# A Propagation Model for Rough Sea Surface Conditions using the Parabolic Equation with the Shadowing Effect

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Abstract - In this article, an accurate and fast approach is proposed to calculate the electromagnetic wave propagation characteristics over the sea surface under tropospheric ducting conditions. The method is based on the parabolic equation and an asymptotic model of a rough sea surface and is used to calculate the electromagnetic characteristics and to model the sea surface reflection. In the proposed model, termed the extremum approximation of the shadowing effect (EA of SE) model, the shadowing effect is considered and simplified to improve the accuracy and shorten the computation time. The probability density functions and the propagation factors of different models are compared, the influence of the rough sea surface and the shadowing effect on the electromagnetic wave propagation is analyzed and the model accuracy and efficiency are evaluated. Some comparisons are made with experimental data. The results show that the average error is about 1 dB less after the shadowing effect is considered; and the proposed approach shortens the computation time about 600 times while maintaining a high accuracy.

*Index Terms* — Electromagnetic propagation, parabolic equation, rough sea surface reflection, shadowing effect, tropospheric duct.

### I. INTRODUCTION

Electromagnetic (EM) waves are reflected, refracted, and diffused when propagating in the troposphere over a rough sea surface. This can result in the alteration of the radar performance, such as over-the-horizon radar detection and differences in the radar shadow [1,2]. In an informationalized battlefield at sea, the real-time evaluation of the radar performance effectively improves the radar usage and battlefield survivability. Therefore, it is important to model, predict, and analyze the EM propagation conditions rapidly.

It is computationally expensive to calculate precisely the EM field in a large region under certain tropospheric conditions and over a rough sea surfaces to satisfy military needs. However, the parabolic equation (PE) model, an approximation model of the Helmholtz scalar wave equation, can be used for rapid and accurate EM field calculations [3,4]. The rough sea surface reflection can be modeled with an asymptotic method based on the Kirchhoff approximation, in which the reflection field of the rough sea surface is regarded as the summation of the coherent field reflected by the sea surface at different heights. The probability density function (PDF) of the sea wave height is used to obtain the effective reflection coefficient of the rough sea surface [5].

The Ament and Miller-Brown (MB) models [6,7] are two routinely used rough surface asymptotic models integrated with the PE model [4]. The advantages of these models are their fast running times and high accuracies [8]. However, Fabbro et al. [9] demonstrated that the inaccuracies of these models are due to the non-consideration of the shadowing effect; therefore, the authors proposed the shadowing effect (SE) model, which is an another rough surface asymptotic model based on Ament model. Freund et al. [10] also analyzed the influence of the shadowing effect on the EM field and verified the accuracy of the model by simulations. However, this SE model requires integral calculations, which are time-consuming.

In this study, the influence of the shadowing effect on the EM propagation characteristics is analyzed and the extremum approximation of the shadowing effect (EA of SE) model is proposed to simplify the modeling of the shadowing effect. The proposed model is used by the PE to calculate the EM wave field in a complex sea environment. The simulation shows that the results are different after the shadowing effect is considered and the new approach has a similar result but a shorter computation time than the SE model. A comparison with the experimental data indicates that the consideration of the shadowing effect increases the precision of the results, and that the proposed method has higher computational efficiency than the SE model and a similar accuracy.

# II. ELECTROMAGNETIC PROPAGATION OVER THE SEA

### A. PE model

The PE model is a numerical computation method that is an approximation of the Helmholtz scalar wave equation [11,12]. The method can be used for the rapid calculation of EM wave propagation characteristics in a complex environment. Therefore, it is routinely used when EM propagation problems are investigated in a large region with complex environmental conditions. Using the Feit-Fleck approximation, the wide angle formulation of the PE is [13]:

$$\frac{\partial}{\partial x}u(x,z) = i\left(\sqrt{k_0^2 + \frac{\partial^2}{\partial z^2}} - k_0\right)u(x,z), \qquad (1)$$
$$+ik_0(n-1)u(x,z)$$

where *n* is the refractive index,  $k_0$  is the free-space wave number, *x* and *z* are the horizontal range and altitude respectively, and u(x, z) is the wave function. The relationship between u(x, z) and the scalar field  $\Phi(x, z)$  is:

$$u(x,z) = \Phi(x,z)e^{-ik_0x}, \qquad (2)$$

where  $\Phi(x, z)$  represents the electric or magnetic field for the horizontal or vertical polarization, respectively.

By using a Fourier transform with z at the range x and  $x+\Delta x$ , the split-step Fourier transform (SSFT) method for the PE can be deduced as:

$$u(x + \Delta x, z) = e^{ik_0 \Delta x(n-1)} \mathfrak{I}^{-1} \left[ e^{i\Delta x \left( \sqrt{k_0^2 - p^2} - k_0 \right)} \mathfrak{I} \left( u(x, z) \right) \right], (3)$$

where  $\Im(\bullet)$  and  $\Im^{-1}(\bullet)$  represent the Fourier transform and its inverse operator. *p* is a parameter in the spectral domain and is related to the EM propagation angle  $\theta$ as  $p = k_0 \sin \theta$ .  $\Delta x$  is the step length. By using the SSFT,  $u(x + \Delta x, z)$  can be calculated if the u(x, z) at the range *x* is known. The Discrete Mixed Fourier Transform (DMFT) can also be used instead of the SSFT in Eq. (3) to calculate electromagnetic wave under impedance boundary conditions [14]. For calculation the EM propagation over the rough sea surface, the effective reflection coefficient is used in DMFT [8].

#### **B.** Reflection from a rough surface

In the asymptotic models of a rough sea surface, the reflection coefficient are corrected using the Kirchhoff approximation. If  $\Gamma_0$  represents the reflection coefficient of a smooth sea surface,  $\xi$  is the random variable of the

surface height,  $p(\xi)$  is the PDF of  $\xi$ ,  $\alpha$  is the grazing incidence angle, the effective reflection coefficient can be written as:

$$\Gamma_e = \rho \Gamma \,, \tag{4}$$

where the function  $\rho$  is the roughness reduction factor, which is defined as:

$$\rho(\alpha) = \int_{-\infty}^{+\infty} \exp(-2ik_0\xi\sin\alpha)p(\xi)d\xi .$$
 (5)

### C. Asymptotic model of a rough surface

In the Ament approximation, the PDF of the sea surface heights is regarded as having a Gaussian distribution with zero mean and variance  $\sigma_{\xi}^2$ , written as:

$$p_{\rm A}(\xi) = \frac{1}{\sigma_{\xi}\sqrt{2\pi}} \exp\left(-\frac{\xi^2}{2\sigma_{\xi}^2}\right). \tag{6}$$

By inserting Eq. (6) into Eq. (4), one obtains:

$$\Gamma_{\rm A}\left(\alpha,\sigma_{\xi}\right) = \exp\left(-0.5\gamma^2\sigma_{\xi}^2\right)\Gamma_0,\tag{7}$$

where  $\gamma = 2k_0 \sin \alpha$ .

The PDF of the sea surface height is different in the MB model. The height of the rough sea surface is assumed to take the form  $\xi = H \sin \varphi$ , where the magnitude *H* has a Gaussian distribution with zero mean and variance  $\sigma_{\xi}^2$  and the phase  $\varphi$  is uniformly distributed in the interval  $\left[-\pi/2, \pi/2\right]$ . Therefore, the PDF of the surface height is:

$$p_{\rm MB}\left(\xi\right) = \frac{1}{2\pi^{3/2}\sigma_{\xi}} \exp\left(-\frac{\xi^2}{8\sigma_{\xi}^2}\right) K_0\left(\frac{\xi^2}{8\sigma_{\xi}^2}\right),\qquad(8)$$

where  $K_0(\bullet)$  is a modified Bessel function of the second kind of order zero. By inserting Eq. (8) into Eq. (4), one can obtain the effective reflection coefficient of the MB model:

$$\Gamma_{\rm MB} = \exp\left(-0.5\gamma^2 \sigma_{\xi}^2\right) I_0\left(0.5\gamma^2 \sigma_{\xi}^2\right) \Gamma, \qquad (9)$$

where  $I_0(\bullet)$  is the modified Bessel function of the first kind of order zero.

#### D. Shadowing effect and its extremum approximation

The SE model is based on the Ament model. If  $\alpha$  is the grazing incidence angle, the PDF of the illumination surface height is written as [10,15]:

$$p_{\rm SHD}(\xi;\alpha) = p_{\rm A}(\xi)S(\xi,\alpha,\sigma_{\xi},\sigma_{r}), \qquad (10)$$

where  $\sigma_r$  is the root mean square slope of the surface;  $S(\xi;\alpha)$  is a function called the shadow factor and is defined as:

$$S(\xi, \alpha, \sigma_{\xi}, \sigma_{r}) = \frac{2B}{1 - \exp(-2B)} \exp\left[-B \operatorname{erfc}\left(\frac{\xi}{\sqrt{2}\sigma_{\xi}}\right)\right], \quad (11)$$
$$B = \frac{\sigma_{r}}{2\sqrt{2\pi}\tan\alpha}.$$

Inserting Eq. (11) into Eq. (5) yields the roughness reduction factor with the shadowing effect:

$$\rho_{\text{SHD}}(\alpha, \sigma_{\xi}) = \int_{-\infty}^{+\infty} \exp(-i\gamma\xi) p_{\text{SHD}}(\xi) d\xi$$
$$= \frac{2B}{1 - \exp(-2B)} \int_{-\infty}^{+\infty} \exp(-i\gamma\xi) \qquad (12)$$
$$\times \exp\left[-B \operatorname{erfc}\left(\frac{\xi}{\sqrt{2}\sigma_{\xi}}\right)\right] p_{\text{A}}(\xi) d\xi$$

Therefore, the reflection coefficient of the shadowing effect for the coherent reflected field can be written as:

$$\Gamma_{\rm SHD} = \rho_{\rm SHD} \left( \alpha, \sigma_{\xi} \right) \Gamma_0 \,. \tag{13}$$

It is evident from Eq. (13) that the SE model is computationally expensive because an integral operator is needed for the calculation of the effective reflection coefficient. Since the PDF of the illumination surface height has an approximately Gaussian distribution, the PDF of the SE model can be simplified by assuming a Gaussian distribution [9]. However, the calculation of the mean and variance still require integral operators using the illumination height PDF  $p_{\text{SHD}}(\xi; \alpha)$ . Therefore, an extremum approximation method is proposed in this study to obtain the reflection coefficient of the shadowing effect in a relatively short time.

Because the mean and variance of the Gaussian distribution are related to the extremum of the PDF, the illumination surface height PDF  $p_{SHD}(\xi;\alpha)$  can be Gaussian fitted at its extreme point.

If 
$$h = \frac{\xi}{\sqrt{2}\sigma_{\xi}}$$
, the  $p_{\text{SHD}}(\xi;\alpha)$  can be expressed as:

$$p_{\text{SHD}}(\xi;\alpha) = f(h) = A \exp\left[-B \operatorname{erfc}(h) - h\right], \quad (14)$$

where the parameter A is:

$$A = \frac{2B}{1 - \exp(-2B)} \frac{1}{\sqrt{2\pi}\sigma_{\xi}}.$$
 (15)

If calculating the derivative of f(h) and making f'(h) = 0; then the parameter *h* must satisfy:

$$\frac{B}{\sqrt{\pi}}\exp\left(-h^2\right) - h = 0.$$
 (16)

Transform Eq. (16) and it can be expressed as:

$$-2\ln\frac{\sqrt{\pi}h}{B} \times \exp\left(-2\ln\frac{\sqrt{\pi}h}{B}\right) = 2\frac{B^2}{\pi}.$$
 (17)

If 
$$k = -2\ln \frac{\sqrt{\pi}h}{B}$$
, Eq. (17) can be written as:  
 $k \exp(k) = 2B^2/\pi$ . (18)

The extreme point h in Eq. (14) can be calculated using the Lambert function:

$$k = \mathcal{L}\left(2B^2/\pi\right),\tag{19}$$

where  $L(\bullet)$  represents the Lambert function.

Therefore, the mean  $m'_{\xi}$  and root mean square  $\sigma'_{\xi}$  can be written as:

$$\begin{cases} m'_{\xi} = \sqrt{2}\sigma_{\xi}h\\ \sigma'_{\xi} = \frac{1}{\sqrt{2\pi}p_{SHD}(m'_{\xi};\alpha)}, \end{cases}$$
(20)

Then, the PDF of the illumination surface height can be described as:

$$p_{\rm E}\left(\xi \mid \alpha\right) = \frac{1}{\sigma_{\xi}'\sqrt{2\pi}} \exp\left(-\frac{\left(\xi - m_{\xi}'\right)^2}{2\sigma_{\xi}'^2}\right),\qquad(21)$$

and the effective reflection coefficient is:

$$\Gamma_{\rm E} = \exp\left(-iQm'_{\xi} - 0.5\gamma^2 {\sigma'_{\xi}}^2\right)\Gamma_0.$$
<sup>(22)</sup>

The phase of the effective reflection coefficient is related to the mean value of the surface height and its magnitude is related to the variance of the surface height. The Ament model and the MB model are only corrected with regard to the magnitude of the reflection coefficient but not the phase. However, in the SE model and the proposed model, the magnitude and phase of the coefficient are corrected.

# **III. SIMULATION AND VERIFICATION**

In order to determine the precision and efficiency of the EA of SE model, the PDF and the propagation factor (PF) of the models are compared using a simulation. The simulation is performed in MATLAB 2016a on a computer with an Intel i5 3.2 GHz CPU.

# A. Comparison of the PDF

The PDFs of the models are calculated and compared. Assuming that the grazing incidence angle  $\alpha$  is 0.5°, the root mean square of the surface height and slope are calculated using the Elfouhaily wave spectrum at a wind speed of 7 m/s [16]. Figure 1 shows the PDFs of the different models.

In Fig. 1, the symbols  $p_A$ ,  $p_{MB}$ ,  $p_{SHD}$ , and  $p_E$  represents the PDFs of the Ament model, MB model, SE model, and EA of SEmodel respectively. The mean value and the root mean square of the  $p_A$  and  $p_{MB}$  are 0 and 0.3199. The mean values of the  $p_{SHD}$  and  $p_E$  are 0.4090 and 0.3989 and the root mean square values are

0.2233 and 0.2106 respectively. The results in Fig. 1 show that the mean value is higher for the  $p_{\rm SHD}$  than the  $p_{\rm A}$ . That is because the PDF of the Ament model only takes into account the wind speed (surface height variance  $\sigma_{\xi}^2$ ) but the PDF of the SE model also takes in account the grazing incidence angle  $\alpha$  and only the illumination surface is considered. As the value of  $\alpha$  decreases, the illumination surface decreases, which results in an increase in the surface height mean value and a decrease in the root mean square of the  $p_{\rm SHD}$ , as shown in Fig. 2. The curves of the  $p_{\rm SHD}$  and  $p_{\rm E}$  are similar but there are some differences because the PDF of the SE model does not have a perfect Gaussian distribution.



Fig. 1. PDF of surface height.



Fig. 2. Mean value and root mean square vs grazing incidence angle.

#### **B.** Comparison of the PF under simple conditions

A horizontally polarized 3-GHz line source at a height of 10 m was used to compute the PF over the sea using a geometrical optics (GO) two-ray model. In the simulation, the propagation range is 1 km, the height is 50 m, the number of height grid points is 250, the wind speed is 7 m/s, and the atmospheric effects and the earth's curvature are ignored. The PFs of the four models are shown in Fig. 3.



Fig. 3. Propagation factors under simple conditions.

The symbols  $\eta_{\rm A}$ ,  $\eta_{\rm MB}$ ,  $\eta_{\rm SHD}$ , and  $\eta_{\rm E}$  represent the PFs calculated by the Ament model, MB model, SE model, and the proposed model respectively. It is evident that, after the shadowing effect is considered, the results of  $\eta_{\text{SHD}}$  and  $\eta_{\text{A}}$  exhibit clear differences in the heights and maximums of the peaks and nulls. The heights of the PF peaks and nulls are determined by the phase of the reflection coefficient, which is different in the Ament model and the SE model. The mean value of the surface height is zero in the Ament model; therefore, its phase of the reflection coefficient is equal to that of a smooth surface. However, the phase is different in the SE model because only the illumination surface is considered, which results in different heights of the peaks and nulls. The maximums of the peaks and nulls are determined by the magnitude of the reflection coefficient. In the SE model, only the reflection field from the illuminated surface is considered and this results in an increase in magnitude of the effective reflection coefficient. The increase in the reflection field results in an increase in the maximums of the peaks and nulls. The heights of the peaks and nulls are identical in  $\,\eta_{\rm A}\,$  and  $\,\eta_{\rm MB}\,$  but the maximums are different, which means the phases of the reflection coefficient are identical in  $\,\eta_{\rm A}\,$  and  $\,\eta_{\rm MB}\,$  but the magnitude is larger in  $\eta_{\mathrm{MB}}$  . A comparison between  $\eta_{\rm SHD}$  and  $\eta_{\rm MB}$  shows that the maximum of  $\eta_{\rm SHD}$  is larger at a low altitude but smaller at a high altitude. This is attributed to the smaller grazing incidence angle  $\alpha$  at a low altitude, which results in a larger effective reflection coefficient and maximums of the peaks and nulls in  $\eta_{\rm SHD}$  than in  $\eta_{\rm MB}$ ; the opposite result is observed at high altitude.  $\eta_{\text{SHD}}$  and  $\eta_{\text{E}}$  have similar results with an average error of 0.07 dB, which indicates that the EA of SE model has a similar precision as the SE model.

The computation times of the four models  $\eta_A$ ,  $\eta_{\rm MB}$ ,  $\eta_{\rm SHD}$ , and  $\eta_{\rm E}$  are 0.002 s, 0.008 s, 8.94 min, and 0.781 s respectively. Because the integral operator is needed in the SE model, its computation time is much greater than that of the other models, which means the EA of SE model improves the efficiency while considering the shadowing effect.

# C. Comparison of the PFs under complex conditions

The PE model and the asymptotic model of a rough surface are used to calculate the PF under tropospheric ducting conditions and a rough sea surface. A Gaussian antenna pattern is used with a  $3^{\circ}$  lobe width,  $0^{\circ}$  elevation, and 9 GHz frequency at 10 m height. The wind speed is 5 m/s and the duct height is 15 m. Because the EM energy is influenced by the tropospheric duct, some of the EM energy is trapped in the duct layer, which results in the increase in the PF over the sea. Four asymptotic models are used in the PE model to calculate the PF at a range of 100 km; the PFs are plotted versus the height and are shown in Fig. 4.



Fig. 4. Propagation factors under complex conditions.

The symbols  $\eta_A$ ,  $\eta_{MB}$ ,  $\eta_{SHD}$ , and  $\eta_E$  represent the PFs calculated by the PE with the Ament model, MB model, SE model, and the proposed model respectively. Figure 4 shows that  $\eta_A$  and  $\eta_{MB}$  have similar curves. However, the  $\eta_{\rm MB}$  , which maximum is 10.277 dB, is slightly larger than the  $\eta_A$ , which maximum is 10.252 dB, as its magnitude of the reflection coefficient is larger in the MB model than in the Ament model, which results in a stronger reflection field in  $\eta_{\mathrm{MB}}$  . It is also observed that the  $\eta_{\rm SHD}$  is larger than the  $\eta_{\rm A}$  and  $\eta_{\rm MB}$ . The reason is that, when the EM energy propagates in the tropospheric duct, the grazing incidence angle  $\alpha$  is small; therefore, the magnitude of the effective reflection coefficient is larger than that of the Ament model and MB model and the maximum of the  $\eta_{SHD}$  is about 1 dB larger than that of the  $\eta_A$ . However, the computation time of the  $\eta_{\rm SHD}$  (11.43 min) is much longer than that of the  $\eta_{\rm A}$  (0.495 s). The  $\eta_{\rm E}$  and  $\eta_{\rm SHD}$  have similar curves but the  $\eta_{\rm E}$  has a shorter computation time (1.158 s), which means the approximation model is a more efficient model with similar results.

### **IV. EXPERIMENT AND VERIFICATION**

The experimental data were obtained at Dachen Island in China in 2007 and are used for the verification of the models [17]. The atmospheric conditions and the tropospheric duct were measured using a weather balloon and a meteorograph. The radar transmitting antenna operating in the X band was mounted on the beach at a height of 15 m. A horn antenna was used as the receiving antenna and was mounted on a boat at a height of 2 m. A meteorograph was used on the boat for measuring the wind speeds, humidity, temperature, and pressure. The wind speed was 2–4 m/s during the experiment. The PE model and the asymptotic model of a rough surface were used to calculate the propagation loss and the results are shown in Fig. 5.



Fig. 5. Range versus propagation loss.

In Fig. 5, the blue points are the experimental results and the symbols  $L_A$  and  $L_{MB}$  represent the Ament model and MB model, whose curves are identical because the effective reflection coefficients are very close to the smooth surface reflection coefficient due to a low wind speed. The symbols  $L_{SHD}$  and  $L_{E}$ represent the SE model and the proposed model, whose curves are also identical. The effective reflection coefficients of the two models are close to the smooth surface reflection coefficient in magnitude but different in the phases, which results in the differences in the propagation losses for  $L_A$  and  $L_{MB}$ . The average errors of these models are shown in Table 1. Compared with the experimental results, the SE model and the proposed model have average errors that are nearly 1 dB lower than those of the other models. The computation time of the proposed model is about 600 times shorter than that of the SE model.

Model	L <sub>A</sub>	L <sub>MB</sub>	L <sub>SHD</sub>	L <sub>E</sub>
Average Error (dB)	3.47	3.47	2.54	2.55
Computation Time	0.340 s	0.320 s	10.14 min	1.074 s

Table 1: Computation time and average error

The results are close to the experimental results after the shadowing effect is considered but errors still remain. The errors may be caused by the following:

1) Measurement errors. The weather sensors in the weather balloon have a slow reaction time due to a 1-Hz sampling rate. The accuracy of the pressure sensor is 0.3 hpa, which means that the accuracy of the height calculated by the barometric formula is  $\pm 3$  m. Therefore, an error exists in the refractive index profile, even after using the noise reduction method.

2) Errors caused by the horizontal inhomogeneity and turbulence of the tropospheric duct. There are some random changes in the refractive index profile at different ranges and heights, which results in the errors of the propagation loss.

3) Model errors. The parameter of the grazing incidence angle  $\alpha$  is needed at different ranges in the PE model and the SE model. The angle  $\alpha$  is obtained by a ray-tracing method and errors may occur because it is an approximation method.

## V. CONCLUSION

The Ament model and MB model are routinely used in the PE model for the rapid prediction of the EM propagation characteristics under tropospheric ducting conditions over the sea surface. However, the shadowing effect caused by the rough sea surface is not considered in these models. The SE model has a higher accuracy but is computationally expensive. In this study, a new approximation model is proposed to simplify the shadowing effect; this model has a higher computational efficiency and similar accuracy as the SE model. A comparison with the other models and the experimental results indicate that the EA of SE model used in conjunction with the PE is a fast and accurate model for the calculation of the EM propagation characteristics. Therefore, the proposed model satisfied the needs of the real-time evaluation of the radar performance to improve the battlefield survivability.

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