Stable Partial Inductance Calculation for Partial Element Equivalent Circuit Modeling

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Abstract **─** Accurate and stable analytical solutions for partial element calculation in partial element equivalent circuit (PEEC) modeling are desirable due to fast and simple usage. Conventional analytical formulae based on integral Neumann formula may give miscalculations when we compute the partial inductance of three-dimensional structures with large spacing. In this paper, a novel model-order reduction (MOR) method is presented for mutual inductance calculation in order to improve its accuracy and stability. Mutual inductances of higher order models are represented by relatively lower order models. The criteria for MOR are revealed and a code implementation routine is described. The numerical accuracy, stability and calculation cost are investigated comparing with conventional procedures. Numerical experiments show that the MOR method can guarantee numerical stability and reduce calculation complexity simultaneously, and it has advantages to be implemented in PEEC modeling for large complex electronic systems.

Index Terms **─** Model-order reduction (MOR), numerical stability, partial element equivalent circuit (PEEC), partial inductance.

I. INTRODUCTION

A multitude of mixed electromagnetic and circuit problems of ever-increasing frequencies in complex electronic systems make electromagnetic compatibility (EMC) an increasingly troublesome issue, and numerical modeling techniques provide cost effective solutions. The partial element equivalent circuit (PEEC) method was first introduced by Ruehli in the 1970s [1] and now is a promising numerical method for electromagnetic (EM) modeling of various engineering problems, e.g., EMC, EM interference (EMI), and signal integrity (SI) of high-speed digital circuits [2-4]. The main advantage of PEEC is its ability to provide a circuit interpretation of the electric field integral equation in terms of partial elements, namely resistances, partial inductances, and coefficients of potential [3]. It is especially suitable for mixed electromagnetic and circuit problems for its ease to integrate the field solver with real circuit elements [5]. Different from other integral equation (IE) based EM modeling methodologies, PEEC is a full spectrum method valid from dc to the maximum frequency determined by the meshing [6].

The impact of partial element accuracy on quasi-static PEEC model stability has been discussed in [6], and the major sources of inaccurate partial element values were found to be the utilization of unsuitable calculation routines and poor geometrical meshing. The instabilities associated with a full-wave PEEC model is an important and complicated issue, and reasons for its instabilities have been revealed in [7-11] and several full-wave PEEC models attempts to improve the stability of time domain solutions have been proposed [8-13] in recent years. In this paper, we mainly focus on the stability and accuracy of partial mutual inductance calculation by analytical routines for quasi-static PEEC modeling.

Closed-form formulae for mutual inductance calculations were first presented in Grover's book [14], and Ruehli improved it for better computer implementation. Ruehli used filament approximation to calculate mutual inductance of three-dimensional multi-conductor interconnection structures [15]. This approximation represented a conductor in terms of a set of filaments in the direction of current flow. Following this procedure, Ruey-Beei Wu further developed his approach later by quadrature formulae [16]. Hoer and Love derived a closed-form formula for the mutual inductance of any pair of parallel rectangular conductors [17]. However, the accuracy of these formulae depends much on the ratio of cross sectional size to segment distance and the aspect ratio of each conductor.

A closed-form mutual inductance equation developed by Zhong and Koh [18] draws attention to the numerical stability. In the following discourse, we use "Zhong's procedure" to refer strictly to this method. In their procedure, the mutual inductance between two parallel rectangular conductors was expressed as a weighted sum of the self inductances of 64 virtual conductors which are defined by one corner point of the first conductor and one corner point of the second conductor. This formula is numerically more stable than any others before, especially for large-aspect-ratio structures. Unfortunately, it still suffers from numerical stability problems in some particular cases, e.g., the two conductors with large spacing.

The proposed method concerns the EMI modeling for large complex electronic systems with PEEC method, e.g., cable-cable and cable-component coupling behaviors in automotives. High accuracy and stable analytical solutions for partial element calculation in PEEC modeling are desirable due to fast and simple usage. Normally interfering sources and susceptive parts sometimes are localized far away (e.g., one or two meters) in the large systems, and the conventional mutual inductance formulae may give inaccurate values (see Fig. 3). In this paper, a model-order reduction (MOR) scheme is developed to improve the numerical stability of mutual inductance with analytical procedures. Mutual inductance of higher order models are represented by relatively lower order models according to compact MOR criteria. Reduced-order models can guarantee accuracy and reduce calculation complexity simultaneously.

This paper is organized as follows. Analytical formulae for mutual inductance calculation of various structures are derived in Section II. Mathematic preliminary and the MOR criteria together with the working flow of a code implementation are described in Section III. Numerical experiments are discussed in Section IV. In this section, we represent the miscalculation phenomenon, and stable values obtained by our MOR method are, also, presented as a contrast. The numerical accuracy, stability and calculation cost are analyzed in this section, as well. Finally, Section V ends with conclusions and discussions.

II. FORMULATIONS

A. Mutual inductance of a primary PEEC cell

Conventional discretization cells for large-scale PEEC modeling are three dimensional structures [19]. Figure 1 illustrates a calculation model of two rectangular conductors with relevant geometrical parameters. This is a general configuration without any restrictions on the alignment of the two conductors and we can calculate the mutual inductance by a closed-form analytical formula (1).

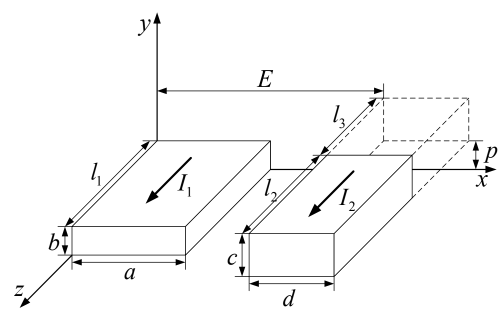


Fig. 1. Two rectangular conductors.

$$M = \frac{\mu}{4\pi}\frac{1}{abcd} \cdot [[[f(X,Y,Z)]_{E+d-a,E}^{E+d,E-a}(X)]_{p+c-b,p}^{p+c,p-b}(Y)]_{l_3+l_2-l_1,l_3}^{l_3+l_2,l_3-l_1}(Z), \tag{1a}$$

where

$$[[[f(X,Y,Z)]_{Q_2,Q_4}^{Q_1,Q_3}(X)]_{R_2,R_4}^{R_1,R_3}(Y)]_{S_2,S_4}^{S_1,S_3}(Z) = \sum_{i=1}^{4}\sum_{j=1}^{4}\sum_{k=1}^{4}(-1)^{i+j+k+1} f(Q_i,R_j,S_k) \tag{1b}$$

$$
\begin{aligned}
&f(X,Y,Z)=(\frac{Y^2Z^2}{4}-\frac{Y^4}{24}-\frac{Z^4}{24})X\ln(X+\sqrt{X^2+Y^2+Z^2})\\
&(\frac{X^2Z^2}{4}-\frac{X^4}{24}-\frac{Z^4}{24})Y\ln(Y+\sqrt{X^2+Y^2+Z^2})\\
&(\frac{X^2Y^2}{4}-\frac{Y^4}{24}-\frac{X^4}{24})Z\ln(Z+\sqrt{X^2+Y^2+Z^2})\\
&+\frac{1}{60}(X^4+Y^4+Z^4-3X^2Y^2-3Y^2Z^2-3X^2Z^2)\sqrt{X^2+Y^2+Z^2}\\
&-\frac{X^3YZ}{6}\arctan\frac{YZ}{X\sqrt{X^2+Y^2+Z^2}}\\
&-\frac{XY^3Z}{2}\arctan\frac{XZ}{Y\sqrt{X^2+Y^2+Z^2}}\\
&-\frac{XYZ^3}{6}\arctan\frac{XY}{Z\sqrt{X^2+Y^2+Z^2}}.
\end{aligned}
\tag{1c}
$$

B. Mutual inductance of various structures

Using primitive integral Neumann formula (2) [20], we can deduce closed-form mutual inductance formulae for various structures as Table 1 shows by the integration technique. In (2), a denotes the cross section which is perpendicular to the direction of current flow, and l means the length of cell. In Table 1, we express the multiple summations in the similar way as (1b)

$$
L_{p_{km}}=\frac{\mu}{4\pi}\frac{1}{a_k a_m}\int_{a_k}\int_{l_k}\int_{a_m}\int_{l_m}\frac{dl_k\cdot dl_m}{r_{km}}da_k da_m \quad . \tag{2}
$$

The model complexity can be measured by its degree of freedom, for example, the degree of freedom of two three-dimensional rectangular conductors is six, and is two for two filaments. In the following discussion, we use the notation nD to refer to the models with n degrees of freedom.

It was shown in [6, 11] that the major source of errors in partial inductance calculation is the numerical dispersion due to high-order terms creating large numbers while the values of other lower-order terms are relatively small. From the expressions in Table 1, we see that the formulae corresponding to lower order models are more stable, and simpler inductance expressions usually result in less calculation cost. The main idea of our MOR method is to present the mutual inductance of a higher order model with a relatively lower order model.

Table 1: Partial mutual inductance formulae of various structures

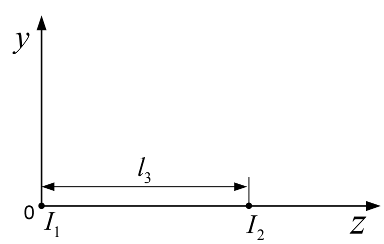
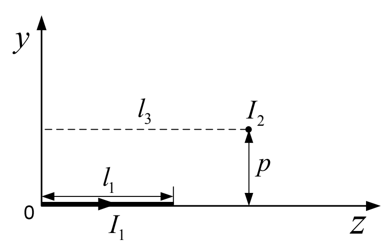
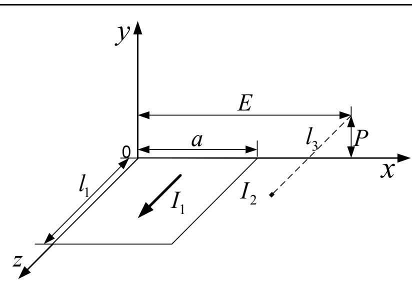
Order	Configurations	Formulae
0D		$M=\frac{\mu}{4\pi}\frac{1}{l_3}$
1D		$M=\frac{\mu}{4\pi}[f(Z)]_{l3-l1}^{l3}(Z)$ $f(Z)=\ln(Z+\sqrt{Z^2+P^2})$
2D-1		$M=\frac{\mu}{4\pi}\frac{1}{a}[[f(X,Z]_{E-a}^{E}(X)]_{l_3-l_1}^{l_3}(Z)$ $f(X,Z)=X\ln(Z+\sqrt{X^2+P^2+Z^2})+Z\ln(X+\sqrt{X^2+P^2+Z^2})$ $-P\arctan(\frac{XZ}{P\sqrt{X^2+P^2+Z^2}})$

Table 1: Partial mutual inductance formulae of various structures (continued)

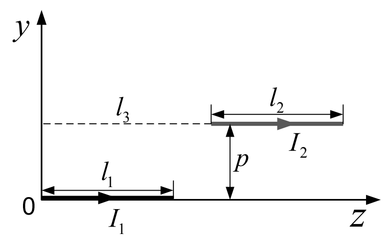
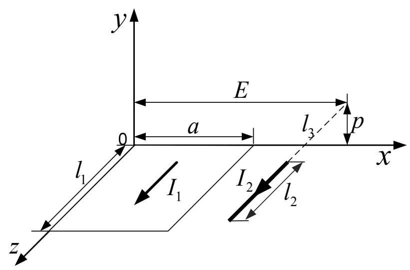
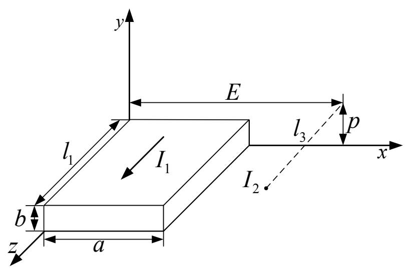
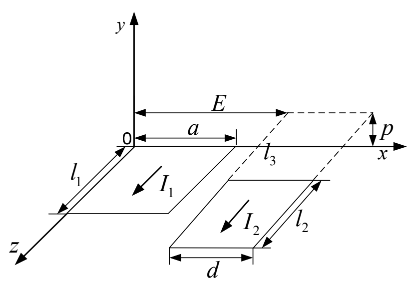
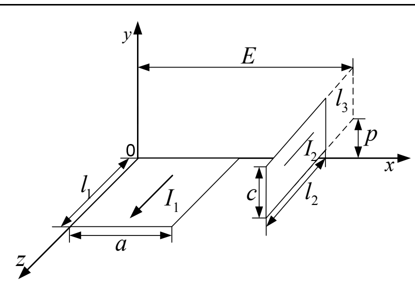
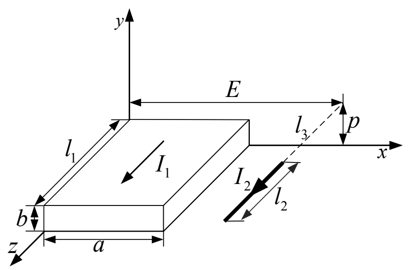
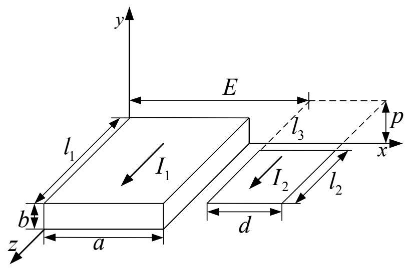
2D-2		$M=\frac{\mu}{4\pi}[f(Z)]_{l_3+l_2-l_1,l_3}^{l_3-l_1,l_3+l_2}(Z)$ $f(Z)=Z\ln(Z+\sqrt{Z^2+P^2})-\sqrt{Z^2+P^2}$
3D-1		$M=\frac{\mu}{4\pi}\frac{1}{a}[[f(X,Z)]_{E-a}^{E}(X)]_{l_3+l_2-l_1,l_3}^{l_3-l_1,l_3+l_2}(Z)$ $f(X,Z)=\frac{Z^2-P^2}{2}\ln(X+\sqrt{X^2+P^2+Z^2})-\frac{X}{2}\sqrt{X^2+P^2+Z^2}$ $-ZP\arctan(\frac{XZ}{P\sqrt{X^2+P^2+Z^2}})+XZ\ln(Z+\sqrt{X^2+P^2+Z^2})$
3D-2		$M=\frac{\mu}{4\pi}\frac{1}{ab}[[[f(X,Y,Z)]_{E-a}^{E}(X)]_{p-b}^{p}(Y)]_{l_3-l_1}^{l_3}(Z)$ $f(X,Y,Z)=YZ\ln(X+\sqrt{X^2+P^2+Z^2})+XZ\ln(Y+\sqrt{X^2+P^2+Z^2})$ $+XY\ln(Z+\sqrt{X^2+P^2+Z^2})-\frac{X^2}{2}\arctan(\frac{YZ}{X\sqrt{X^2+P^2+Z^2}})$ $-\frac{Y^2}{2}\arctan(\frac{XZ}{Y\sqrt{X^2+P^2+Z^2}})-\frac{Z^2}{2}\arctan(\frac{XY}{Z\sqrt{X^2+P^2+Z^2}})$
4D-1		$M=\frac{\mu}{4\pi}\frac{1}{ad}[[f(X,Z)]_{E+d-a,E}^{E+d,E-a}(X)]_{l_3+l_2-l_1,l_3}^{l_3-l_1,l_3+l_2}(Z)$ $f(X,Z)$ $=\frac{X^2-P^2}{2}Z\ln(Z+\sqrt{X^2+P^2+Z^2})-X\lvert P\rvert Z\arctan(\frac{XZ}{P\sqrt{X^2+P^2+Z^2}})$ $+\frac{Z^2-P^2}{2}X\ln(X+\sqrt{X^2+P^2+Z^2})-\frac{X^2-2P^2+Z^2}{6}\sqrt{X^2+P^2+Z^2}$
4D-2		$M=\frac{\mu_0}{4\pi}\frac{1}{ac}[[[f(X,Y,Z)]_{E-a}^{E}(X)]_{P}^{P+c}(Y)]_{l_3+l_2-l_1,l_3}^{l_3-l_1,l_3+l_2}$ $f(X,Y,Z)=(\frac{Z^2}{2}-\frac{Y^2}{6})Y\ln(X+\sqrt{X^2+Y^2+Z^2})$ $+(\frac{Z^2}{2}-\frac{X^2}{6})X\ln(Y+\sqrt{X^2+Y^2+Z^2})+XYZ\ln(Z+\sqrt{X^2+Y^2+Z^2})$ $-\frac{XY}{3}\sqrt{X^2+Y^2+Z^2}-\frac{Z^3}{6}\arctan(\frac{XY}{Z\sqrt{X^2+Y^2+Z^2}})$ $-\frac{X^2Z}{2}\arctan(\frac{YZ}{X\sqrt{X^2+Y^2+Z^2}})-\frac{Y^2Z}{2}\arctan(\frac{XZ}{Y\sqrt{X^2+Y^2+Z^2}})$

Table 1: Partial mutual inductance formulae of various structures (continued)

4D-3		$M = \frac{\mu_0}{4\pi}\frac{1}{ab}[[[f(X,Y,Z)]_{E-a}^{E}(X)]_{p-b}^{p}(Y)]_{l_3+l_2-l_1,l_3}^{l_3+l_2,l_3-l_1}(Z)$ $f(X,Y,Z)$ is as above 4D-2 model (perpendicular thin tapes)
5D		$M = \frac{\mu}{4\pi}\frac{1}{abd}[[[f(X,Y,Z]_{E+d-a,E}^{E+d,E-a}(X)]_{p-b}^{p}(Y)]_{l_3+l_2-l_1,l_3}^{l_3+l_2,l_3-l_1}(Z)$ $f(X,Y,Z)$ $= (\frac{X^2}{2}-\frac{Y^2}{6})YZ\ln(Z+\sqrt{X^2+Y^2+Z^2}) - \frac{X^3Z}{6}\arctan\frac{YZ}{X\sqrt{X^2+Y^2+Z^2}}$ $+(\frac{Z^2}{2}-\frac{Y^2}{6})XY\ln(X+\sqrt{X^2+Y^2+Z^2}) - \frac{XZ^3}{6}\arctan\frac{XY}{Z\sqrt{X^2+Y^2+Z^2}}$ $-\frac{XY^2Z}{2}\arctan\frac{XZ}{Y\sqrt{X^2+Y^2+Z^2}} - \frac{3X^2-2Y^2+3Z^2}{24}Y\sqrt{X^2+Y^2+Z^2}$ $+\frac{1}{24}(6X^2Z^2-X^4-Z^4)\ln(Y+\sqrt{X^2+Y^2+Z^2})$

III. MODEL-ORDER REDUCTION METHOD

A. Criteria for MOR method

The lemma for order reduction in the appendix is the mathematical foundation of our MOR method. Regarding the mutual inductance calculation, a specific MOR criterion in one dimension with the size of a is as (3) describes according to the lemma.

$$a\left|\frac{d}{dX_1}\int_E^{E+d}\int_0^b\int_p^{p+c}\int_0^{l_1}\int_{l_3}^{l_3+l_2}\frac{1}{R}dX_2dY_1dY_2dZ_1dZ_2\right| \le \varepsilon \quad , \quad (3)$$

where $R=\sqrt{(X_1-X_2)^2+(Y_1-Y_2)^2+(Z_1-Z_2)^2}$.

As a result of (4), the criterion (3) is equivalent with (5a).

$$a\left|\frac{d}{dX_1}\int_E^{E+d}\int_0^b\int_p^{p+c}\int_0^{l_1}\int_{l_3}^{l_3+l_2}\frac{1}{R}dX_2dY_1dY_2dZ_1dZ_2\right| \le abcdl_1l_2\cdot\max(\left|(X_1-X_2)dX_1\right|/R^3) \quad (4)$$

$$V_1V_2\cdot\max|Q|\cdot a/(\min R)^3 \le \varepsilon. \quad (5a)$$

Here, V_1 and V_2 denote the volumes of two rectangular cells ($V_1 = a\cdot b\cdot l_1$, $V_2 = c\cdot d\cdot l_2$). Following this procedure, the quantified MOR criteria in the other five dimensions (i.e., with sizes of b, l_1, d, c, l_2 in Fig. 1) are obtained as (5b) - (5f) describe.

$$V_1V_2\cdot\max|R|\cdot b/(\min R)^3 \le \varepsilon \quad (5b)$$

$$V_1V_2\cdot\max|S|\cdot l_1/(\min R)^3 \le \varepsilon \quad (5c)$$

$$V_1V_2\cdot\max|Q|\cdot d/(\min R)^3 \le \varepsilon \quad (5d)$$

$$V_1V_2\cdot\max|R|\cdot c/(\min R)^3 \le \varepsilon \quad (5e)$$

$$V_1V_2\cdot\max|S|\cdot l_2/(\min R)^3 \le \varepsilon. \quad (5f)$$

The value of ε depends on the permitted error and will influence the adoption of reduced-order models. Normally, smaller values of ε will produce more rigorous reduced-order models, while larger values will generate approximate model-order reduction.

In the previous formulae (5), elements of the arrays Q, R, and S are summation limits of X, Y, and Z variables in (1), respectively, e.g., $Q=[E+d, E+d-a, E-a, E]$. The min$R$ means the minimum distance of two cells which can be calculated by an arithmetic expression (6).

$$\min R = \sqrt{\min\_x^2+\min\_y^2+\min\_z^2} \quad (6)$$

where

$$\min\_x = \begin{cases}\min|Q| & Q=|Q|,\ or\ Q=-|Q| \\ 0 & \text{otherwise}\end{cases}$$

$$\min\_y = \begin{cases}\min|R| & R=|R|,\ or\ R=-|R| \\ 0 & \text{otherwise}\end{cases}$$

$$\min\_z = \begin{cases} \min|S| & S = |S|,\ or\ S = -|S| \\ 0 & \text{otherwise} \end{cases}.$$

In practical applications, when the dimensions in the direction of current (i.e., l_1 or l_2 in Fig. 1) are eliminated according to the MOR criteria, the inductance value is equal to the value obtained by the simplified model multiplied by the eliminated length; otherwise, the inductance value of the primitive model is equivalent with the reduced-order model.

B. Code implementation

In this section, we propose a scheme of the working flow of our MOR procedure, which can be easily implemented in MATLAB or other computation environments. We assume the meshing or spatial discretization is carried out according to $\lambda_{\min}/20$-rule [6]. The code implementation can be carried out, step by step, as follows:

1. Evaluation of the following quantities:

a) Find the minimum distance of two cells (i.e., min$R$) according to (6);

b) Find the LHS terms of (5) using matrix approach simultaneously and denote them as $ratio(i)$ (i=1, 2, 3, 4, 5, 6, corresponding to dimensions of a, b, l_1, d, c, l_2);

2. Determination of model-order reduction basing on the MOR criteria

An array *flag* is used to record a binary decision of MOR for corresponding dimensions; i.e., we compare $ratio(i)$ of each dimension with the user-set threshold value ε.

$if\,(ratio(i) \le \varepsilon)\ \ then\ \ flag(i) = 0\ \ \ else\ \ flag(i) = 1$;

Then, a fast hierarchical decision algorithm is applied to choose the suitable reduced-order calculation model. The sum of all elements of the array *flag* is equal to the order of the reduced-order model; e.g., if $sum(flag[\,]) = 4$, then the primitive model can be represented by a 4D model.

3. Coordinate transformation

A certain model-order value usually corresponds to several primary structures. For instance, there are 15 different 2D models, and all of them can be categorized as two primitive models (see the 2D-1 model and 2D-2 model in Table 1) with some coordinate transformation. We take a specific case ($flag = [0\ 0\ 0\ 0\ 1\ 1]$) to illustrate three procedures of the coordinate transformation.

a) Determination of specific reduced-order model;

The sum of *flag* indicates that the reduced-order model is a 2D model which is composed of a point and a thin tape as Fig. 2(a) shows, so it can be categorized to the primitive 2D-1 model illustrated in Table 1 using coordinate transformation.

b) Matching cells;

The point in Fig. 2(a) is localized at the origin of coordinates which is different from 2D-1 model in Table 1; hence, we need to exchange all the dimensional parameters of the two cells to obtain a model as Fig. 2(b).

c) Matching coordinates;

The thin tape in primitive 2D-1 model in Table 1 is perpendicular to Y-axis, In order to use the related formula we need to exchange X and Y coordinates of cell with current I_2.

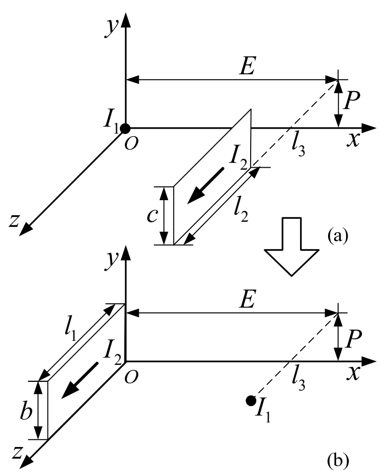


Fig. 2. A coordinate transformation example.

4. Correction of inductance values

In the above case, since the dimension l_1 in the direction of current I_1 is eliminated according to the MOR criteria, the inductance value is equal to the value obtained by the reduced-order model (see 2D-1 model in Table 1) multiplied by the eliminated length l_1.

IV. NUMERICAL RESULTS

In the following experiments, the cross-sectional dimensions of two rectangular cells are fixed at 0.5 mm×0.5 mm and 0.8

mm×0.8 mm. The two cells are aligned at one terminal and their lengths are 4 mm and 6 mm, respectively. We set the threshold value ε of 1e-21 in the numerical experiments. Double precision is used in the following results, and all the numerical experiments are conducted under the hardware condition of an ordinary 64-bit computer with a 2.4GHz processor and a 3GB RAM (DDR 3).

To represent the miscalculation phenomenon, mutual inductance values are extracted using the primitive formula (1) and our MOR approach. The inductance values and reduced-orders of adopted models respect to different lateral separation are shown in Fig. 3. The mutual inductance should decrease smoothly as the distance increases; nevertheless, from the data in Fig. 3, it is evident that the primitive formula (1) produces totally wrong inductance values, including even negative results. On the contrary, the proposed MOR method is numerically stable.

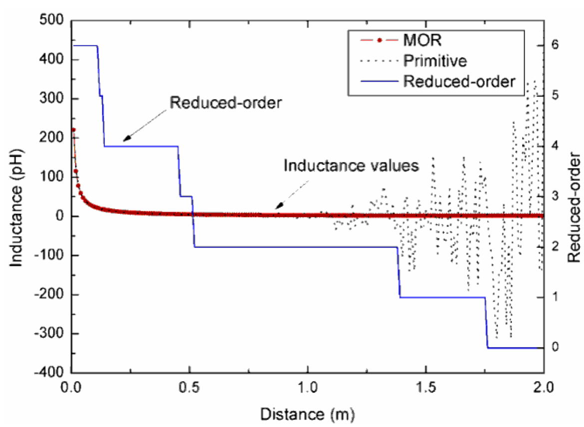


Fig. 3. Mutual inductance values and reduced-orders respect to varying distance.

A. Accuracy

The fast multipole method (FMM) is applied in general PEEC framework to calculate partial elements in a more efficient way without compromising the accuracy [5]. A widely-used tool FastHenry uses a specific multipole-accelerated generalized minimal residual (GMRES) algorithm to compute inductance matrix [21]. To investigate the accuracy of our procedure, mutual inductance values are calculated using our MOR method and FastHenry. The geometrical parameters in the above case are maintained in this study. Figure 4 shows the results.

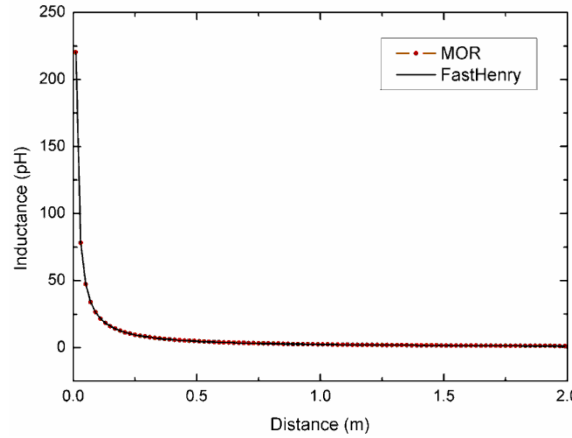


Fig. 4. Mutual inductance values by MOR method and FastHenry.

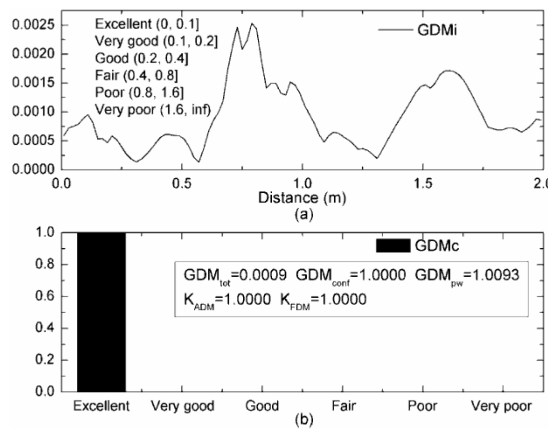


Fig. 5. Feature selective validation of mutual inductance values by MOR method and FastHenry.

The feature selective validation (FSV) [22-24] technique is an effective algorithm that can be used to validate computational electromagnetics (CEM). We use the FSV technique to compare details of the two inductance datasets. Figure 5(a) shows point-by-point comparisons for a global difference measure (GDM). Figure 5(b) illustrates the proportion of the point-by-point analyses that falls into the six natural language descriptor categories (i.e., excellent, very good, good, fair, poor, very poor). Other synthetic FSV parameters are also given in Fig. 5(b). The feature selective validation indicates that our MOR method has an excellent agreement with FastHenry.

B. Stability

Zhong's procedure is numerically more stable than the conventional formulae in [15-17].

Mutual inductance values are extracted by our MOR method and Zhong's procedure in case that the lateral separation between two cells varies up to five meters. Figure 6 shows the computation results. It is evident that our MOR method is numerically more stable than Zhong's procedure for the structures with large spacing.

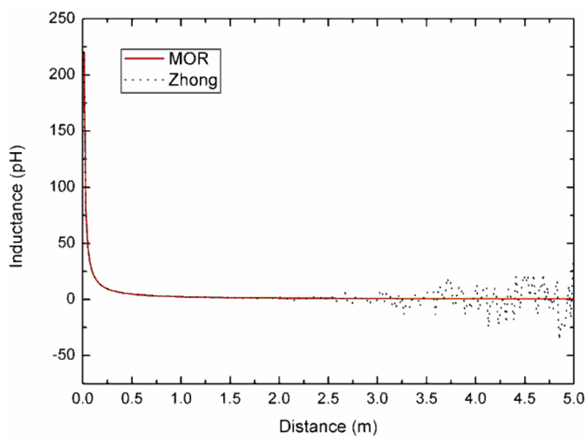


Fig. 6. Mutual inductance values by MOR method and Zhong's procedure.

C. Calculation cost

Table 2 shows the total consumed time of various models in 100,000 independent experiments. The number of summation loops of a 0D model's formula is only 1/64 of a 6D model's formula. Since a particular matrix approach is used to accelerate 64 summation loops in our Matlab code, the realistic consumed time of the 0D model is 43.45% of the 6D model. As a conclusion from Table 2, our MOR procedure not only improves the accuracy and stability, but also reduces the calculation complexity.

Zhong's formula consumes nearly the same time as the primitive 6D model; because it is actually a result of 64 expressions of self inductance [15, 16] which is as complex as the primitive function (1c). That is to say, our MOR method is, also, faster than Zhong's procedure.

Table 2: Consumed time of various models

Model	Cost time (s)	Normalized percent
6D	26.763	100.00%
5D	22.432	83.82%
4D-1	22.341	83.48%
4D-2	26.345	98.44%
4D-3	26.015	97.21%
3D-1	22.939	85.71%
3D-2	25.336	94.67%
2D-1	16.565	61.90%
2D-2	13.895	51.92%
1D	12.189	45.54%
0D	11.629	43.45%

D. A transmission line structure experiment

In the following, we present a two-conductor transmission line structure to show the influence of inductance values on the accuracy of PEEC modeling. The dimensions of the structure are illustrated in Fig. 7. The two conductors are with the same dimensions, and a common reference ground provides the possibility of generating interference due to crosstalk. An excitation source consisting of a source resistance R_S (50 ohms) and a source voltage V_S (1 volt) is connected to a load R_L (50 ohms) via a generator conductor and a reference ground. A receptor conductor connects the other two terminations, represented by resistors R_{NE} (50 ohms) and R_{FE} (50 ohms).

The crosstalk can be viewed as a transfer function between the input V_S and the outputs V_{NE} and V_{FE} (induced voltages at two ends of the receptor conductor). The frequency domain response of the crosstalk transfer ratio at the near-end (i.e., $20\times\log_{10}(V_{NE}/V_S)$) was calculated by both MTL theory [25] and PEEC method.

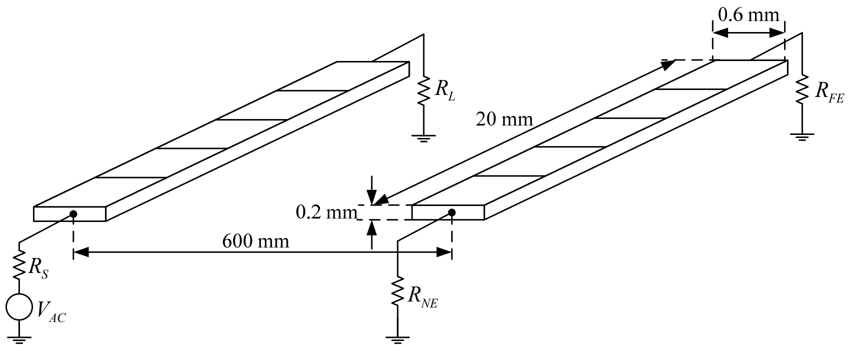


Fig. 7. A two-conductor transmission line structure.

$$L_1 = \begin{bmatrix} 2283.7737 & 532.9164 & 209.1607 & 135.8845 & 101.0547 & 2.6667 & 2.6666 & 2.6664 & 2.6661 & 2.6657 \\ 532.9164 & 2283.7737 & 532.9164 & 209.1607 & 135.8845 & 2.6666 & 2.6667 & 2.6666 & 2.6664 & 2.6661 \\ 209.1607 & 532.9164 & 2283.7737 & 532.9164 & 209.1607 & 2.6664 & 2.6666 & 2.6667 & 2.6666 & 2.6664 \\ 135.8845 & 209.1607 & 532.9164 & 2283.7737 & 532.9164 & 2.6661 & 2.6664 & 2.6666 & 2.6667 & 2.6666 \\ 101.0547 & 135.8845 & 209.1607 & 532.9164 & 2283.7737 & 2.6657 & 2.6661 & 2.6664 & 2.6666 & 2.6667 \\ 2.6667 & 2.6666 & 2.6664 & 2.6661 & 2.6657 & 2283.7737 & 532.9164 & 209.1607 & 135.8845 & 101.0547 \\ 2.6666 & 2.6667 & 2.6666 & 2.6664 & 2.6661 & 532.9164 & 2283.7737 & 532.9164 & 209.1607 & 135.8845 \\ 2.6664 & 2.6666 & 2.6667 & 2.6666 & 2.6664 & 209.1607 & 532.9164 & 2283.7737 & 532.9164 & 209.1607 \\ 2.6661 & 2.6664 & 2.6666 & 2.6667 & 2.6666 & 135.8845 & 209.1607 & 532.9164 & 2283.7737 & 532.9164 \\ 2.6657 & 2.6661 & 2.6664 & 2.6666 & 2.6667 & 101.0547 & 135.8845 & 209.1607 & 532.9164 & 2283.7737 \end{bmatrix} [pH] \qquad (7)$$

$$L_2 = \begin{bmatrix} 2283.7737 & 532.9164 & 209.1607 & 135.8845 & 101.0547 & 9.0350 & -7.5292 & 16.5642 & 3.0117 & -6.0233 \\ 532.9164 & 2283.7737 & 532.9164 & 209.1607 & 135.8845 & 3.0117 & 9.0350 & -7.5292 & 16.5642 & 3.0117 \\ 209.1607 & 532.9164 & 2283.7737 & 532.9164 & 209.1607 & 15.0584 & 3.0117 & 9.0350 & -7.5292 & 16.5642 \\ 135.8845 & 209.1607 & 532.9164 & 2283.7737 & 532.9164 & -10.5409 & 15.0584 & 3.0117 & 9.0350 & -7.5292 \\ 101.0547 & 135.8845 & 209.1607 & 532.9164 & 2283.7737 & 10.5409 & -10.5409 & 15.0584 & 3.0117 & 9.0350 \\ 9.0350 & 3.0117 & 15.0584 & -10.5409 & 10.5409 & 2283.7737 & 532.9164 & 209.1607 & 135.8845 & 101.0547 \\ -7.5292 & 9.0350 & 3.0117 & 15.0584 & -10.5409 & 532.9164 & 2283.7737 & 532.9164 & 209.1607 & 135.8845 \\ 16.5642 & -7.5292 & 9.0350 & 3.0117 & 15.0584 & 209.1607 & 532.9164 & 2283.7737 & 532.9164 & 209.1607 \\ 3.0117 & 16.5642 & -7.5292 & 9.0350 & 3.0117 & 135.8845 & 209.1607 & 532.9164 & 2283.7737 & 532.9164 \\ -6.0233 & 3.0117 & 16.5642 & -7.5292 & 9.0350 & 101.0547 & 135.8845 & 209.1607 & 532.9164 & 2283.7737 \end{bmatrix} [pH] \qquad (8)$$

$$R + jL = \begin{bmatrix} 0.0027964 + 1.7682\mathrm{e}-008\mathrm{j} & 6.3109\mathrm{e}-030 + 6.6661\mathrm{e}-011\mathrm{j} \\ 6.3109\mathrm{e}-030 + 6.6661\mathrm{e}-011\mathrm{j} & 0.0027964 + 1.7682\mathrm{e}-008\mathrm{j} \end{bmatrix}.$$

Impedance matrix (R+jL) used in the MTL model is computed by the method of moment (MoM).

In order not to introduce the influence of capacitance accuracy, we use a quasi-static *PEEC* (L_p, R) model [2], and the capacitive coupling is not considered in the transmission line model either.

In the PEEC procedure, each conductor is discretized into five even conductive cells along its length. The inductance matrices L_1 and L_2 are calculated by our MOR method and primitive formula (1) respectively. We see some differences in the mutual inductance values which are in the off-diagonal terms. It is clear that the primitive formula (1) produce inaccurate inductances including even negative values.

Figure 8 shows the near-end crosstalk transfer function in frequency domain from 1 kHz to 1 MHz. Comparing with the MTL model, it is obvious that the inaccurate partial inductance values cause errors while good agreement is obtained by the MOR inductance calculation routine. This numerical experiment indicates that our MOR method can improve the stability and accuracy of partial inductance values and thus accurate PEEC results.

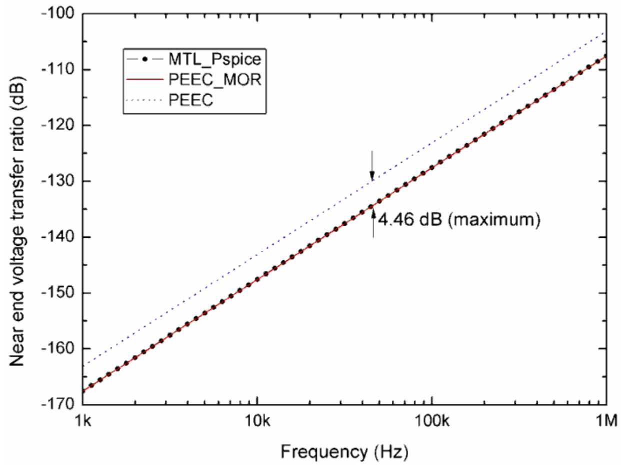


Fig. 8. Frequency-domain representation of the near-end crosstalk transfer function.

V. CONCLUSIONS AND DISCUSSIONS

The proposed novel model-order reduction (MOR) method concerns the stability and accuracy improvement of conventional analytical partial mutual inductance calculation routines. Compact analytical criteria of the MOR approach are presented in this paper and the working flow for a code implementation allows convenient utilization in PEEC modeling. Numerical experiments indicate that the MOR method has its advantages of improving the inductance calculation stability and reducing the calculation complexity, especially for three-dimensional structures with large spacing. This method is more appropriate for PEEC modeling for large complex electronic systems.

This paper only focuses on the analytical inductance calculation for the consideration of fast and simple utilization. Actually, combining analytical and numerical calculation routine (e.g., Gauss-Legendre numerical integration) can be thought of as one option.

APPENDIX

A. A lemma for order reduction

$F(x)$ is a primitive function of $f(x)$. Mathematically, the definite integral of $f(x)$ with limits *a* and *b* respect to *x* equals the difference of $F(x)$ evaluated at *b* and *a*, which can be described as (A-1). This implies that $f(x)$ is with respect to a lower order of *x* comparing with $F(x)$.

$$\int_a^b f(x)dx = F(b) - F(a) \quad \text{(A-1)}$$

Suppose the conditions a) and b) can be satisfied simultaneously, it is easy to proof that $(b-a)\cdot f(x)$ is an approximation to $F(b)-F(a)$ for any value of x in the interval [*a*, *b*]; thus, a lower order formula $f(x)$ can represent the higher order formula $F(x)$.

a) $f(x)$ is a continuous monotonic function in an interval [a, b];

b) $|(b-a)f'(x)| \le \varepsilon$, ε is an extremely small positive value;

B. Brief proof of the lemma

From the view of the limitation theory, we can draw a conclusion as (A-2) if the conditions a) and b) can be satisfied simultaneously.

$$\lim_{x\in[a,b]} (b-a)f'(x) = 0 \quad \text{(A-2)}$$

(A-2) implies two cases:

Case 1: $b-a \to 0$, this means the limitation of integral interval is zero;

$$\int_a^b f(x)dx = F(b)-F(a) \approx (b-a)f(x) \to 0 \quad \text{(A-3)}$$

Case 2: $f'(x) \to 0$, that is, $f(x)$ is a constant function with respect to $x \in [a,b]$;

$$\int_a^b f(x)dx = F(b)-F(a) \approx (b-a)f(x) \quad \text{(A-4)}$$

Considering (A-3) and (A-4), we can draw the conclusion as the lemma describes.

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