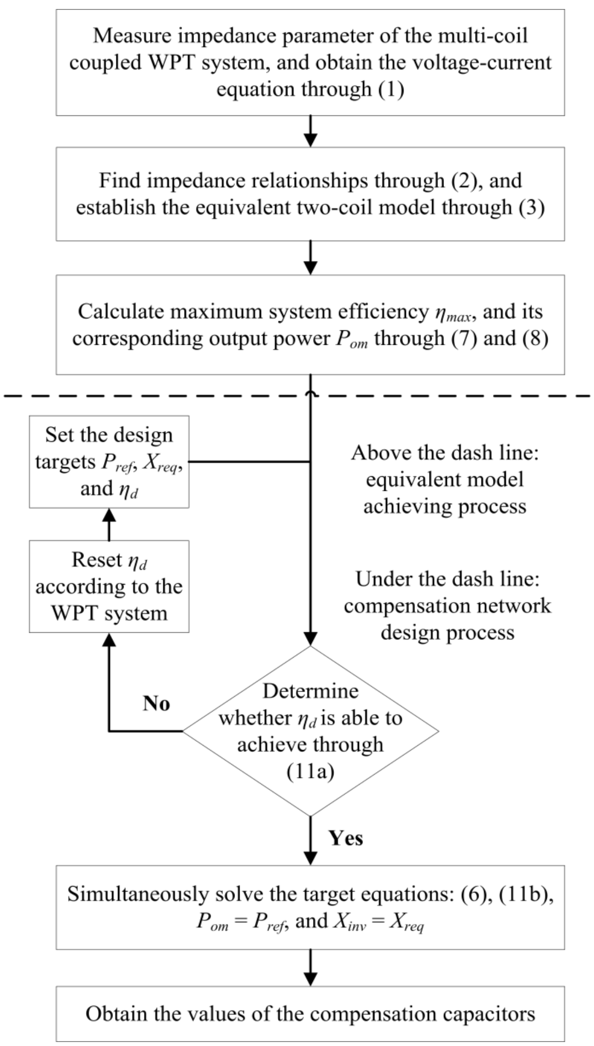

Fig. 4. Flow chart of the proposed equivalent model achievement and compensation network design method.

The solution procedure based on the proposed equivalent model and the compensation network design method are shown in Fig. 4. According to the equivalent model achieving process shown in Fig. 4, we can build an equivalent relationship between the part in dashed box in Fig. 3 and the one in dashed box in Fig. 2. This conversion will change the four-order impedance matrix to a two-order one. Through operations of the impedance matrixes and their elements, the relationships are obtained and given by Equation (12):

$$
\begin{aligned}
Z_{e1} &= Z_{11} - \frac{Z_{12}{}^2 Z_{33} + Z_{13}{}^2 Z_{22} - 2Z_{12}Z_{13}Z_{23}}{Z_{22}Z_{33} - Z_{23}{}^2}, \\
Z_{e2} &= Z_{44} - \frac{Z_{24}{}^2 Z_{33} + Z_{34}{}^2 Z_{22} - 2Z_{23}Z_{24}Z_{34}}{Z_{22}Z_{33} - Z_{23}{}^2}, \\
Z_{em} &= Z_{14} - \frac{Z_{12}Z_{24}Z_{33} + Z_{13}Z_{34}Z_{22} - Z_{23}(Z_{12}Z_{34} + Z_{13}Z_{24})}{Z_{22}Z_{33} - Z_{23}{}^2}.
\end{aligned}
\tag{12}
$$

Equation (12) suggests that primary side impedance Z_{e1} of the equivalent two-coil model is decided by the self-impedances of drive coil, transmit coil, receive coil, and the mutual-impedances among them. But it does not have a relationship with the load coil. Similarly, secondary side impedance Z_{e2} has no relationship with the drive coil. Mutual-impedance Z_{em} is independent of self-impedances of drive coil and load coil. It should be noticed mutual-impedance between the two equivalent coils contains resistance component R_{em}, because self-impedance Z_{22} and Z_{33} are included in the transfer relationship of the last equation in Equation (12).

B. Compensation capacitor design and experimental verifications

In order to verify the proposed general equivalent model and the compensation network design method, a WPT prototype for EV charging is developed. Where, a full bridge inverter is used as the power source, and the system load is a diode rectifier with resistor. The drive coil and transmit coil are set up together in the ground side, wound by Litz wires. Also, ferromagnetic materials are adopted to constraint electromagnetic field. The load coil and receive coil share the same structure in the vehicle side. The size of the coils is 40cm × 40cm, and the vertical distance between ground side and vehicle side coils is 20cm.

The prototype is designed with rated output power 3.3kW on the input DC voltage 300V. However, we are worried that the electric stress may be too large to harm the system or devices in large misalignment distances, if the prototype works in the rated power. Therefore, the input voltage is reduced to 100V with the output power 366.7W for most of the following experiments and analysis. The equivalent load resistance R_L is 23.2Ω, and system operation frequency is 100kHz. Values of other parameters are shown as follows: L_1=13.7uH, L_2=53.1uH,

L_3=44.2uH, L_4=12.1uH, M_{12}=21.6uH, M_{34}=19.1uH. Also, measured results of M_{13}, M_{14}, M_{23}, and M_{24} are given in Fig. 5, which all change with the lateral misalignment distance.

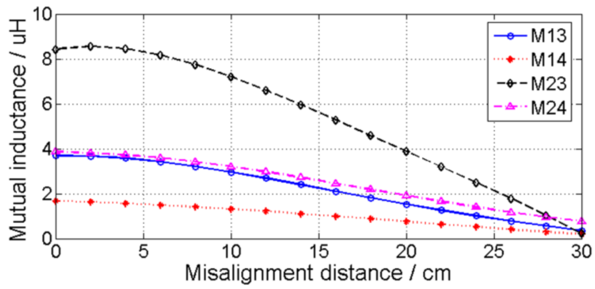

Fig. 5. Measured values of coil mutual-inductances on conditions of lateral misalignments.

Based on the parameter values in the prototype, the compensation capacitors have been designed, according to the design process shown in Fig. 4. Since there are four compensation capacitors in the four-coil WPT system, we can set four target equations. The first one is the target equation to get maximum efficiency, based on Equation (6); the second is P_{om} = 366.7W to make the system output rated power, based on Equation (8); the third is the target equation to adjust the inverter load reactive power, based on Equation (9); the last one is the target equation to keep system efficiency at 85% in the misalignment distance 20cm, to improve system misalignment tolerance, based on Equation (11). Simultaneously solving these four target equations and then fine tuning in practice, finally the values of the compensation capacitors can be obtained as follows: C_1=41.2nF, C_2=39.6nF, C_3=49.5nF, C_4=39.2nF.

On the basis of the designed compensation capacitors, the prototype has been completed, and the experimental waveforms are given in Fig. 6. It suggests the prototype works well on the conditions of both coil alignment and misalignment. Further tests show that the measured system efficiency (DC source to load resistor) is 92.5% on the condition of coil alignment, which meets the requirement of the first target equation. System output power is 367.8W, which approximately meets the requirement of the second target equation. System input reactance is very small on the condition of coil alignment, which has minimized the inverter load reactive power, and reduced electric stress of the power electrics devices. Measured system efficiency is 84.3% on the condition of misalignment distance 20cm, which is a little lower than the requirement of the last target equation. This is caused by some ignored factors, such as cable stray resistances, device on-off losses, etc.

To sum up, these experimental results have proved the effectiveness of the proposed general equivalent model and the compensation network design method.

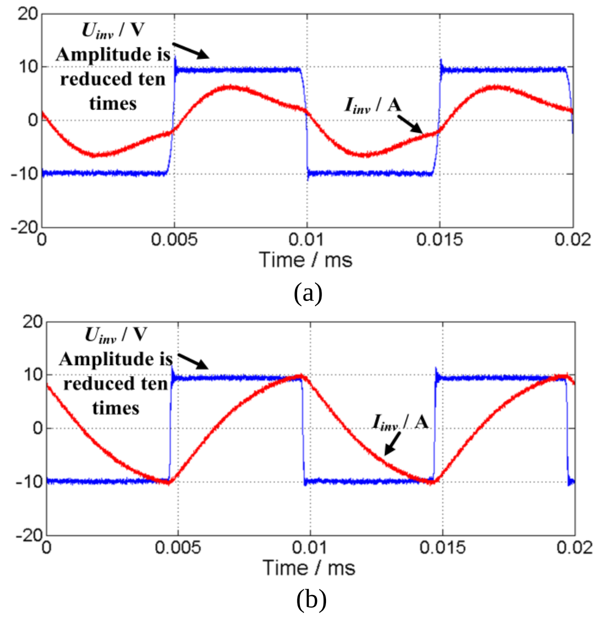

Fig. 6. Experimental waveforms of the inverter output voltages and currents. (a) On the condition of coil alignment; (b) on the condition of coil misalignment distance 20cm.

C. Equivalent characteristic analysis

Based on the parameter values of the developed WPT prototype, we can discuss the equivalent characteristics between the original multi-coil coupled model and the proposed equivalent model. In this section, system efficiency, input and output variables are used to analyze the equivalent features. Only power losses of the coils and compensation networks are considered. So, system efficiency shown in this section will be a little higher than the experimental results.

Calculated efficiencies according to the original four-coil coupled model and the proposed equivalent two-coil model are given in Fig. 7. Additionally, the result of existing simplified four-coil model [13], which ignores the mutual-inductances M_{13}, M_{14}, and M_{24}, is also given in Fig. 7 for comparison.

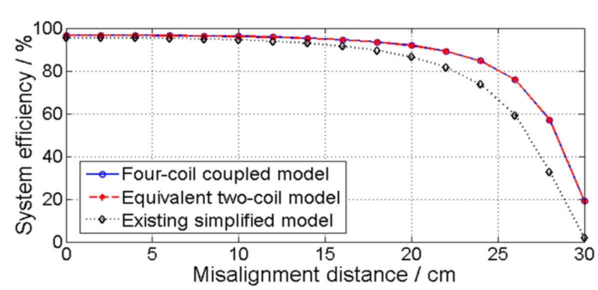

Fig. 7. Comparison of system efficiency calculation results among the original four-coil coupled model, the proposed equivalent two-coil model and the existing simplified four-coil model.

Figure 7 shows that the proposed equivalent two-coil model can get the same accurate system efficiencies as the original four-coil coupled model. This has been verified on the conditions of both coil alignment and misalignment. Because all the parameters in the four-coil coupled model are considered during the equivalent process, the equivalent two-coil model contains all information of the original model. Figure 7 also indicates that all system efficiency calculation results reduce with the misalignment distance, while the result of existing simplified model reduces faster. The existing simplified four-coil model has an acceptable deviation on the condition of coil alignment, only about 3%. However, with the increasing of misalignment distance, the deviation becomes bigger significantly, even reaches to more than 24% in the misalignment distance 28cm.

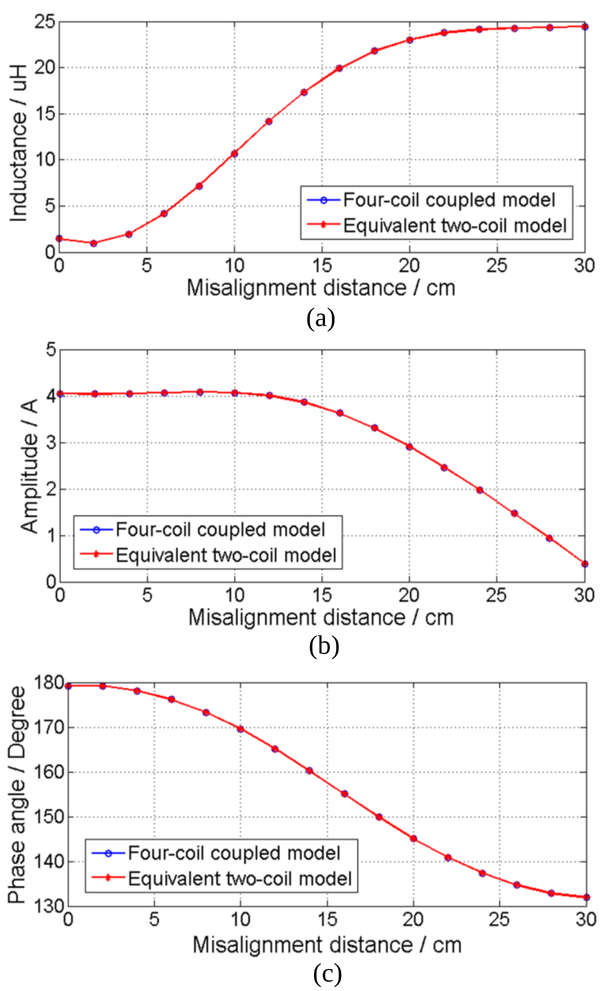

Fig. 8. Comparison of calculation results between the original four-coil coupled model and the proposed equivalent two-coil model. (a) Inverter load inductance L_{inv}; (b) amplitude of load current I_4; (c) phase angle of load current I_4.

Meanwhile, the calculation time based on the four-coil coupled model is 0.798ms, while the one with the proposed equivalent two-coil model is 0.058ms, according to the counting function in the software MATLAB. This result suggests the calculation time based on the proposed equivalent two-coil model is ten times less than the one based on the four-coil model. Its advantage on the calculation speed is very obvious, and will make it more effective in actual applications.

Moreover, other comparisons between the original four-coil coupled model and the proposed equivalent two-coil model are given in Fig. 8, including the calculation results of inverter load inductance L_{inv}, amplitude and phase angle of load current I_4. They suggest that the proposed equivalent two-coil model can get the same accurate input and output variables as the original four-coil coupled model. This has further proved its effectiveness.

Finally, the effect of mutual-resistance R_{em}, which is an obvious difference between the proposed model and the traditional two-coil WPT system model, is analyzed on the conditions of both coil alignment and misalignment. The efficiency calculation results with and without R_{em} are both shown in Fig. 9 for comparison.

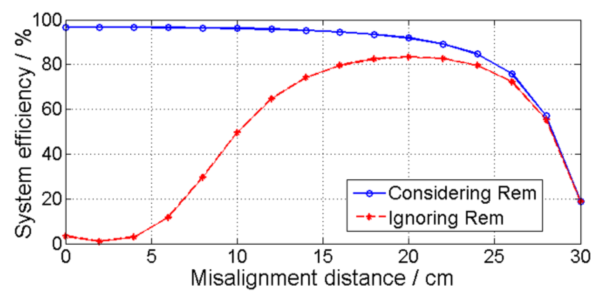

Fig. 9. Comparisons between calculated system efficiencies considering and ignoring R_{em}.

Figure 9 suggests that if ignoring R_{em}, the calculation results will have deviations, especially on conditions of coil alignment and small misalignment. So, R_{em} is an important parameter which affects the accuracy of efficiency calculation. Essentially, R_{em} is a parameter indicating the coupling degree between ground side and vehicle side coils. The stronger the coupling degree is, the greater its effect will be. When lateral misalignment distance increases, the coupling degree becomes weaker, and the effect of R_{em} becomes smaller. Hence, when ignoring R_{em}, the calculation deviation will decrease with the misalignment distance as shown in Fig. 9.

D. Numerical simulation analysis

Some issues arising with the proposed model and method are discussed based on numerical simulations. Firstly, a numerical simulation model is established through the finite element analysis software COMSOL, according to the developed WPT prototype. The schematic of the 2D numerical simulation is shown in Fig. 10.

Then, the magnetic flux distribution is analyzed based on the 2D numerical simulation. Here, the rated output power 3.3kW is considered for this simulation, and the result is shown in Fig. 11. It suggests the major magnetic flux is confined within the air-gap between the coils, because of the ferromagnetic materials. The maximum magnetic flux density can be larger than 2mT, and the magnetic flux density in most of the space between the coils is larger than 1mT. However, the magnetic flux density will decrease fast in the space out of the air-gap between the coils. After a short distance away, it will reduce to less than 100uT.

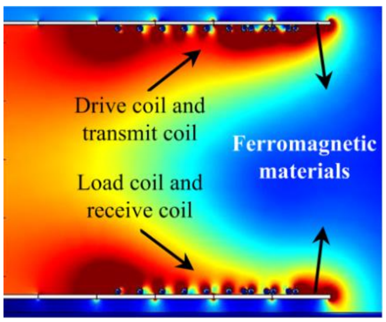

Fig. 10. 2D numerical simulation schematic of the developed WPT prototype.

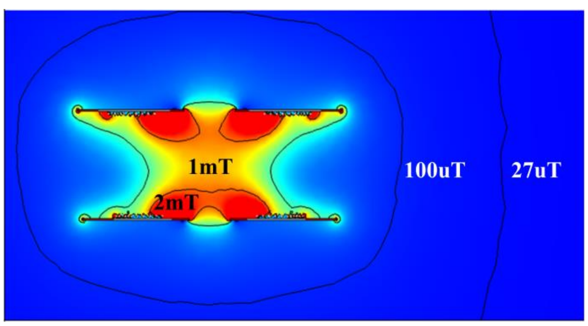

Fig. 11. Magnetic flux distribution simulation result of the developed WPT prototype.

Furthermore, the magnetic field exposure issue of WPT system is discussed for human safety concern, according to the International Commission on Non-Ionizing Radiation Protection (ICNIRP) guidelines [17]. The magnetic field exposure level should be under 27uT, at the frequency of 100kHz. As shown in the simulation result, the 27uT threshold line shifts outward from 573cm. Considering the scenario of EV wireless charging, most passage cars are more than 1500mm wide, which naturally keeps people around at least 700mm away. So, the ICNIRP guidelines can be met if the WPT prototype is set on the symmetrical centerline of the vehicular chassis.

Finally, the current distribution in the Litz wire used for the coils is analyzed, as well as the frequency effect. Here, a usually used frequency tuning range of 90kHz - 110kHz is adopted to show the frequency influence, considering system operation frequency 100kHz as the central frequency. Current distributions in Litz wire at different frequencies can be obtained from numerical simulation conducted by the software COMSOL. The simulation results are shown in Fig. 12, on the conditions of different frequencies.

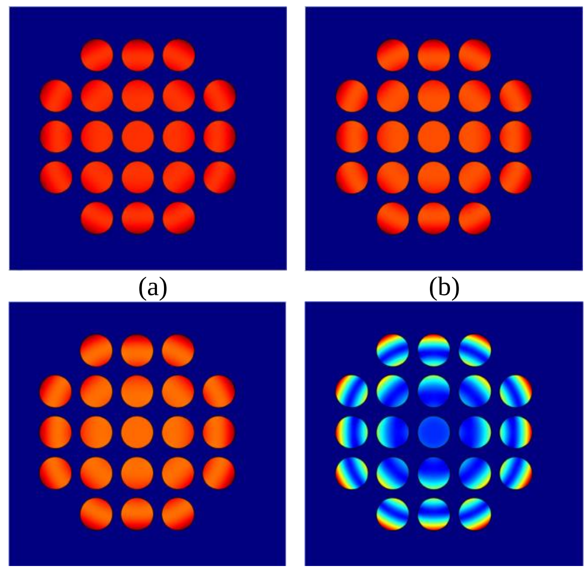

Fig. 12. Simulation results of frequency influence on the Litz wire current distribution. (a) Frequency: 90kHz; (b) frequency: 100kHz; (c) frequency: 110kHz; (d) frequency: 1MHz.

Figure 12 suggests the current distribution in Litz wire becomes more and more inhomogeneous, when frequency increases from 90kHz to 110kHz. But this inhomogeneity and the change of current distribution are both not very obvious in the frequency range of 90kHz - 110kHz. So, the numerical simulation results indicate that skin effect and proximity effect only have small effects on current distribution in the range usually used for frequency tuning. However, if the frequency continues increasing, the influence of skin effect and proximity effect will be significant as shown in Fig. 12 (d). In this case, AC resistance of the Litz wire will greatly increase and lead to substantial power losses.

E. Efficiency improvement

On the basis of the above analysis, a varied capacitor compensation method is presented to improve system efficiency on conditions of coil misalignments. As in Section IV.C, only the power losses of coils and compensation networks are considered.

Parameter values of the equivalent two-coil model will change with coil misalignment distance, and lead to efficiency reduction. If the load coil compensation capacitor C_4 is adjusted according to different misalignment distances, the influence of misalignment could be partly compensated. The optimal values of C_4 can be calculated through Equation (10), and the results are shown in Fig. 13. It suggests that the optimal compensation capacitor $C_{4\ opt}$ decreases with coil misalignment distance and has a large change range.

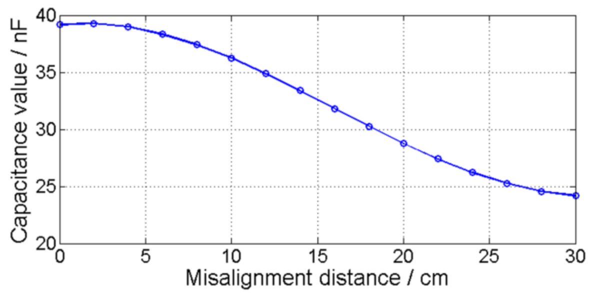

Fig. 13. Calculation results of the optimal load coil compensation capacitor $C_{4\ opt}$.

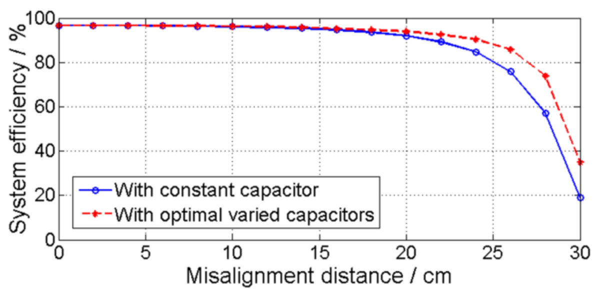

Fig. 14. Efficiency comparisons between adopting the constant capacitor and the optimal varied capacitors.

The varied capacitor compensation method is used in WPT system, and the results are shown in Fig. 14. It suggests system efficiency can be improved through this method, especially in the misalignment distances larger than 20cm. Further calculation indicates that the maximum efficiency improvement can reach up to 16%. These results have proved the effectiveness of the proposed efficiency improvement method.

V. CONCLUSION

This paper presents a novel general equivalent model for multi-coil WPT system analysis and its application on compensation network design. The proposed model has good accuracy because it contains complete information of the multi-coil WPT system. Also, it can lead to concise results, and fast calculation speed. These advantages make the compensation network design easier to be conducted, and has been verified by a developed WPT prototype on the conditions of both coil alignment and misalignment. The results show that the proposed equivalent model can get the same accurate efficiencies, input and output variables as the original multi-coil model; meanwhile, the compensation network design method works well. Based on the actual parameter values, the equivalent characteristics are analyzed, as well as the magnetic flux distribution, the magnetic field exposure issue, and the current distribution in coil Litz wire. Finally, a varied capacitor compensation method is presented to improve system efficiency. The proposed model and methods in this paper will be helpful for multi-coil WPT system analysis and design.

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