Equivalent Electromagnetic Currents on Infinite Stratified Homogeneous Bi-anisotropic Media Backed by A PEC Layer

Sen Tian[1], Zhuo Li[1, 2], Chang Qing Gu[1], and Dan Ping Ding[3]

[1] College of Electronic Information Engineering
Nanjing University of Aeronautics and Astronautics, Nanjing 210016, P. R. China

[2] State Key Laboratory of Millimeter Waves
Southeast University, Nanjing 210096, P. R. China

[3] Nanjing Panda Handa Company Limited
Nanjing 210002, P. R. China

Abstract - In this work, the equivalent electromagnetic (EM) currents on the surface of stratified homogeneous bi-anisotropic media backed by a perfect electric conductor (PEC) layer are derived and investigated. By using the representation of Maxwell's equations with a first-order state-vector differential equation, the tangential field components and the corresponding equivalent EM currents at the interface between the outmost bi-anisotropic media layer and the free space is derived analytically and can be easily degenerated into the single anisotropic and isotropic cases. This work is considered as a further step in the study of the EM characteristics of stratified complex media and the obtained results may provide a way for approximately fast calculation of the EM scattering from PEC targets coated by stratified homogeneous bi-anisotropic media. Simulation results are given to validate our analysis and conclusions.

Index Terms **–** Bi-anisotropic media, equivalent electromagnetic current, high frequency method, scattering.

I. INTRODUCTION

In modern warfare, stealth technology has been becoming one of the most important technologies for military targets. Besides shape stealth, coating is the most commonly used method. The equipped aircrafts and warships are usually coated with one-layer or multi-layer radar absorption materials (RAM), which are commonly anisotropic or bi-anisotropic. To obtain the scattering characteristics and the evaluation of these stealth targets, several full-wave methods have been employed, in which the finite-difference time-domain (FDTD) method [1-3], the couple dipole approximation method [4], the method of moments (MoM) [5], and the finite element method (FEM) are the most representative ones. However, these methods are not suitable for electrically large EM scattering problems owing to limited computational resources. Asymptotic methods, such as the physical optics (PO) method with its extension the physical theory of diffraction (PTD), the geometrical optics (GO) method with its extension the geometrical theory of diffraction (GTD), and the shooting and bouncing ray (SBR) method, can solve electrically large problems at the cost of losing precision since these approximate methods do not exactly capture the EM characteristics of complex media. Our work here concentrates on the hybridization of the full-wave and the asymptotic methods based on the generalized field equivalence principle. As we know, the PO method utilizes the induced PO currents on the surface of PEC objects to calculate the scattered field based on the high frequency approximation that the induced equivalent electric current $\vec{J}$ at a certain point is solely determined by the incident magnetic field $\vec{H}$ and the normal vector $\hat{n}$ at that point with the formula $\vec{J} = 2\hat{n}\times\vec{H}$. Another assumption is also made in this approximation that each illuminated point is regarded as an infinite plane without considering the curvature at that point. Thus, the mutual coupling of two arbitrary induced currents is not taken into consideration. This approximation is well suited for electrically large EM scattering problems. So in this work, we inherit this approximation idea for the extraction of equivalent electric and magnetic currents on the surface of an infinite stratified homogeneous bi-anisotropic media backed by a PEC layer. These analytical equivalence currents give us an alternative solution for fast approximate estimation of the EM scattering from complex coated PEC targets through asymptotic methods.

In 1950, Abelès [6] provided a solution called characteristic matrix to solve the problem of EM wave propagation in isotropic stratified medium in a single direction. Later in the 1970s, Teitler and Henvis [7] applied Abelès's characteristic matrix method to the stratified anisotropic media. Then, Berreman [8] introduced a 4×4 matrix technique to solve the problem of reflection and transmission of EM waves in anisotropic material. Graglia [9] used integro-differential equations to solve the scattering problem in linear, lossy, anisotropic media. References [10] provided another approach that employs a first-order state-vector differential equation representation of Maxwell's equations and uses a 4×4 transition matrix relating the tangential field components at the input and output planes of the anisotropic region to represent the solution. References [11-13] provide several analytical methods to the propagation problem in stratified anisotropic media. All of the above works only considered the reflection and transmission of EM waves in the stratified anisotropic material.

In this paper, we focus on the equivalent EM currents extraction on the surface of anisotropic or bi-anisotropic media backed by a PEC layer, which can be employed in the estimation of EM scattering from electrically large PEC targets coated by stratified anisotropic or bi-anisotropic media. In Section II and III, the EM wave propagation in multiple layered infinite bi-anisotropic media is studied, in which the tangential EM components in bi-anisotropic material are derived from Maxwell's equation in the form of a 4×4 matrix. After imposing the boundary condition that the tangential EM components are continuous across the boundary of two different media and the tangential electric field is zero on the surface of the PEC layer, the relation of the tangential EM field components of the outmost and inmost layer of the stratified bi-anisotropic is obtained. Then the total tangential EM fields on the surface of the outmost anisotropic media layer are derived analytically and the equivalent EM currents on the surface of the outmost layer of bi-anisotropic are obtained. Finally, we show in Section III that these equivalent currents can be easily degenerated into the single anisotropic and isotropic cases. Simulation results are given in Section IV to validate our conclusions.

II. PROPAGATION EQUATION IN STRATIFIED BIANISOTROPIC MATERIAL

In this paper, the infinite stratified homogeneous bi-anisotropic coating is viewed as a one-dimensional issue. As shown in Fig.1, that a monochromatic plane wave is obliquely incident from free space to an infinite stratified homogeneous bi-anisotropic media backed with a PEC layer. Each layer is assumed to be infinite towards x and y directions. The position of each layer along the z direction is set as $z = d_n\,(n = 0,1,2\cdots,N)$.

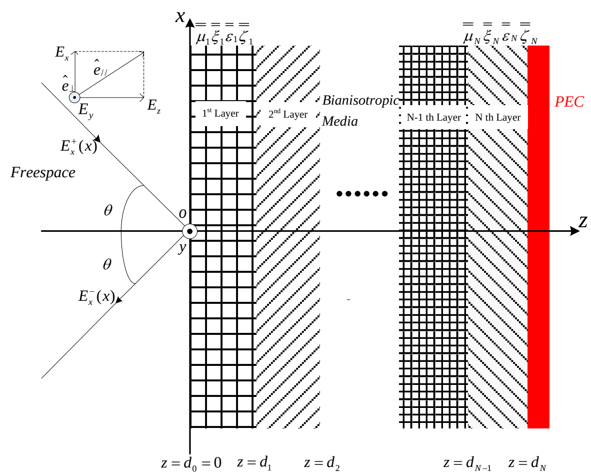


Fig. 1. A monochromatic plane wave is obliquely incident from free space to an infinite stratified homogeneous bi-anisotropic media backed with a PEC layer.

Imposing the phase matching condition in each layer, the x-component of wave vector k_n should be equal and denoted by k_x, in which the subscript n stands for the nth layer. Using the $e^{j\omega t}$ time convention, the phasor fields in the nth layer can be written in a separable product form as below

$$\begin{aligned}\vec{E}_n(x,z) &= \vec{E}_n(z)e^{-jk_x x},\\ \vec{H}_n(x,z) &= \vec{H}_n(z)e^{-jk_x x},\end{aligned} \tag{1}$$

where $k_x = k_0 \sin\theta$ is the x-component of k_0. k_0 is the incident wave number in free space and θ is the incident angle. For bi-anisotropic materials, the source-free Maxwell's equations can be expressed as

$$
\begin{aligned}
-\nabla \times \vec{E}_n &= j\omega \bar{\bar{\mu}}_n \cdot \vec{H}_n + j\omega \bar{\bar{\varsigma}}_n \cdot \vec{E}_n, \\
\nabla \times \vec{H}_n &= j\omega \bar{\bar{\xi}}_n \cdot \vec{H}_n + j\omega \bar{\bar{\varepsilon}}_n \cdot \vec{E}_n,
\end{aligned} \tag{2}
$$

where $\bar{\bar{\varepsilon}}_n$, $\bar{\bar{\mu}}_n$ are the dyadic permittivity and permeability, $\bar{\bar{\varsigma}}_n$, $\bar{\bar{\xi}}_n$ are the dyadic magneto electric parameters as follows:

$$
\bar{\bar{\varepsilon}}_n = \begin{bmatrix} \varepsilon_{n11} & \varepsilon_{n12} & \varepsilon_{n13} \\ \varepsilon_{n21} & \varepsilon_{n22} & \varepsilon_{n23} \\ \varepsilon_{n31} & \varepsilon_{n32} & \varepsilon_{n33} \end{bmatrix},
\bar{\bar{\mu}}_n = \begin{bmatrix} \mu_{n11} & \mu_{n12} & \mu_{n13} \\ \mu_{n21} & \mu_{n22} & \mu_{n23} \\ \mu_{n31} & \mu_{n32} & \mu_{n33} \end{bmatrix},
\bar{\bar{\varsigma}}_n = \begin{bmatrix} \varsigma_{n11} & \varsigma_{n12} & \varsigma_{n13} \\ \varsigma_{n21} & \varsigma_{n22} & \varsigma_{n23} \\ \varsigma_{n31} & \varsigma_{n32} & \varsigma_{n33} \end{bmatrix},
\bar{\bar{\xi}}_n = \begin{bmatrix} \xi_{n11} & \xi_{n12} & \xi_{n13} \\ \xi_{n21} & \xi_{n22} & \xi_{n23} \\ \xi_{n31} & \xi_{n32} & \xi_{n33} \end{bmatrix}. \tag{3}
$$

Substituting Eq. (1) into Eq. (2) and canceling the common exponential factor, six first-order differential equations can be derived. By eliminating the *z*-component of the EM field, the above six equations can be reduced to four independent equations. If we define a state vector in terms of the transverse field components of $\vec{E}_n$ and $\bar{H}_n$

$$
\Psi_n(z) = \begin{bmatrix} \Psi_n^{(1)}(z) \\ \Psi_n^{(2)}(z) \\ \Psi_n^{(3)}(z) \\ \Psi_n^{(4)}(z) \end{bmatrix} = \begin{bmatrix} E_{nx}(z) \\ E_{ny}(z) \\ H_{nx}(z) \\ H_{ny}(z) \end{bmatrix}, \tag{4}
$$

the four independent first-order differential equations can be expressed as

$$
\frac{d}{dz}\Psi_n(z) = \Gamma_n \cdot \Psi_n(z), \tag{5}
$$

in which the complex elements of the 4×4 Γ matrix are given in Appendix(I).

The solutions of Eq. (5) have the form of non-uniform plane waves as follow

$$
\Psi_n(z) = B_n E_n A_n, \tag{6}
$$

where $E_n = Diag[e^{\lambda_{n1}z}, e^{\lambda_{n2}z}, e^{\lambda_{n3}z}, e^{\lambda_{n4}z}]$, in which λ_{nj} (j=1,2,3,4) are the eigenvalues of the Γ_n matrix and can be easily found by solving the roots of the unitary complex quartic equation in Appendix (II). B_n is a 4×4 matrix with its elements $b_{nij} = \Delta_{nij}/\Delta_{n1j}$, ($i, j$=1, 2,3,4) see Δ_{nij} in the Appendix (III). In general anisotropic or bi-anisotropic case, λ_{nj} (j=1,2,3,4) are two pairs of conjugate complex roots, which represent the type I and type II waves going in the positive and negative *z* directions [19]. And the column vectors of B_n are the corresponding eigenvectors. In isotropic cases, λ_{nj} (j=1, 2, 3, 4) are two identical pairs of conjugate complex roots and B_n has four linearly independent eigenvectors. So, B_n is always reversible in any case. $A_n = \left[a_1^{(n)}, a_2^{(n)}, a_3^{(n)}, a_4^{(n)}\right]^T$ is the unknown column vector to be determined, in which $a_i^{(n)}$ (i=1, 2, 3, 4) are the unknown coefficients of the tangential fields in the *n*th layer. It can be seen from Eq. (6) that once the A_n is known, the tangential components of EM wave in the *n*th layer can be evaluated analytically.

III. DERIVATION OF THE EM FIELDS AND EQUIVALENT CURRENTS ON THE SURFACE OF STRATIFIED BIANISOTROPIC MEDIA BACKED BY A PEC LAYER

Imposing the boundary condition at the interface between the *nth* and (n+1) th layer that the tangential EM fields are continuous $\Psi_n(z=d_n) = \Psi_{n+1}(z=d_n)$, the following matrix equations can be derived

$$
\begin{aligned}
B_n E_{Rn} A_n &= B_{(n+1)} E_{L(n+1)} A_{(n+1)}, \\
E_{L(n+1)}^{-1} B_{(n+1)}^{-1} B_n E_{Rn} A_n &= A_{(n+1)},
\end{aligned} \tag{7}
$$

where

$$
\begin{aligned}
E_{Rn} &= Diag[e^{\lambda_{n1}d_n}, e^{\lambda_{n2}d_n}, e^{\lambda_{n3}d_n}, e^{\lambda_{n4}d_n}], \\
E_{Ln} &= Diag[e^{\lambda_{n1}d_{n-1}}, e^{\lambda_{n2}d_{n-1}}, e^{\lambda_{n3}d_{n-1}}, e^{\lambda_{n4}d_{n-1}}].
\end{aligned} \tag{8}
$$

By using the boundary condition at N-1 interfaces (from $z = d_1$ to $z = d_{n-1}$) repeatedly, the relation of the tangential electric fields between the outmost and inmost layers can be derived as

$$E_{LN}^{-1} B_N^{-1} B_{(N-1)} E_{R(N-1)} \cdots E_{L2}^{-1} B_2^{-1} B_1 E_{R1} A_1 = A_N. \tag{9}$$

On the surface of the PEC layer, tangential electric fields are naturally set to zero that $B_{N/2} E_{RN} A_N = 0$, in which $B_{N/2}$ is a 2×4 matrix with the elements from the first two rows of matrix B_N. Thus, we can get $C \times A_1 = 0$, where

$$C = B_{N/2} E_{RN} E_{LN}^{-1} B_N^{-1} B_{(N-1)} E_{R(N-1)} \cdots E_{L2}^{-1} B_2^{-1} B_1 E_{R1} \tag{10}$$

is a 2×4 matrix.

From Maxwell's equations, the transverse EM fields in the left half free space can be expressed as the sum of the incident wave and the reflected wave

$$\begin{aligned} E_x(x,z) &= E_x^+(x) e^{-jk_z z} + E_x^-(x) e^{jk_z z}, \\ E_y(x,z) &= E_y^+(x) e^{-jk_z z} + E_y^-(x) e^{jk_z z}, \\ H_x(x,z) &= \eta_1 [-E_y^+(x) e^{-jk_z z} + E_y^-(x) e^{jk_z z}], \\ H_y(x,z) &= \eta_2 [E_x^+(x) e^{-jk_z z} - E_x^-(x) e^{jk_z z}], \end{aligned} \tag{11}$$

where $E_x^+(x) = E_{x0}^+ e^{-jk_x x}$, $E_y^+(x) = E_{y0}^+ e^{-jk_x x}$ are the transverse electric fields of incident wave at $z = d_0 = 0$, which are known, and $E_x^-(x) = E_{x0}^- e^{jk_x x}$, $E_y^-(x) = E_{y0}^- e^{jk_x x}$ are the reflected electric fields, which are unknown. $k_z = k_0 \cos\theta$ is the incident field wave propagation vector in the z direction. $\eta_1 = \cos\theta / \eta_0$, $\eta_2 = 1/(\eta_0 \cos\theta)$, $\eta_0 = 120\pi\,\Omega$ is the wave impedance in free space. By using the boundary condition that the tangential electric fields must be continuous across the interface (z=0) of free space and the outmost layer of the stratified bi-anisotropic media, the following matrix can be obtained

$$\begin{aligned} B_1 E_{L1} A_1 &= [E_x(x,z=0), E_y(x,z=0), \\ & H_x(x,z=0), H_y(x,z=0)]^T. \end{aligned} \tag{12}$$

Considering Eqs. (10 and 12) and through some matrix operations, the unknown $E_x^-(x)$ and $E_y^-(x)$ can be expressed as

$$\begin{bmatrix} a_1^{(1)} \\ a_2^{(1)} \\ a_3^{(1)} \\ a_4^{(1)} \\ E_x^-(x) \\ E_y^-(x) \end{bmatrix} = Q^{-1} \begin{bmatrix} E_x^+(x) \\ E_y^+(x) \\ -\eta_1 E_y^+(x) \\ \eta_2 E_x^+(x) \\ 0 \\ 0 \end{bmatrix},$$

$$Q = \begin{bmatrix} b_{111} & b_{112} & b_{113} & b_{114} & -1 & 0 \\ b_{121} & b_{122} & b_{123} & b_{124} & 0 & -1 \\ b_{131} & b_{132} & b_{133} & b_{134} & 0 & -\eta_1 \\ b_{141} & b_{142} & b_{143} & b_{144} & \eta_2 & 0 \\ c_{11} & c_{12} & c_{13} & c_{14} & 0 & 0 \\ c_{21} & c_{22} & c_{23} & c_{24} & 0 & 0 \end{bmatrix}, \tag{13}$$

and c_{ij} (i=1,2;j=1,2,3,4) are the elements of matrix C in Eq.(10).

Finally, the relation between the incident and reflected tangential electric fields can be simplified as

$$\begin{bmatrix} E_x^-(x) \\ E_y^-(x) \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} E_x^+(x) \\ E_y^+(x) \end{bmatrix}, \tag{14}$$

where

$$s_{11} = (q_1 + \eta_2 q_2)/|Q|, \qquad s_{12} = (q_3 + \eta_1 q_4)/|Q|,$$
$$s_{21} = (q_5 + \eta_2 q_6)/|Q|, \qquad s_{22} = (q_7 + \eta_1 q_8)\ /|Q|,$$

and $q_j\,(j = 1, 2 \cdots 8)$ can be found in Appendix (III).

So far the reflected tangential EM fields on the surface of the outmost layer are obtained, which can be added to the incident fields to obtain the total tangential EM fields on the surface of the outmost layer. Thus, the equivalent EM currents can be derived as follows:

$$\begin{aligned} \vec{J}_s &= \hat{n} \times \vec{H}_{total}|_{z=0} = \hat{y}[-s_{21} E_x^+(x) \\ &+ (1 - s_{22}) E_y^+(x)] \eta_1 \\ &+ \hat{x}[(1 - s_{11}) E_x^+(x) - s_{12} E_y^+(x)] \eta_2, \\ \vec{J}_{ms} &= \vec{E}_{total} \times \hat{n}|_{z=0} = \hat{y}[(1 + s_{11}) E_x^+(x) \\ &+ s_{12} E_y^+(x)] \eta_1 - \hat{x}[s_{21} E_x^+(x) \\ &+ (1 + s_{22}) E_y^+(x)] \eta_2. \end{aligned} \tag{15}$$

Equation (15) can also be written in the following general forms

$$\vec{J}_s = \hat{e}_\perp[-s_{21}E^i_{//}\cos\theta + (1-s_{22})E^i_\perp]\frac{\cos\theta}{\eta_0}$$
$$+\hat{e}_\omega[(1-s_{11})E^i_{//} - \frac{s_{12}}{\cos\theta}E^i_\perp]\frac{1}{\eta_0}, \quad (16)$$
$$\vec{J}_{ms} = \hat{e}_\perp[(1+s_{11})E^i_{//}\cos\theta + s_{12}E^i_\perp]$$
$$-\hat{e}_\omega[s_{21}E^i_{//}\cos\theta + (1+s_{22})E^i_\perp],$$

where $\hat{e}_\omega = \hat{n}\times\hat{e}_\perp$, $E^i_{//}$ and $E^i_\perp$ are the incident electric fields those are parallel and perpendicular to the incidence plane, respectively, as shown in Fig. 1. Observing Eq. (16), the relations between $\vec{J}_s$ and $\vec{J}_{ms}$ can be expressed as

$$\vec{J}_{ms} = -\eta_0\hat{n}\times(\bar{\bar{Z}}_s\cdot\vec{J}_s), \quad (17)$$

where

$$\bar{\bar{Z}}_s = Z_\perp\hat{e}_\perp\hat{e}_\perp + Z_{//}\hat{e}_\omega\hat{e}_\omega \quad (18)$$

is the normalized dyadic surface impedance to the free space and

$$Z_\perp = \frac{(1+s_{22})E^i_\perp + s_{21}E^i_{//}\cos\theta}{[(1-s_{22})E^i_\perp - s_{21}E^i_{//}\cos\theta]\cos\theta},$$
$$Z_{//} = \frac{[(1+s_{11})E^i_{//}\cos\theta + s_{12}E^i_\perp]\cos\theta}{(1-s_{11})E^i_{//}\cos\theta - s_{12}E^i_\perp}. \quad (19)$$

We remark that Eq. (16) - Eq. (19) give the final results of the equivalent EM currents on the surface of stratified homogeneous bi-anisotropic media backed with a PEC layer, which can be degenerated into the single anisotropic and isotropic cases easily.

Now, we consider the single anisotropic case when the PEC layer is coated with stratified uniaxial or biaxial anisotropic material and the permittivity $\bar{\bar{\varepsilon}}_n$ and permeability $\bar{\bar{\mu}}_n$ are diagonal matrixes

$$\bar{\bar{\varepsilon}}_n = \varepsilon_0\times Diag[\varepsilon^r_{n11}, \varepsilon^r_{n22}, \varepsilon^r_{n3}],$$
$$\bar{\bar{\mu}}_n = \mu_0\times Diag[\mu^r_{n11}, \mu^r_{n22}, \mu^r_{n3}]. \quad (20)$$

In this case, the matrix B_n can be simplified as [17, 18]

$$B_n = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & \frac{\lambda_{n3}}{\Gamma_{n23}} & \frac{\lambda_{n4}}{\Gamma_{n23}} \\ \frac{\lambda_{n1}}{\Gamma_{n14}} & \frac{\lambda_{n2}}{\Gamma_{n14}} & 0 & 0 \end{bmatrix}, \quad (21)$$

where Γ_{nij} (i, j=1,2,3,4) is the element of matrix Γ_n in Eq. (5).

If there is only one anisotropic layer, the following expressions can be obtained that

$$s_{11} = (Z_{//} - \cos\theta)/(Z_{//} + \cos\theta),$$
$$s_{22} = (Z_\perp\cos\theta - 1)/(Z_\perp\cos\theta + 1), \quad (22)$$
$$s_{12} = s_{21} = 0,$$

$$Z_{//} = j\sqrt{\mu^r_{22}\varepsilon^r_{33} - \sin^2\theta}$$
$$\tan(k_0 d\sqrt{\mu^r_{22}\varepsilon^r_{11} - \varepsilon^r_{11}\sin^2\theta/\varepsilon^r_{33}})/\sqrt{\varepsilon^r_{11}\varepsilon^r_{33}},$$
$$Z_\perp = j\sqrt{\mu^r_{11}\mu^r_{33}}$$
$$\tan(k_0 d\sqrt{\mu^r_{11}\varepsilon^r_{22} - \mu^r_{11}\sin^2\theta/\mu^r_{33}})/\sqrt{\mu^r_{33}\varepsilon^r_{22} - \sin^2\theta}. \quad (23)$$

Then Eq. (16) can be simplified as

$$\vec{J}_s = \hat{e}_\perp T_\perp E^i_\perp/(\eta_0 Z_\perp) + (\hat{n}\times\hat{e}_\perp)T_{//}E^i_{//}/(\eta_0 Z_{//}),$$
$$\vec{J}_{ms} = \hat{e}_\perp T_{//}E^i_{//} - (\hat{n}\times\hat{e}_\perp)T_\perp E^i_\perp, \quad (24)$$

where $T_{//} = 2Z_{//}\cos\theta/(Z_{//}+\cos\theta)$, and $T_\perp = 2Z_\perp\cos\theta/(Z_\perp\cos\theta+1)$.

Comparing Eq. (24) with the Eq. (17) and Eq. (18) in [14], we can observe that our generalized form of equivalent EM currents can be degenerated into the single anisotropic case naturally. By comparing Eq. (24) with Eq. (12) and Eq. (13) in [15], we can see that our results can also be degenerated into the isotropic case easily. This equivalent EM current can be employed for RCS prediction of coated targets in free space and half space [16].

IV. VALIDATION AND SIMULATION RESULTS

In this section, several simulation results are given for validation of the deduced analytical equivalent EM currents $\vec{J}_s$ and $\vec{J}_{ms}$. We remark that only the four s parameters ($s_{11,}s_{12,}s_{21,}s_{22}$) are calculated for comparison considering that the s parameters are identical to $\vec{J}_s$ and $\vec{J}_{ms}$ if the incident fields are known based on Eq. (15).

First, we consider a two-layered isotropic lossy media case and a two-layered anisotropic lossy media case. The parameters of isotropic lossy case is as follows

$d_1 = 0.002m,$

$\bar{\bar{\varepsilon}}_1 = \varepsilon_0 \times Diag[15 - j4, 15 - j4, 15 - j4]$

$\bar{\bar{\mu}}_1 = \mu_0 \times Diag[2 - j1.2, 2 - j1.2, 2 - j1.2];$

$d_2 = 0.006m,$

$\bar{\bar{\varepsilon}}_2 = \varepsilon_0 \times Diag[4 - j0.8, 4 - j0.8, 4 - j0.8]$

$\bar{\bar{\mu}}_2 = \mu_0 \times Diag[1.5 - j0.4, 1.5 - j0.4, 1.5 - j0.4];$

and the parameters of the uniaxial anisotropic lossy case is as follows, the frequency is 10 GHz

$d_1 = 0.002m,$

$\bar{\bar{\varepsilon}}_1 = \varepsilon_0$

$\times Diag[10 - j2, 10 - j2, 29.39 - j0.94],$

$\bar{\bar{\mu}}_1 = \mu_0$

$\times Diag[2.24 - j1.68, 2.24 - j1.68, 3.52 - j16];$

$d_2 = 0.006m,$

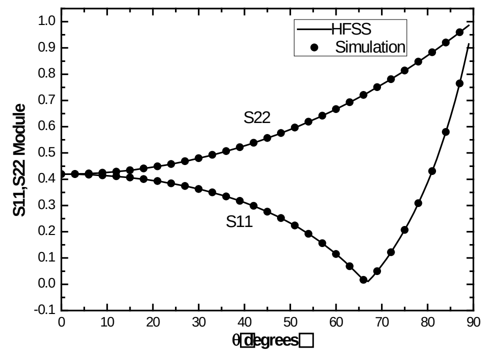
$\bar{\bar{\varepsilon}}_2 = \varepsilon_0$

$\times Diag[25.59 - j3.89, 25.59 - j3.89, 8.19 - j1.30],$

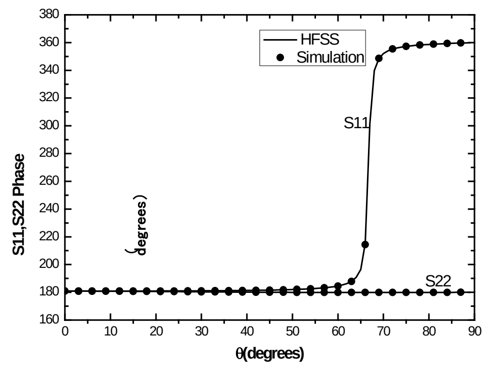
$\bar{\bar{\mu}}_2 = \mu_0$

$\times Diag[2.16 - j1.68, 2.16 - j1.68, 1.39 - j0.56];$

In both of these two cases $s_{12} = s_{21} = 0$. When the incident wave is TE wave, $E_x^+(x) = 0$, $E_y^-(x) = s_{22}E_y^+(x)$, when the incident wave is TM wave, $E_y^+(x) = 0$, $E_x^-(x) = s_{11}E_x^+(x)$, the simulation results of the modulus and phase of s_{11}, s_{22} changed with the incident angle θ are shown in Fig. 2 and Fig. 3 for comparison with HFSS. It is obvious that our results agree quite well with those of HFSS and the analytical calculation in this paper takes no time while the numerical results obtained by HFSS takes more than 5 minutes even calculating one incident angle.



(a)



(b)

Fig. 2. Comparison of the s parameters (s_{11} and s_{22}) simulated in this paper with HFSS in the two layered lossy isotropic case. (a) Module and (b) phase.

Then, we consider a single layered bi-anisotropic case, the parameters are as follows, the frequency is 10 GHz,

$d_1 = 0.006m,$

$\bar{\bar{\varepsilon}}_1 = \varepsilon_0 \times$

$Diag[25.59 - j1.68, 25.59 - j1.68, 8.19 - j1.30],$

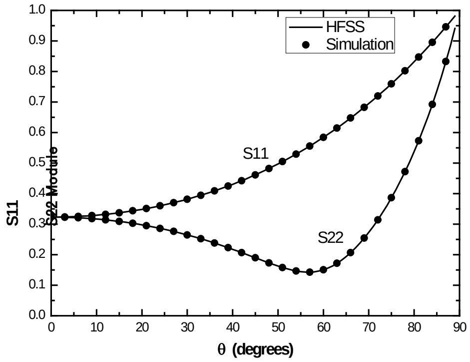
$\bar{\bar{\mu}}_1 = \mu_0 \times$

$Diag[2.16 - j1.68, 2.16 - j1.68, 8.19 - j1.30],$

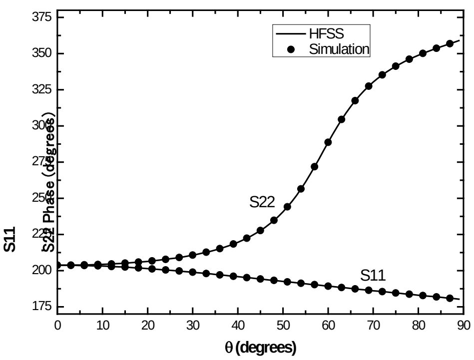
$\bar{\bar{\varsigma}}_1 = \sqrt{\varepsilon_0\mu_0} \times Diag[1.5 - j1.2, 1.5 - j1.2, 1 + j0.2],$

$\bar{\bar{\xi}}_1 = \sqrt{\varepsilon_0\mu_0} \times Diag[1 - j0.8, 1 - j0.8, 1.1 - j0.5].$

As shown in Fig. 4, s_{12} and s_{21} is not zero in this case for bi-anisotropic media is rotational, which causes cross polarization.



(a)



(b)

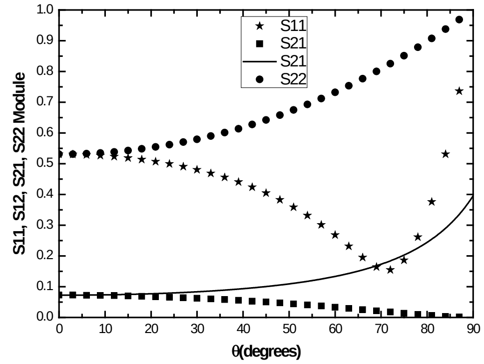
Fig. 3. Comparison of the *s* parameters (s_{11} and s_{22}) simulated in this paper with HFSS in the two layered lossy uniaxial anisotropic case. (a) Module and (b) phase.

V. CONCLUSIONS

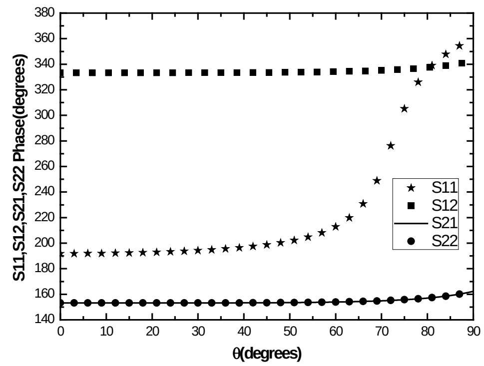
In this work, the analytical expressions of equivalent EM currents on the surface of stratified homogeneous bi-anisotropic media backed by a PEC layer are derived, which can be degenerated into the single anisotropic and isotropic cases. Some simulation results are given to validate our conclusions. These equivalent currents are straight forward and very general for derivation and coding, which can be employed in the quick approximate estimation of EM scattering from electrically large PEC targets coated by stratified isotropic, anisotropic, or bi-anisotropic material.

VI. ACKNOWLEDGEMENT

The work is supported by the No. K201003 Foundation of State Key Laboratory of Millimeter Waves, Southeast University, P. R. China.



(a)



(b)

Fig. 4. The simulation results of the four s parameters (s_{11}, s_{12}, s_{21} and s_{22}) of the bi-anisotropic case. (a) Module and (b) phase.

APPENDIX

APPENDIX (I)

$$\Gamma_{n11} = -j\omega \left(\begin{array}{l} \varsigma_{n21} + \varsigma_{n23} \dfrac{\xi_{n33}\varsigma_{n31} - \mu_{n33}\varepsilon_{n31}}{\mu_{n33}\varepsilon_{n33} - \xi_{n33}\varsigma_{n33}} \\ +\mu_{n23} \dfrac{\varepsilon_{n31}\varsigma_{n33} - \varsigma_{n31}\varepsilon_{n33}}{\mu_{n33}\varepsilon_{n33} - \xi_{n33}\varsigma_{n33}} \end{array} \right)$$

$$-jk_x \frac{\xi_{n33}\varsigma_{n31} - \mu_{n33}\varepsilon_{n31}}{\mu_{n33}\varepsilon_{n33} - \xi_{n33}\varsigma_{n33}}$$

$$\Gamma_{n12} =$$

$$-j\omega \left(\begin{array}{l} \varsigma_{n22} + \varsigma_{n23} \dfrac{\omega(\xi_{n33}\varsigma_{n32} - \mu_{n33}\varepsilon_{n32}) - \xi_{n33}k_x}{\omega(\mu_{n33}\varepsilon_{n33} - \xi_{n33}\varsigma_{n33})} \\ +\mu_{n23} \dfrac{\omega(\varepsilon_{n32}\varsigma_{n33} - \varsigma_{n32}\varepsilon_{n33}) + \varepsilon_{n33}k_x}{\omega(\mu_{n33}\varepsilon_{n33} - \xi_{n33}\varsigma_{n33})} \end{array} \right)$$

$$-jk_x\frac{\omega(\xi_{n33}\varsigma_{n32}-\mu_{n33}\varepsilon_{n32})-\xi_{n33}k_x}{\omega(\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33})}$$

$$\Gamma_{n13}=-j\omega\begin{pmatrix}\mu_{n21}+\mu_{n23}\dfrac{\varsigma_{n33}\xi_{n31}-\varepsilon_{n33}\mu_{n31}}{\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33}}\\+\varsigma_{n23}\dfrac{\mu_{n31}\xi_{n33}-\xi_{n31}\mu_{n33}}{\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33}}\end{pmatrix}$$

$$-jk_x\frac{\xi_{n33}\mu_{n31}-\mu_{n33}\xi_{n31}}{\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33}}$$

$$\Gamma_{n14}=$$

$$-j\omega\begin{pmatrix}\mu_{n22}+\mu_{n23}\dfrac{\omega(\varsigma_{n33}\xi_{n32}-\varepsilon_{n33}\mu_{n32})+\varsigma_{n33}k_x}{\omega(\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33})}\\+\varsigma_{n23}\dfrac{\omega(\mu_{n32}\xi_{n33}-\xi_{n32}\mu_{n33})+\mu_{n33}k_x}{\omega(\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33})}\end{pmatrix}$$

$$-jk_x\frac{\omega(\xi_{n33}\mu_{n32}-\mu_{n33}\xi_{n32})-\mu_{n33}k_x}{\omega(\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33})}$$

$$\Gamma_{n21}=j\omega\begin{pmatrix}\varsigma_{n11}+\varsigma_{n13}\dfrac{\xi_{n33}\varsigma_{n31}-\mu_{n33}\varepsilon_{n31}}{\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33}}\\+\mu_{n13}\dfrac{\varepsilon_{n31}\varsigma_{n33}-\varsigma_{n31}\varepsilon_{n33}}{\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33}}\end{pmatrix}$$

$$\Gamma_{n22}=$$

$$j\omega\begin{pmatrix}\varsigma_{n12}+\varsigma_{n13}\dfrac{\omega(\xi_{n33}\varsigma_{n32}-\mu_{n33}\varepsilon_{n32})-\xi_{n33}k_x}{\omega(\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33})}\\+\mu_{n13}\dfrac{\omega(\varepsilon_{n32}\varsigma_{n33}-\varsigma_{n32}\varepsilon_{n33})+\varepsilon_{n33}k_x}{\omega(\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33})}\end{pmatrix}$$

$$\Gamma_{n23}=j\omega\begin{pmatrix}\mu_{n11}+\mu_{n13}\dfrac{\mu_{n33}\varepsilon_{n31}-\varepsilon_{n33}\mu_{n31}}{\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33}}\\+\varsigma_{n13}\dfrac{\mu_{n31}\xi_{n33}-\xi_{n31}\mu_{n33}}{\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33}}\end{pmatrix}$$

$$\Gamma_{n24}=$$

$$j\omega\begin{pmatrix}\mu_{n12}+\mu_{n13}\dfrac{\omega(\varsigma_{n33}\xi_{n32}-\varepsilon_{n33}\mu_{n32})+\varsigma_{n33}k_x}{\omega(\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33})}\\+\varsigma_{n13}\dfrac{\omega(\mu_{n32}\xi_{n33}-\mu_{n32}\xi_{n33})+\mu_{n33}k_x}{\omega(\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33})}\end{pmatrix}$$

$$\Gamma_{n31}=j\omega\begin{pmatrix}\mu_{n21}+\mu_{n23}\dfrac{\xi_{n33}\varsigma_{n31}-\mu_{n33}\varepsilon_{n31}}{\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33}}\\+\xi_{n23}\dfrac{\varepsilon_{n31}\varsigma_{n33}-\varsigma_{n31}\varepsilon_{n33}}{\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33}}\end{pmatrix}$$

$$+jk_x\frac{\varepsilon_{n33}\varsigma_{n31}-\varsigma_{n33}\varepsilon_{n31}}{\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33}}$$

$$\Gamma_{n32}=$$

$$j\omega\begin{pmatrix}\varepsilon_{n22}+\varepsilon_{n23}\dfrac{\omega(\xi_{n33}\varsigma_{n32}-\mu_{n33}\varepsilon_{n32})-\xi_{n33}k_x}{\omega(\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33})}\\+\xi_{n23}\dfrac{\omega(\varepsilon_{n32}\varsigma_{n33}-\varsigma_{n32}\varepsilon_{n33})-\varepsilon_{n33}k_x}{\omega(\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33})}\end{pmatrix}$$

$$+jk_x\frac{\omega(\xi_{n33}\mu_{n32}-\mu_{n33}\xi_{n32})-\mu_{n33}k_x}{\omega(\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33})}$$

$$\Gamma_{n33}=j\omega\begin{pmatrix}\xi_{n21}+\xi_{n23}\dfrac{\varsigma_{n33}\xi_{n31}-\varepsilon_{n33}\mu_{n31}}{\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33}}\\+\varepsilon_{n23}\dfrac{\mu_{n31}\xi_{n33}-\xi_{n31}\mu_{n33}}{\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33}}\end{pmatrix}$$

$$+jk_x\frac{\varepsilon_{n33}\mu_{n31}-\varsigma_{n33}\xi_{n31}}{\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33}}$$

$$\Gamma_{n34}=$$

$$j\omega\begin{pmatrix}\xi_{n22}+\xi_{n23}\dfrac{\omega(\xi_{n32}\varsigma_{n33}-\mu_{n32}\varepsilon_{n33})+\varsigma_{n33}k_x}{\omega(\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33})}\\+\varepsilon_{n23}\dfrac{\omega(\xi_{n33}\mu_{n32}-\xi_{n32}\mu_{n33})-\mu_{n33}k_x}{\omega(\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33})}\end{pmatrix}$$

$$+jk_x\frac{\omega(\varepsilon_{n33}\mu_{n32}-\varsigma_{n33}\xi_{n32})-\varsigma_{n33}k_x}{\omega(\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33})}$$

$$\Gamma_{n41}=-j\omega\begin{pmatrix}\varepsilon_{n11}+\varepsilon_{n13}\dfrac{\xi_{n33}\varsigma_{n31}-\mu_{n33}\varepsilon_{n31}}{\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33}}\\+\varepsilon_{n13}\dfrac{\varepsilon_{n31}\varsigma_{n33}-\varsigma_{n31}\varepsilon_{n33}}{\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33}}\end{pmatrix}$$

$$\Gamma_{n42}=$$

$$-j\omega\begin{pmatrix}\varepsilon_{n12}+\varepsilon_{n13}\dfrac{\omega(\xi_{n33}\varsigma_{n32}-\mu_{n33}\varepsilon_{n32})-\xi_{n33}k_x}{\omega(\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33})}\\+\xi_{n13}\dfrac{\omega(\varepsilon_{n32}\varsigma_{n33}-\varsigma_{n32}\varepsilon_{n33})+\varepsilon_{n33}k_x}{\omega(\mu_{n33}\varepsilon_{n33}-\xi_{n33}\varsigma_{n33})}\end{pmatrix}$$

$$\Gamma_{n43} = -j\omega\left(\begin{array}{l}\xi_{n11} + \xi_{n13}\dfrac{\varsigma_{n33}\varepsilon_{n31} - \varepsilon_{n33}\mu_{n31}}{\mu_{n33}\varepsilon_{n33} - \xi_{n33}\varsigma_{n33}} \\ +\varepsilon_{n13}\dfrac{\mu_{n31}\xi_{n33} - \xi_{n31}\mu_{n33}}{\mu_{n33}\varepsilon_{n33} - \xi_{n33}\varsigma_{n33}}\end{array}\right)$$

$$\Gamma_{n44} = -j\omega\left(\begin{array}{l}\xi_{n12} + \xi_{n13}\dfrac{\omega(\varsigma_{n33}\xi_{n32} - \varepsilon_{n33}\mu_{n32}) + \varsigma_{n33}k_x}{\omega(\mu_{n33}\varepsilon_{n33} - \xi_{n33}\varsigma_{n33})} \\ +\varepsilon_{n13}\dfrac{\omega(\mu_{n32}\xi_{n33} - \mu_{n33}\xi_{n32}) - \mu_{n33}k_x}{\omega(\mu_{n33}\varepsilon_{n33} - \xi_{n33}\varsigma_{n33})}\end{array}\right)$$

APPENDIX (II)

$$\lambda_n^4 - a_n\lambda_n^3 + b_n\lambda_n^2 + c_n\lambda_n + d_n = 0$$

$$a_n = \Gamma_{n11} + \Gamma_{n22} + \Gamma_{n33} + \Gamma_{n44}$$

$$\begin{aligned} b_n = {} & \Gamma_{n11}\Gamma_{n44} - \Gamma_{n41}\Gamma_{n14} + \Gamma_{n22}\Gamma_{n44} \\ & +\Gamma_{n33}\Gamma_{n44} - \Gamma_{n31}\Gamma_{n13} - \Gamma_{n42}\Gamma_{n24} + \\ & \Gamma_{n22}\Gamma_{n11} + \Gamma_{n11}\Gamma_{n33} - \Gamma_{n32}\Gamma_{n23} + \Gamma_{n22}\Gamma_{n33} \end{aligned}$$

$$\begin{aligned} c_n = {} & \Gamma_{n32}\Gamma_{n23}\Gamma_{n44} + \Gamma_{n11}\Gamma_{n32}\Gamma_{n23} - \Gamma_{n11}\Gamma_{n22}\Gamma_{n33} \\ & +\Gamma_{n41}\Gamma_{n22}\Gamma_{n14} + \Gamma_{n42}\Gamma_{n24}\Gamma_{n33} \\ & +\Gamma_{n31}\Gamma_{n13}\Gamma_{n22} + \Gamma_{n31}\Gamma_{n13}\Gamma_{n44} \\ & -\Gamma_{n11}\Gamma_{n22}\Gamma_{n44} - \Gamma_{n11}\Gamma_{n44}\Gamma_{n33} + \Gamma_{n11}\Gamma_{n42}\Gamma_{n24} \\ & -\Gamma_{n22}\Gamma_{n33}\Gamma_{n44} - \Gamma_{n41}\Gamma_{n12}\Gamma_{n24} - \Gamma_{n42}\Gamma_{n23}\Gamma_{n34} \\ & +\Gamma_{n41}\Gamma_{n14}\Gamma_{n33} - \Gamma_{n41}\Gamma_{n13}\Gamma_{n34} - \Gamma_{n31}\Gamma_{n12}\Gamma_{n23} \end{aligned}$$

$$\begin{aligned} d_n = {} & \Gamma_{n41}\Gamma_{n22}\Gamma_{n13}\Gamma_{n34} - \Gamma_{n41}\Gamma_{n22}\Gamma_{n14}\Gamma_{n33} \\ & +\Gamma_{n11}\Gamma_{n22}\Gamma_{n33}\Gamma_{n44} - \Gamma_{n41}\Gamma_{n32}\Gamma_{n13}\Gamma_{n24} \\ & +\Gamma_{n41}\Gamma_{n32}\Gamma_{n14}\Gamma_{n23} - \Gamma_{n31}\Gamma_{n13}\Gamma_{n22}\Gamma_{n44} \\ & -\Gamma_{n11}\Gamma_{n32}\Gamma_{n23}\Gamma_{n44} + \Gamma_{n41}\Gamma_{n12}\Gamma_{n24}\Gamma_{n33} \\ & +\Gamma_{n11}\Gamma_{n42}\Gamma_{n23}\Gamma_{n34} - \Gamma_{n11}\Gamma_{n42}\Gamma_{n24}\Gamma_{n33} \\ & -\Gamma_{n41}\Gamma_{n12}\Gamma_{n23}\Gamma_{n34} + \Gamma_{n31}\Gamma_{n12}\Gamma_{n23}\Gamma_{n44} \\ & +\Gamma_{n31}\Gamma_{n42}\Gamma_{n13}\Gamma_{n24} - \Gamma_{n31}\Gamma_{n42}\Gamma_{n14}\Gamma_{n23} \end{aligned}$$

APPENDIX (III)

$$\Delta_{n1j} = \begin{vmatrix} \Gamma_{n12} & \Gamma_{n13} & \Gamma_{n14} \\ \Gamma_{n22} - \lambda_{n1} & \Gamma_{n23} & \Gamma_{n24} \\ \Gamma_{n32} & \Gamma_{n33} - \lambda_{n1} & \Gamma_{n34} \end{vmatrix},$$

$$\Delta_{n2j} = \begin{vmatrix} \lambda_{n2} - \Gamma_{n11} & \Gamma_{n13} & \Gamma_{n14} \\ -\Gamma_{n21} & \Gamma_{n23} & \Gamma_{n24} \\ -\Gamma_{n31} & \Gamma_{n33} - \lambda_{n2} & \Gamma_{n34} \end{vmatrix},$$

$$\Delta_{n3j} = \begin{vmatrix} \Gamma_{n12} & \lambda_{n3} - \Gamma_{n11} & \Gamma_{n14} \\ \Gamma_{n22} - \lambda_{n3} & -\Gamma_{n21} & \Gamma_{n24} \\ \Gamma_{n32} & -\Gamma_{n31} & \Gamma_{n34} \end{vmatrix},$$

$$\Delta_{n4j} = \begin{vmatrix} \Gamma_{n12} & \Gamma_{n13} & \lambda_{n4} - \Gamma_{n11} \\ \Gamma_{n22} - \lambda_{n4} & \Gamma_{n23} & -\Gamma_{n21} \\ \Gamma_{n32} & \Gamma_{n33} - \lambda_{n4} & -\Gamma_{n31} \end{vmatrix}.$$

APPENDIX (IV)

$$q_1 = \begin{vmatrix} b_{121} & b_{122} & b_{123} & b_{124} & -1 \\ b_{131} & b_{132} & b_{133} & b_{134} & -\eta_1 \\ b_{141} & b_{142} & b_{143} & b_{144} & 0 \\ c_{11} & c_{12} & c_{13} & c_{14} & 0 \\ c_{21} & c_{22} & c_{23} & c_{24} & 0 \end{vmatrix},$$

$$q_2 = \begin{vmatrix} b_{111} & b_{112} & b_{113} & b_{114} & 0 \\ b_{121} & b_{122} & b_{123} & b_{124} & -1 \\ b_{131} & b_{132} & b_{133} & b_{134} & -\eta_1 \\ c_{11} & c_{12} & c_{13} & c_{14} & 0 \\ c_{21} & c_{22} & c_{23} & c_{24} & 0 \end{vmatrix},$$

$$q_3 = -\begin{vmatrix} b_{111} & b_{112} & b_{113} & b_{114} & 0 \\ b_{131} & b_{132} & b_{133} & b_{134} & -\eta_1 \\ b_{141} & b_{142} & b_{143} & b_{144} & 0 \\ c_{11} & c_{12} & c_{13} & c_{14} & 0 \\ c_{21} & c_{22} & c_{23} & c_{24} & 0 \end{vmatrix},$$

$$q_4 = \begin{vmatrix} b_{111} & b_{112} & b_{113} & b_{114} & 0 \\ b_{121} & b_{122} & b_{123} & b_{124} & -1 \\ b_{141} & b_{142} & b_{143} & b_{144} & 0 \\ c_{11} & c_{12} & c_{13} & c_{14} & 0 \\ c_{21} & c_{22} & c_{23} & c_{24} & 0 \end{vmatrix},$$

$$q_5 = -\begin{vmatrix} b_{121} & b_{122} & b_{123} & b_{124} & 0 \\ b_{131} & b_{132} & b_{133} & b_{134} & 0 \\ b_{141} & b_{142} & b_{143} & b_{144} & \eta_2 \\ c_{11} & c_{12} & c_{13} & c_{14} & 0 \\ c_{21} & c_{22} & c_{23} & c_{24} & 0 \end{vmatrix},$$

$$q_6 = \begin{vmatrix} b_{111} & b_{112} & b_{113} & b_{114} & -1 \\ b_{121} & b_{122} & b_{123} & b_{124} & 0 \\ b_{131} & b_{132} & b_{133} & b_{134} & 0 \\ c_{11} & c_{12} & c_{13} & c_{14} & 0 \\ c_{21} & c_{22} & c_{23} & c_{24} & 0 \end{vmatrix},$$

$$q_7 = \begin{vmatrix} b_{111} & b_{112} & b_{113} & b_{114} & -1 \\ b_{131} & b_{132} & b_{133} & b_{134} & 0 \\ b_{141} & b_{142} & b_{143} & b_{144} & \eta_2 \\ c_{11} & c_{12} & c_{13} & c_{14} & 0 \\ c_{21} & c_{22} & c_{23} & c_{24} & 0 \end{vmatrix},$$

$$q_8 = \begin{vmatrix} b_{111} & b_{112} & b_{113} & b_{114} & -1 \\ b_{121} & b_{122} & b_{123} & b_{124} & 0 \\ b_{141} & b_{142} & b_{143} & b_{144} & \eta_2 \\ c_{11} & c_{12} & c_{13} & c_{14} & 0 \\ c_{21} & c_{22} & c_{23} & c_{24} & 0 \end{vmatrix}.$$

REFERENCES

[1] A. Taflove and M. E. Brodwin, "Numerical Solution of Steady-State Electromagnetic Scattering Problems using the Time-Dependent Maxwell's Equations," *IEEE Transaction on Microwave Theory Technology*, MTT-23, pp. 623-630, 1975.

[2] A. Taflove and K. R. Umashankar, "Radar Cross Section of General Three-Dimensional Scatters," *IEEE Transaction Electromagnetic Compatibility*, MTT-23, pp. 623-630, 1975.

[3] J. Schneider and S. Hudson, "The Finite-Difference Time-Domain Method Applied to Anisotropic Material," *IEEE Transaction on Antennas and Propagation*, vol. 41, no. 7, pp. 994-999, 1993.

[4] V. V. Varadan, A. Lakhtakia, and V. K. Varadan, "Scattering by Three-Dimensional Anisotropic Scatters," *IEEE Transaction on Antennas and Propagation*, vol. 37 no. 6, pp. 800-802, 1989.

[5] R. D. Graglia, P. L. E. Uslenghi, and R. S. Zich, "Moment Method with Isoparametric Elements for Three-Dimensional Anisotropic Scatters," *Proceedings of the IEEE*, vol. 77, no. 5, pp. 750-760, 1989.

[6] F. Abelès, Ann Phy. (Paris) vol. 5, pp. 596-640, 1950.

[7] S. Teitler and B. W. Henvis, "Refraction in Stratified, Anisotropic Media," *J. Opt. Soc. Am.* vol. 60, no. 6, pp. 830-834, June 1970.

[8] D. W. Berreman, "Optics in Stratified and Anisotropic Media: 4×4 Matrix Formulation," *J. Opt. Soc. Am.* vol. 62, no. 4, pp. 502-510, Apr. 1972.

[9] R. D. Graglia and P. L. E. Uslenghi, "Electromagnetic Scattering from Anisotropic Materials, Part I: General Theory," *IEEE Trans.Ant.Progat*., vol. AP-32, no. 8, pp. 867-869, Aug. 1984.

[10] M. A. Morgan, "Electromagnetic Scattering by Stratified Inhomogeneous Anisotropic Media," *IEEE Transaction on Antennas and Propagation*, vol. AP-35, no. 2, pp. 191-197, 1987.

[11] L. M. Barkovskii and G. N. Borzdov, "Electromagnetic Wave in Absorbing Plane-Layered Anisotropic and Gyrotropic Media," *J. App. Spectrosc.*, vol. 23, no. 1, pp. 985-991, Sept. 1976.

[12] L. M. Barkovskii and G. N. Borzdov, "Reflection of Electromagnetic Waves from Layered Continuously Inhomogeneous Anisotropic Media: Multiple Reflection Method," *Opt. Spectosc. (USSR)*, vol. 45, no. 4, pp. 701-705, 1978.

[13] L. M. Barkovskii, "Construction of Input Wave Impedance of Layered Anisotropic Media by the Direct Tensor Method," *Sov. Phys. Crystallogr*, vol. 23, no. 6 pp. 647-650.

[14] P. K. Huang and H. C. Yin, "Equivalent Currents on an Anisotropic Material Backed by a Metal Surface and their Relation," *Journal of Systems Engineering and Electronics*, vol. 11, no. 4, pp. 1-10, 2000.

[15] P. Corona, "Backscattering by Loaded and Unloaded Dihedral Corners," *IEEE Transaction on Antennas and Propagation*, vol. 35, no. 10, pp. 1148-1153, 1987.

[16] X. F. Li, Y. J. Xie, R. Yang, and Y. Y. Wang, "High-Frequency Analysis of Scattering from Complex Targets in Half Space," *Applied*

Computational Electromagnetic Society (ACES) Journal, vol. 25, no. 5, pp. 433-443, 2010.

[17] L. Zhao, Y. Zuo, and Y. J. Feng, "Electromagnetic Wave Reflection of Anisotropic Media with a Metallic Substrate," *Chinese Journal of Radio Science*, vol. 24, no. 5, pp. 804-807, 2009.

[18] H. P. Guo, G. X. Liu, H. C. Yin, and P. K. Huang, "Analytical Expression of Equivalent Currents on Stratified Anisotropic Material Backed by Metal Surface," *Chinese Journal of Radio Science*. vol. 17, no. 6, pp. 605-608, 2002.

[19] W. C. Chew, *Wave and Fields in Inhomogeneous Media*, IEEE Press, 2nd, New York, 1995.