

IMPLEMENTATION OF PASSIVE AND ACTIVE CIRCUIT ELEMENTS IN CYLINDRICAL FINITE-DIFFERENCE TIME-DOMAIN FORMULATION

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Abstract—The implementation of passive and active linear circuit elements in cylindrical finite-difference time-domain technique (CFDTD) is developed. The updating equations for active element such as a voltage sources and passive elements such as resistors, capacitors and inductors are represented for elements located along one of the cylindrical coordinate’s independent parameters. The analytical solutions for simple circuit configurations are computed and used as a reference to verify and confirm the accuracy of the developed CFDTD formulation and numerical results.

Keywords—cylindrical finite-difference time-domain, passive and active linear circuit elements.

I. INTRODUCTION

Finite-difference time-domain (FDTD) is a technique which is used to solve Maxwell’s equations in time domain EM by using numerical approximation of the derivatives, mainly in the Cartesian coordinates system [1]. The FDTD is an easy technique since it does not require some complicated functions such as the Green’s functions to find the solution. Also, it handles complex geometries without further complications. In addition to these advantages, by using a source waveform like a Gaussian pulse, the solution of a wide spectrum of frequencies will be covered by solving the problem only one time. For cylindrical structure problems and especially near to the axis of rotation the FDTD formulation in cylindrical coordinates system (CFDTD) is required to yield high accuracy [2]. There are applications in which the CFDTD can be used such as in geophysics and petroleum engineering. One of these applications is the wireline logging which is a process of recording the formation’s rock properties [3].

This summary introduces the formulation and verification of integrating passive and active circuit elements in the CFDTD formulation to facilitate the simulation of the radiating elements such as dipoles and loops excited with voltage sources required for such applications.

II. CFDTD UPDATING EQUATIONS

Field components where circuit elements are present in the computational domain needs special updating equations. Here, only the E_z updating equation will be modified for a circuit element located between the nodes carrying this E_z component.

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Starting with the E_z updating as shown below:

$$\begin{aligned}
 E_z^{n+1}(i, j, k) = & C_{eze}(i, j, k) \times E_z^n(i, j, k) \\
 & + (C_{ez\phi 1}(i, j, k) \times H_\phi^{n+\frac{1}{2}}(i, j, k) \\
 & - C_{ez\phi 2}(i, j, k) \times H_\phi^{n-\frac{1}{2}}(i-1, j, k)) \\
 & + C_{ez\rho}(i, j, k) \times (H_\rho^{n+\frac{1}{2}}(i, j, k) - H_\rho^{n-\frac{1}{2}}(i, j-1, k)) \\
 & + C_{ezj}(i, j, k) \times J_{iz}^{n+\frac{1}{2}}(i, j, k),
 \end{aligned} \tag{1}$$

where

$$C_{ezj}(i, j, k) = -\frac{2\Delta t}{2\varepsilon_z(i, j, k) + \Delta t\sigma_z^e(i, j, k)},$$

and all other coefficients can be expressed from (3) in [2]. The parameters i, j, k represent the nodes in the $\rho, \phi,$ and z directions, respectively, while $\Delta\rho$ and $\Delta\phi$ are the step size in the ρ and ϕ directions, respectively. The time step Δt is given by (28) in [2]. All other fields updating equations can be expressed using the same methodology applied for E_z .

III. UPDATING EQUATIONS FOR PASSIVE AND ACTIVE ELEMENTS

The passive elements such as resistors, inductors and capacitors and active elements such as voltage sources are important components in any electromagnetics simulation. Here, any of these elements will be placed between nodes in z direction. Fig. 1 shows a general form of a simple circuit diagram where V_s and Z_s are placed between two nodes and Z_L is placed one node a way in the ρ direction. In this paper, three cases will be presented. The first case, Z_s is a resistor with a sinusoidal waveform as a voltage source. The second case, Z_s is a resistor with a capacitor in series and in the third case Z_s is a resistor with an inductor in series where a unit-step function uses as a voltage source in the second and third cases. The Z_L for all cases is a resistor. For any of these cases, the updating equation for E_z in (1) can be modified with the help of term $J_{iz}^{n+\frac{1}{2}}(i, j, k)$. For the first case, we have:

For the first case, we have:

$$J_{iz}^{n+\frac{1}{2}}(i, j, k) = \frac{\Delta z}{2A_i R_s} (E_z^{n+1}(i, j, k) + E_z^n(i, j, k)) + \frac{V_s}{A_i R_s}, \tag{2}$$

for the second case, we used the recursive convolution method [1] and we get:

$$Q_{iz}^{n+\frac{1}{2}}(i, j, k) = Q_{iz}^{n-\frac{1}{2}}(i, j, k) \times e^{-\frac{\Delta t}{CR_s}} + (\Delta z E_z^n(i, j, k) + V_s), \tag{3}$$

$$J_{iz}^{n+\frac{1}{2}}(i, j, k) = \frac{1}{A_i R_s} (\Delta z E_z^n(i, j, k) + V_s) + (e^{-\frac{\Delta t}{CR_s}} - 1) \times Q_{iz}^{n+\frac{1}{2}}(i, j, k),$$

and for the third case, we get:

$$J_{iz}^{n+\frac{1}{2}}(i, j, k) = \frac{2L - R_s \Delta t}{2L + R_s \Delta t} J_{iz}^{n-\frac{1}{2}}(i, j, k) \tag{4}$$

$$+ \frac{2\Delta z \Delta t}{A_i (2L + R_s \Delta t)} E_z^n(i, j, k) + \frac{2\Delta z \Delta t}{A_i (2L + R_s \Delta t)} V_s,$$

where A_i is the area normal to $E_z(i, j, k)$ is given by:

$$A_i = i\Delta\rho^2 \Delta\phi. \tag{5}$$

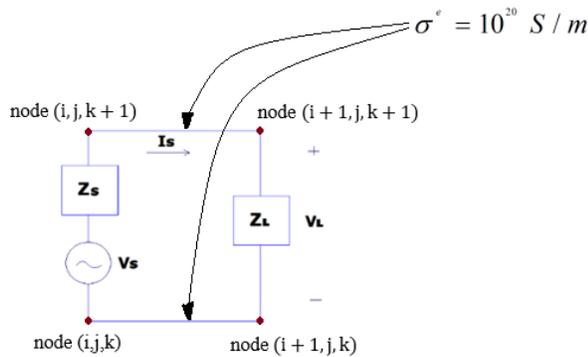


Fig. 1. Simple circuit diagram in CFDTD.

For all cases, the resistors are $R_s = R_L = 50\Omega$, the capacitor is $C = 1nF$ and the inductor is $L = 2.5\mu H$. In Fig. 2 and Fig. 3, the sampled voltage across the load using the CFDTD simulation of the circuit and the corresponding analytical solution show very good agreement.

IV. CONCLUSOIN

The validation of integrating passive and active circuit elements into the formulation and simulations using the cylindrical finite-difference time-domain is achieved. The

developed code has been used to test other element positional possibilities in the CFDTD grid. Work is in progress to include nonlinear elements in this cylindrical formulation to address a larger array of electromagnetic problems.

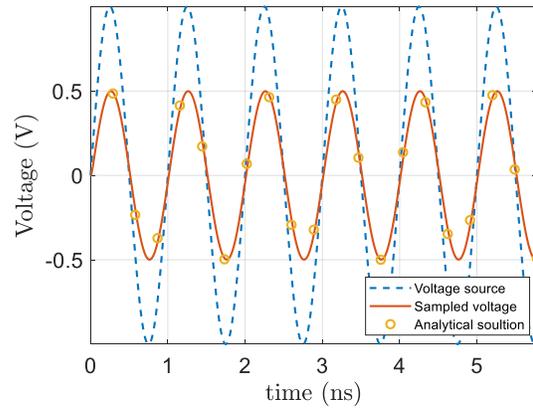


Fig. 2. Voltage across $Z_L = R_L$ using $Z_s = R_s$.

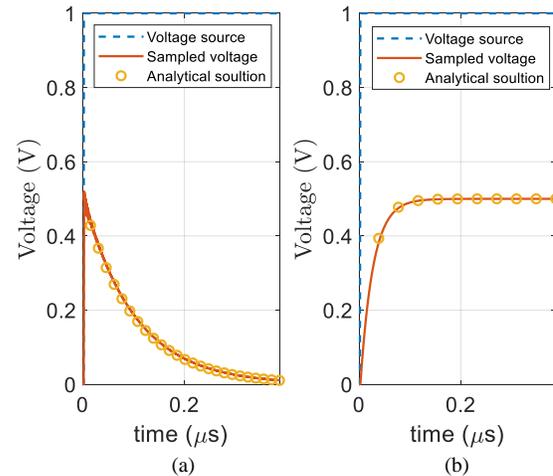


Fig. 3. Voltage across $Z_L = R_L$ using (a) $Z_s = R_s + Z_{Cap}$; (b) $Z_s = R_s + Z_{Ind}$.

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