An Equivalent Dipole-Moment Method Based Multilevel Fast Multipole Algorithm for Dielectric Objects

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Abstract ─ An equivalent dipole-moment method (EDM) based multilevel fast multipole algorithm (MLFMA), namely EDM-MLFMA, is proposed for the electromagnetic scattering from three-dimensional (3D) dielectric objects. In this scheme, the MLFMA is employed to accelerate the interactions of equivalent dipoles in the far regions by expanding the 3D dyadic Green's function into a multipole expression. The memory requirement and CPU time of the conventional EDM are reduced from $O(N^2)$ to $O(N\log N)$, where N is the number of unknowns. Numerical results are presented to validate the efficiency and accuracy of this method.

Index Terms ─ Equivalent dipole-moment method (EDM), multilevel fast multipole algorithm (MLFMA), volume integral equation (VIE).

I. INTRODUCTION

The scattering of electromagnetic (EM) waves from dielectric materials is an important research area because of the wide applications in dielectric radomes, anisotropic substrates, absorbing materials, etc. [1-2]. In the method of moments (MoM), the volume integral equation (VIE) is generally appropriate for dielectric objects with inhomogeneity and anisotropy. However, the conventional MoM converts the VIE to a dense matrix equation, which is very expensive and formidable especially for large scale EM targets. The $O(N^2)$ memory and computational complexity for iterative solvers is required for both computing the MoM impedance matrix elements and solving this dense matrix equation, where N is the number of unknowns.

More recently, the equivalent dipole-moment method (EDM) [3-4] has been developed to efficiently generate the impedance matrix elements for surface integral equation (SIE). Later, the EDM was extended to deal with the isotropic media [5] and anisotropic media [6-7]. The EDM is based on the commonly used Rao-Wilton-Glisson (RWG) [8] and Schaubert-Wilton-Glisson (SWG) [9] basis functions. In the EDM, each RWG triangle pair or SWG tetrahedron pair is viewed as a dipole model with an equivalent dipole moment. The main advantage of the EDM is that the impedance matrix element can be expressed in an extremely simplified form. However, the memory requirement and the matrix-solve time do not change, and the complexities are still $O(N^2)$.

In this article, the EDM is speeded up by the multilevel fast multipole algorithm (MLFMA) [10-16] for solving the EM scattering from three-dimensional (3D) dielectric materials in free-space. All the SWG basis functions are modeled as equivalent dipole models and divided into multilevel cubical groups. Through expanding the 3D dyadic Green's function [12] in the formulation of the EDM using the addition theorem [17-18], the interactions between the source and field equivalent dipoles in nonnearby groups are transformed into aggregation, translation and disaggregation operators. Benefiting from the octree-structured grouping, the interpolation and anterpolation, only $O(N)$ impedance elements at the finest level should be calculated and stored, and the computation complexity as well as the memory requirement of the conventional EDM are reduced from $O(N^2)$ to $O(N\log N)$.

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The remainder of the paper is organized as follows. In Section II, the EDM is briefly presented for the VIE. Then we describe how the MLFMA is used to accelerate the EDM in detail. In Section III, some numerical results about the bistatic radar cross section (RCS) are given to verify the efficiency and accuracy of the method. Finally, conclusions are drawn in Section IV.

II. FORMULATIONS

A. Basic principles of the EDM

Considering an arbitrarily shaped 3D scattering target, which consists of electric anisotropic material characterized by permittivity tensor $\overline{\overline{\varepsilon}}(\mathbf{r})$.

The VIE can be constructed by

$$\mathbf{E}^i(\mathbf{r}) = \overline{\overline{\varepsilon}}^{-1}(\mathbf{r}) \cdot \mathbf{D}(\mathbf{r}) + j\omega \mathbf{A}(\mathbf{r}) + \nabla \phi(\mathbf{r}), \quad (1)$$

where

$$\mathbf{A}(\mathbf{r}) = \mu_0 \int_v \mathbf{J}(\mathbf{r}') G(\mathbf{r}, \mathbf{r}') dv' \ , \quad (2)$$

$$\phi(\mathbf{r}) = -\frac{1}{j\omega\varepsilon_0} \int_v \nabla' \cdot \mathbf{J}(\mathbf{r}') G(\mathbf{r}, \mathbf{r}') dv' \ . \quad (3)$$

$\mathbf{E}^i(\mathbf{r})$ denotes the incident electric field. $\mathbf{D}(\mathbf{r})$ denotes the total electric flux density. $\mathbf{J}(\mathbf{r})$ is the equivalent volume current. ε_0 and μ_0 are the free-space permittivity and permeability, respectively. $G(\mathbf{r}, \mathbf{r}') = e^{-jk|\mathbf{r}-\mathbf{r}'|} \Big/ \left(4\pi |\mathbf{r} - \mathbf{r}'|\right)$ stands for the free-space scalar Green's function.

The unknown equivalent volume current $\mathbf{J}(\mathbf{r})$ can be expanded by a set of SWG basis functions as

$$\mathbf{J}(\mathbf{r}) = j\omega \overline{\overline{\kappa}}(\mathbf{r}) \cdot \mathbf{D}(\mathbf{r}) = \sum_{n=1}^{N} I_n \overline{\overline{\kappa}}_n \cdot \mathbf{f}_n(\mathbf{r}), \quad (4)$$

where $\mathbf{f}_n(\mathbf{r})$ represents the n th SWG basis function which is defined on two adjoining tetrahedron elements $T_n^{\pm}$ sharing the n th face S_n in the volume mesh. I_n is the unknown expansion coefficient. N is the number of the faces in the volume mesh. $\overline{\overline{\kappa}}(\mathbf{r}) = \overline{\mathbf{I}} - \overline{\overline{\varepsilon}}_r^{-1}(\mathbf{r})$ is the contrast ratio [19]. $\overline{\mathbf{I}}$ is the unit tensor.

Substituting Eq. (2) - Eq. (4) into Eq. (1) and using the Galerkin's method, finally the matrix equation of the VIE can be obtained by

$$\sum_{n=1}^{N} Z_{mn} I_n = V_m, m = 1 \sim N \ , \quad (5)$$

where

$$Z_{mn} = j\omega \left\langle \mathbf{f}_m(\mathbf{r}), \mathbf{A}_n(\mathbf{r}) \right\rangle + \left\langle \mathbf{f}_m(\mathbf{r}), \nabla \phi_n(\mathbf{r}) \right\rangle + \frac{1}{j\omega\varepsilon_0} \left\langle \mathbf{f}_m(\mathbf{r}), \overline{\overline{\varepsilon}}_r^{-1} \cdot \mathbf{f}_n(\mathbf{r}) \right\rangle \ , \quad (6)$$

are the impedance matrix elements.

$$V_m = \left\langle \mathbf{f}_m(\mathbf{r}), \mathbf{E}^i(\mathbf{r}) \right\rangle \ , \quad (7)$$

are the right-hand side vector elements.

In the conventional MoM, the multipoint Gaussian quadrature is usually used to calculate Eq. (6). However, this process can be simplified in the EDM. The basic idea of the EDM is that the fields radiated by the current in a SWG element are approximated as the fields due to an infinitely small dipole with an equivalent moment [5]. Based on this assumption, the interaction of two basis functions can be replaced by the interaction of two infinitely small dipoles, except they are very close to each other.

The equivalent dipole moment in the n th SWG element can be obtained by the integration of the volume current over the tetrahedron pair $T_n^{\pm}$ [6].

$$\mathbf{m}_m = \int_{T_n^{\pm}} \overline{\overline{\kappa}}_n \cdot \mathbf{f}_n(\mathbf{r}') dv' \approx a_n \overline{\overline{\kappa}}_n^{+} \cdot (\mathbf{r}_{ns}^c - \mathbf{r}_n^{c+}) + a_n \overline{\overline{\kappa}}_n^{-} \cdot (\mathbf{r}_n^{c-} - \mathbf{r}_{ns}^c) \ , \quad (8)$$

where $\mathbf{r}_n^{c\pm}$ and $\mathbf{r}_{ns}^c$ are the position vectors of the centroid of $T_n^{\pm}$ and the centroid of the common face S_n, respectively.

The impedance elements Z_{mn} can be expressed in a very simple form [5-6]

$$Z_{mn} = \frac{\eta e^{-jkR}}{4\pi} \left[\mathbf{m}'_m \cdot \mathbf{m}_n \left(\frac{jk}{R} + C \right) - \left(\mathbf{m}'_m \cdot \hat{\mathbf{R}} \right) \left(\hat{\mathbf{R}} \cdot \mathbf{m}_n \right) \left(\frac{jk}{R} + 3C \right) \right] \ , \quad (9)$$

where $\mathbf{R} = \mathbf{r}_{mn} = \mathbf{r}_m - \mathbf{r}_n$ is the vector from the center point $\mathbf{r}_n$ of the n th equivalent dipole to the center point $\mathbf{r}_m$ of the m th equivalent dipole. $R = |\mathbf{R}|$, $\hat{\mathbf{R}} = \mathbf{R}/R$. η is the intrinsic impedance of medium in free-space, and $C = \left[1 + 1/(jkR)\right] \Big/ R^2$.

$$\mathbf{m}'_m = a_m (\mathbf{r}_m^{c-} - \mathbf{r}_m^{c+}) \ , \quad (10)$$

is the moment of the SWG basis function [9].

It should be mentioned that the matrix elements are computed by the EDM directly when the distance of two SWG elements is greater than the critical distance $0.2\,\lambda_0$ in this paper.

B. Acceleration of the EDM using the MLFMA

Although the mutual impedance elements can be calculated by the EDM efficiently, the computational complexity for performing a matrix vector product (MVP) is still $O(N^2)$ for iterative methods and all the impedance elements must be stored. So the EDM is limited to solve electrically small targets. In this work, the MLFMA is employed to accelerate the MVP and reduce the memory requirement.

To employ the MLFMA, the entire target needs multilevel grouping first, then nonempty cubes are recorded using octree-structured data at all levels. We consider two dipoles m and n, which belong to two far groups j and i, respectively. Equation (9) of the impedance element Z_{mn} can be rewritten as

$$Z_{mn} = jk\eta \mathbf{m}'_m \cdot \bar{\mathbf{G}}(\mathbf{r}_m, \mathbf{r}_n) \cdot \mathbf{m}_n , \quad (11)$$

where

$$\bar{\mathbf{G}}(\mathbf{r}_m, \mathbf{r}_n) = \left[\bar{\mathbf{I}}(1+C) - \hat{\mathbf{R}}\hat{\mathbf{R}}(1+3C) \right] G(\mathbf{r}_m, \mathbf{r}_n) , \quad (12)$$

is the dyadic Green's function in free-space, in which

$$C = \frac{1}{jkR} + \frac{1}{(jkR)^2} . \quad (13)$$

In addition, the vector $\mathbf{R} = \mathbf{r}_{mn}$ can be rewritten as $\mathbf{R} = \mathbf{r}_{ji} + \mathbf{r}_{mj} - \mathbf{r}_{ni}$, in which $\mathbf{r}_{ji} = \mathbf{r}_{O_j} - \mathbf{r}_{O_i}$, $\mathbf{r}_{mj} = \mathbf{r}_m - \mathbf{r}_{O_j}$, $\mathbf{r}_{ni} = \mathbf{r}_n - \mathbf{r}_{O_i}$. $\mathbf{r}_{O_i}$ and $\mathbf{r}_{O_j}$ are the center positions of group i and j, respectively. Since the two groups are the far group pair, the addition theorem can be used to expand the dyadic Green's function as [12]

$$\bar{\mathbf{G}}(\mathbf{r}_m, \mathbf{r}_n) \approx \int [\bar{\mathbf{I}} - \hat{\mathbf{k}}\hat{\mathbf{k}}] e^{-j\mathbf{k}\cdot(\mathbf{r}_{mj} - \mathbf{r}_{ni})} T_L(\mathbf{k}, \mathbf{r}_{ji}) d^2\hat{\mathbf{k}} , \quad (14)$$

where

$$T_L(\mathbf{k}, \mathbf{r}_{ji}) = \frac{k}{16\pi^2} \sum_{l=0}^{L} (-j)^{l+1} (2l+1) h_l^{(2)}(kr_{ji}) P_l(\hat{\mathbf{k}} \cdot \mathbf{r}_{ji})$$

.(15)

$h_l^{(2)}(\cdot)$ is the spherical Hankel function of the second kind, and $P_l(\cdot)$ is the Legendre function of order l.

Substituting Eq. (14) into Eq. (11), and using the identity of $(\bar{\mathbf{I}} - \hat{\mathbf{k}}\hat{\mathbf{k}}) = (\bar{\mathbf{I}} - \hat{\mathbf{k}}\hat{\mathbf{k}}) \cdot (\bar{\mathbf{I}} - \hat{\mathbf{k}}\hat{\mathbf{k}})$, the impedance element Z_{mn} can be represented by

$$Z_{mn} = jk\eta \int \mathbf{R}_m(\hat{\mathbf{k}}) \cdot T_L(\mathbf{k}, \mathbf{r}_{ji}) \mathbf{F}_n(\hat{\mathbf{k}}) d\hat{\mathbf{k}} , \quad (16)$$

in which the vectors $\mathbf{F}_n(\hat{\mathbf{k}})$ and $\mathbf{R}_m(\hat{\mathbf{k}})$ are radiation function and receive function, respectively and are given by

$$\mathbf{F}_n(\hat{\mathbf{k}}) = \mathbf{m}_n \cdot (\bar{\mathbf{I}} - \hat{\mathbf{k}}\hat{\mathbf{k}}) e^{j\mathbf{k}\cdot\mathbf{r}_{ni}} , \quad (17)$$

$$\mathbf{R}_m(\hat{\mathbf{k}}) = \mathbf{m}'_m \cdot (\bar{\mathbf{I}} - \hat{\mathbf{k}}\hat{\mathbf{k}}) e^{-j\mathbf{k}\cdot\mathbf{r}_{mj}} . \quad (18)$$

Physical interpretation of this expansion is that a spherical wave from a dipole in free space is expanded by the sum of an infinite number of plane waves. Using Eq. (16), then the MVP can be computed fast through an aggregation-translation-disaggregation form. The complexities of computation and memory requirement can achieve $O(N \log N)$ profiting from the interpolation, anterpolation, and the grid-tree data structure [10-11]. At the finest level, only $O(N)$ impedance elements should be calculated and stored. Further more, part of them can be efficiently calculated by the EDM.

III. NUMERICAL RESULTS

In this section, we present some numerical results to validate the efficiency and accuracy of the new method. In the following examples, all the simulations are performed on a personal computer with the Pentium(R) Dual CPU E5500 with 2.80 GHz (only one core is used) and 2.0 GB RAM. The GMRES iterative solver is employed to obtain an identical residual error ≤ 0.001 and the block diagonal preconditioner is used.

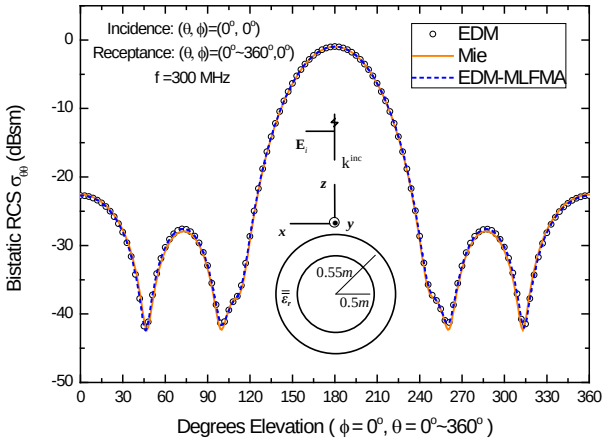

Fig. 1. Bistatic RCSs in $\theta\theta$ polarization of a spherical dielectric shell illuminated by a uniform plane wave with the incident direction of $(\theta, \phi) = (0^\circ, 0^\circ)$.

First, we consider the scattering problem of a spherical dielectric shell. The shell's inner and outer radii are 0.5 m and 0.55 m and the relative permittivity of the shell is $\bar{\bar{\varepsilon}}_r$ =1.5 $\bar{\bar{\mathbf{I}}}$, as shown in the inset of Fig. 1. The shell is discretized into 5309 tetrahedrons, and the total number of unknowns is 12213. The size of the finest group is set to 0.15 m and a 3-level EDM-MLFMA is used. The bistatic RCS for $\theta\theta$ polarization calculated by the EDM-MLFMA agrees well with the conventional EDM and the Mie series solution shown in Fig. 1. At 180°, the RCS value of the Mie series is -0.97987 dBsm, and the results obtained by the EDM-MLFMA and the conventional EDM are -0.97359 and -0.97464 dBsm, respectively. The root mean square (rms) errors of the EDM-MLFMA and the conventional EDM are 0.448 and 0.441 dBsm, respectively.

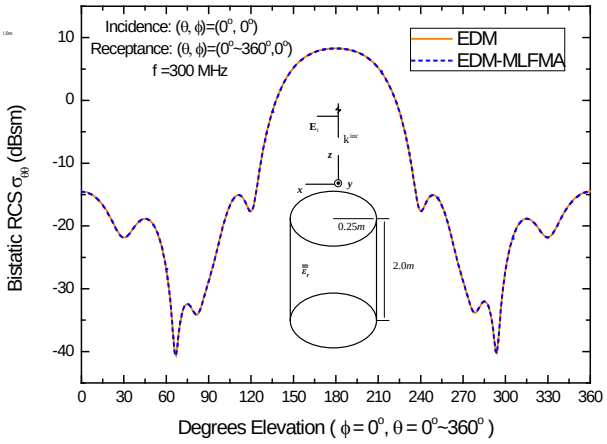

Fig. 2. Bistatic RCSs in $\theta\theta$ polarization of a dielectric cylinder illuminated by a uniform plane wave with the incident direction of $(\theta,\phi)=(0^\circ,0^\circ)$.

Then the bistatic RCS of a dielectric cylinder is considered. The radii and height of the cylinder are 0.25 m and 2.0 m. The relative permittivity of the cylinder is $\bar{\bar{\varepsilon}}_r$ =1.5 $\bar{\bar{\mathbf{I}}}$. The target is discretized into 5668 tetrahedrons, and the total number of unknowns is 11897. A 4-level EDM-MLFMA is used and the size of the group at the finest level is 0.15 m. The bistatic RCS in $\theta\theta$ polarization obtained by the EDM-MLFMA agrees well with the conventional EDM shown in Fig. 2.

Finally the bistatic RCS of an anisotropic dielectric target is considered. The target is constituted by 5 small slabs with the same size 1.0 m × 4.0 m × 0.05 m, but the 5 small slabs have different relative permittivities (see Fig. 3, $\bar{\bar{\varepsilon}}_{r1}$ =[1.5, j, 0; -j, 1.5, 0; 0, 0, 2.0], $\bar{\bar{\varepsilon}}_{r2}$ =(1.75-0.5j) $\bar{\bar{\mathbf{I}}}$ and $\bar{\bar{\varepsilon}}_{r3}$ =[2.0, j, 0; -j, 2.0, 0; 0, 0, 1.5]). The target is discretized into 49470 tetrahedrons, and the total number of unknowns is 115620. A 6-level EDM-MLFMA is used and the group at the finest level is also with the size of 0.15 m. The bistatic RCS in $\theta\theta$ polarization obtained by the EDM-MLFMA is shown in Fig. 3. This target could not be calculated by the conventional EDM, because the memory is not enough. The conventional EDM is estimated to require about 100 GB memory and 4 h CPU time.

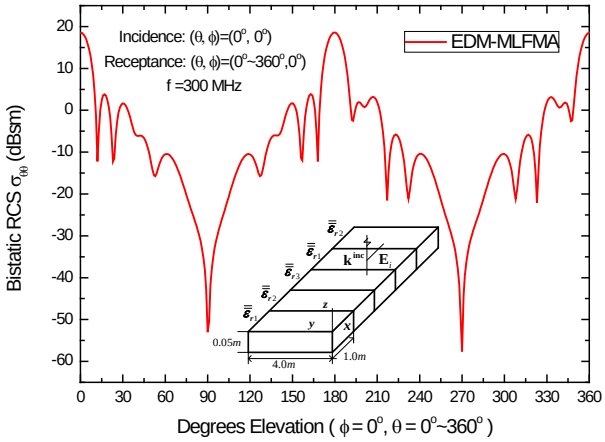

Fig. 3. Bistatic RCSs in $\theta\theta$ polarization of an anisotropic dielectric target illuminated by a uniform plane wave with the incident direction of $(\theta,\phi)=(0^\circ,0^\circ)$.

Table 1 summarizes the CPU time and memory requirment of the above simulations. It can be seen that the EDM-MLFMA saves much CPU time and memory than the conventional EDM.

Table 1: Comparison of CPU time and memory requirement of the conventional EDM and EDM-MLFMA

Method	EDM		EDM-MLFMA	
	Time	Memory	Time	Memory
Problem 1	132 s	1188 MB	51 s	145 MB
Problem 2	138 s	1128 MB	58 s	171 MB
Problem 3	-	-	520 s	1390 MB

IV. CONCLUSION

In this article, the EDM based MLFMA is introduced and applied to solve the VIE in the electromagnetic scattering of dielectric targets.

The new method can reduce both the CPU time and memory requirement of the conventional EDM to $O(N \log N)$. In the future, this method will be extended and applied to solve surface integral equation (SIE) and volume-surface integral equation (VSIE).

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