Nonlinear Schrödinger Equation-Based Adjoint Sensitivity Analysis

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Abstract — A general nonlinear adjoint sensitivity analysis (ASA) approach for the time-dependent nonlinear Schrödinger equation (NLSE) is presented. The proposed algorithm estimates the sensitivities of a desired objective function with respect to all design parameters using only one extra adjoint system simulation. The approach efficiency is shown here through a numerical example.

I. INTRODUCTION

Optical fibers play a vital role in telecommunication systems and networks [1]. The light wave propagation in optical fiber communication systems is described by the time-dependent NLSE. Accurate sensitivity analysis for the NLSE is essential in gradient-based design optimization of optical fiber systems [2]. ASA approaches has been recently emerged as a low-complexity alternative to the classical finite-differences approaches [3]. Regardless of the number of design parameters, an ASA approach estimates the full gradient of the desired objective function or response using at most one extra system simulation. This is contrasted with the computationally expensive finite-differences approximations whose complexity scales linearly with the number of parameters. The need for an ASA algorithm significantly raises in case of design problems with large number of parameters, such as fiber-optic design problems.

Several ASA algorithms based on Maxwell's equations or wave equation have been developed for high frequency structures [3,4]. ASA approaches are also introduced for the linear Schrödinger equation, to evaluate the sensitivities of the semiconductor quantum structures [5]. Recently, we proposed an ASA approach for the linear Schrödinger equation to evaluate the sensitivities of short-reach optical fiber communication systems [6]. In this paper, we extend our linear ASA approach for the general time-dependent nonlinear Schrödinger equation. Using only one extra adjoint simulation, the proposed nonlinear ASA approach estimates the sensitivities for a general long-reach fiber-optic communication system with respect to all design parameters of the fiber.

II. ADJOINT SENSITIVITY APPROACH

The propagation of light through an optical fiber link is described by the normalized NLSE, given by [1]:

$$-\frac{\beta_3}{6}\frac{\partial^3 q}{\partial t^3}+\frac{i\beta_2}{2}\frac{\partial^2 q}{\partial t^2}+\beta_1\frac{\partial q}{\partial t}+\frac{\partial q}{\partial z}+\left(\frac{\alpha}{2}-i\gamma|q|^2\right)q = q_{Tx}\delta(z), \quad (1)$$

where $q(z,t)$ is the complex envelope of the propagated optical field, β_1 is the inverse group speed, β_2 and β_3 are the second- and third-order dispersion coefficients, α is the fiber loss coefficient, and γ is the nonlinear coefficient of the fiber. The signal $q_{Tx}(t)$ is the optical field transmitted signal and $i=\sqrt{-1}$. We discretize the computational domain into M spatial cells. Expressing $q=q_{re}+iq_{im}$, substituting in equation (1), separating the real and imaginary terms, and approximating the spatial derivatives using finite differences, Equation (1) for the whole domain can be casted as follows:

$$\boldsymbol{B}_3\frac{\partial^3\boldsymbol{V}}{\partial t^3}+\boldsymbol{B}_2\frac{\partial^2\boldsymbol{V}}{\partial t^2}+\boldsymbol{B}_1\frac{\partial\boldsymbol{V}}{\partial t}+\boldsymbol{K}_c\boldsymbol{V}+\boldsymbol{A}\boldsymbol{V}+\boldsymbol{\Gamma}\boldsymbol{V}=\boldsymbol{Q}_{in}, \quad (2)$$

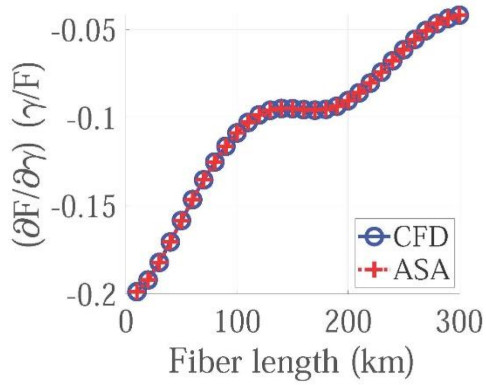
where $\boldsymbol{V}=[\boldsymbol{q}_{re}^T \quad \boldsymbol{q}_{im}^T]^T$ is the system state vector, $\boldsymbol{Q}_{in}(t)=[Re\{q_{Tx}\}\boldsymbol{e}_1^T \quad Im\{q_{Tx}\}\boldsymbol{e}_1^T]^T$ is the excitation vector, and $\boldsymbol{e}_1$ is the 1^{st} elementary column vector. The system matrices are given by: $\boldsymbol{B}_3=-\frac{\beta_3}{6}\boldsymbol{I}_{2M}$, $\boldsymbol{B}_2=\frac{\beta_2}{2}\begin{bmatrix}\boldsymbol{0} & -\boldsymbol{I}_M\\ \boldsymbol{I}_M & \boldsymbol{0}\end{bmatrix}$, $\boldsymbol{B}_1=\beta_1\boldsymbol{I}_{2M}$, $\boldsymbol{A}=\frac{\alpha}{2}\boldsymbol{I}_{2M}$, and $\boldsymbol{\Gamma}(\boldsymbol{V})=\begin{bmatrix}\boldsymbol{0} & \widetilde{\boldsymbol{\gamma}}\\ -\widetilde{\boldsymbol{\gamma}} & \boldsymbol{0}\end{bmatrix}$, where $\boldsymbol{I}_M$ is an identity matrix of size M and $\widetilde{\boldsymbol{\gamma}}$ is a diagonal matrix whose j^{th} entry is given by: $\gamma[q_{re}^2(jh)+q_{im}^2(jh)]$, $j=0,\ 1,\dots,M-1$.

After a lengthy derivation, it can be shown that the adjoint problem corresponding to the original simulation (2) is given by:

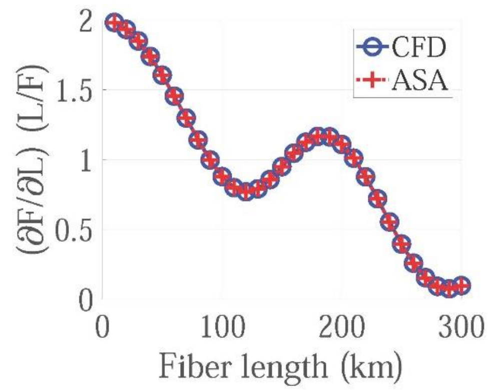
$$-\boldsymbol{B}_3\frac{\partial^3\boldsymbol{\lambda}}{\partial t^3}-\boldsymbol{B}_2\frac{\partial^2\boldsymbol{\lambda}}{\partial t^2}-\boldsymbol{B}_1\frac{\partial\boldsymbol{\lambda}}{\partial t}-\boldsymbol{K}_c\boldsymbol{\lambda}+\boldsymbol{A}\boldsymbol{\lambda}+\boldsymbol{\Gamma}^\lambda\boldsymbol{\lambda}=\boldsymbol{Q}_{in}^\lambda, \quad (3)$$

where $\boldsymbol{\lambda}=[\boldsymbol{\lambda}_{re}^T \quad \boldsymbol{\lambda}_{im}^T]^T$ is the adjoint state vector, $\boldsymbol{Q}_{in}^\lambda=[\partial\psi/\partial\boldsymbol{q}_{re}^T \quad \partial\psi/\partial\boldsymbol{q}_{im}^T]^T$ is the adjoint excitation vector, and ψ is the kernel of the objective function integral [6]. The nonlinear matrix of the adjoint problem $\boldsymbol{\Gamma}^\lambda$ is given by: $\boldsymbol{\Gamma}^\lambda(\boldsymbol{V})=\begin{bmatrix}\boldsymbol{a} & -\boldsymbol{b}\\ \boldsymbol{c} & -\boldsymbol{a}\end{bmatrix}$, where $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ are diagonal matrices whose j^{th} diagonal elements are given as follows:

Submitted On: September 2, 2020
Accepted On: September 5, 2020
https://doi.org/10.47037/2020.ACES.J.351140



(a)



(b)

Fig. 1. Normalized ASA sensitivities of objective function (4) with respect to the fiber nonlinear coefficient γ and the fiber length L, for a sweep of L, as compared to the accurate but computationally expensive CFD.

$a_{jj} = 2\gamma q_{re}(jh) q_{im}(jh)$, $b_{jj} = \gamma[3q_{re}^2(jh) + q_{im}^2(jh)]$, and $c_{jj} = \gamma[q_{re}^2(jh) + 3q_{im}^2(jh)]$, $j = 0,\ 1, \ldots, M-1$.

Once the original and adjoint fields are determined, the sensitivities of the desired objective function with respect to all design parameters are estimated.

III. RESULTS

To illustrate the efficiency of our ASA algorithm, we consider an example of an optical fiber communication system operating at 10 Gbaud. The transmitted data have a Gaussian pulse shape, modulated using the on-off keying format. The transmitted pulse peak power is 2 dBm, and the number of transmitted bits is 32. The fiber parameters are [1]: $\beta_1 = 1.6$ ns/km, $\beta_2 = -21$ ps ^{2}/km, $\beta_3 = 0.1$ ps ^{3}/km, $\alpha = 0.2$ dB/km, and $\gamma = 1.1$ W $^{-1}$km $^{-1}$. A noise-free amplifier is inserted at the end of the fiber to compensate for the loss.

Our objective is to estimate the sensitivities of an objective function of the form:

$$F = \int_{-T_m}^{T_m} \left| q_{Rx} - q_{Ref} \right|^2 \ dt, \qquad (4)$$

where q_{Rx} is the complex envelope field of the received signal, $q_{Ref} = q_{Tx}(t - \beta_1 L)$ is a time delayed version of the transmitted complex envelope field, and T_m is half the computational time window size. This objective function measures the signal distortion due to fiber dispersion and nonlinear effects. The sensitivities of (4) is estimated with respect to 6-design parameters.

Figure 1 shows the normalized sensitivities of F with respect to the parameters γ and L. Due to space limitation, we only show a subset of the obtained sensitivities. Good agreement is achieved with central finite differences (CFD) for all parameters. The ASA algorithm requires only one extra system simulation per gradient evaluation, while the CFD requires 12 extra simulations. The ASA algorithm estimates the sensitivities in 2.45 s, as opposed to 10.74 s taken by the CFD algorithm, i.e., the ASA is faster by 4.4 times.

IV. CONCLUSION

We presented a computationally efficient nonlinear adjoint sensitivity analysis approach for the general time-dependent nonlinear Schrödinger equation. As compared to the computationally expensive central finite-difference approach, the proposed algorithm estimates accurate sensitivities of the desired objective function with respect to all the design parameters but with a much lower computational cost.

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