## Enhancement of Scan Angle Using a Rotman Lens Feeding Network for a Conformal Array Antenna Configuration

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*Abstract* – An antenna with a wide scan angle in a wide frequency band is obtained by feeding a conformal arc array with a modified formulation of Rotman lens design. In this paper, two kinds of Rotman lenses are designed to feed a linear array and a conformal array and their scan angles are compared in a specified frequency band. The phase distributions of the linear array elements are linear in all scan angles, but these phase distributions are nonlinear in the conformal array. Therefore in order to design a Rotman lens for conformal arrays, the conventional Rotman lens design formulations must be modified. For this purpose, first the phase distributions of conformal array elements were obtained using the particle swarm optimization (PSO) process. Then by modifying the conventional Rotman Lens design formulations used for linear arrays, appropriate formulations for conformal arrays are obtained. In the end, by selecting two specified linear and conformal arrays with equal number of elements, their maximum scan angles in a specified frequency band are studied. It is shown that in the same frequency band the maximum scan angle increases about 20% in the Rotman Lens fed conformal array antenna.

Index Term - Conformal array, PSO, Rotman lens.

#### **I. INTRODUCTION**

Using an appropriate beam forming network that can feed array antennas has several applications in radar and satellite communication systems. The main problem of usual beam forming networks is their small bandwidth which causes the locations of radiation beams of the array to vary with frequency. Rotman lens [1] is a wideband feed network that relies on path lengths and is designed based on a true time delay (TTD) response. Using the TTD scheme to feed array antennas prevents the dependence of scanning angles on frequency [2]. To have different scan angles, the Rotman lens should provide linear phase distributions with different slopes, along equi-spaced linear array antenna elements which are independent from frequency. Up to now, many types of Rotman lenses have been designed, constructed and modified with various applications in microwave and millimeter-wave frequency bands [1,3]. Also, various works in Rotman lens design have been done with the goals of low phase error [4] and low insertion loss [5].

There is an increasing demand for conformal arrays in modern systems and that is due to their ability in being mounted on surfaces with different shapes. This causes the aerodynamic drag to reduce considerably and the structures get less visible to the human eye. Another important advantage of conformal arrays which is of vital importance in radar applications is their wide angle coverage. Other benefits include space saving, potential increase in the available aperture, reduction of radar cross-section (RCS), elimination of radome-included bore-sight error, etc. [6].

In this study, an increase in the maximum scan angle is obtained from a conformal array that is fed by a Rotman lens. To this purpose, in the first step, the design formulations of a trifocal Rotman lens [7] which is used to feed linear arrays are modified to feed the conformal array. Next, in order to compare the maximum scan angles, two linear and conformal arrays fed by their corresponding Rotman lenses are examined. Also, to obtain the phase distribution of the conformal array antenna at different scan angles, the PSO procedure has been used.

In Section II, the modified formulations of Rotman lens for conformal array antennas are presented. In Section III, the optimum phase distributions for the modified Rotman lens formulations are obtained based on the PSO method. Finally in Section IV, the designed Rotman lenses are simulated with the EM full wavesimulation software (HFSS) and the simulation results of the linear and conformal arrays are compared. It must be noticed in the mentioned procedures the modified formulations of input and output curvatures of Rotman lens are obtained by suitable code in Matlab software. Then the obtained shapes are imported to the EM full wave simulator HFSS to simulate electromagnetic characteristics of Rotman lens.

### II. ROTMAN LENS FORMULATIONS BASED ON ARRAY ANTENNA CONFIGURATIONS

Different design methods have been reported for Rotman lenses used for linear arrays [1,4,8]. These methods include the trifocal method, quadrufocal method and non-focal method of which the trifocal method is the most frequently used one. Based on the conventional trifocal lens principles, some trifocal Rotman lens formulations have been developed to obtain the desired phase distributions for linear array antennas [7,9,10]. In this section, these formulations are developed such that they can be used for conformal array antennas.

# A. Modified Rotman lens formulations for feeding conformal array antennas

Figure 1 represents a general structure of a Rotman lens to feed a conformal array. Curve E represents the desired configuration of the conformal array antenna.



Fig. 1. Trifocal Rotman lens structure for conformal arrays.

The conventional trifocal Rotman lens for linear arrays produces linear phase distributions for linear array elements. For conformal array antennas, Rotman lens formulations must be modified to produce suitable phase distributions which depend on the configuration of the array antenna. An important point that must be noticed in the new design formulations is determining the required phase distributions to create a set of three design equations. This is done by using a procedure similar to the one used in [1]. Based on the design procedure in [1], the modified formulations for Rotman lens design are presented in Eqs. (1a-c). In these equations,  $l\phi_0$  produces the required phase distribution of the conformal array for zero scan angle and similarly  $l\phi_0 \pm l\phi_1$  create the suitable phase distributions of conformal array to produce the

radiation pattern with maximum scan angles,  $\pm \Psi_a$ , respectively. The optimization procedure to determine these phase distributions are presented in the next section;

$$F_2 P \sqrt{\epsilon_r} + W \sqrt{\epsilon_e} + (l\varphi_0 + l\varphi_1) =$$

$$f_2 \sqrt{\epsilon_r} + W_0 \sqrt{\epsilon_e}, \qquad (1a)$$

$$F_{3}P\sqrt{\epsilon_{r}} + W\sqrt{\epsilon_{e}} + (l\varphi_{0} - l\varphi_{1}) =$$
(1b)

$$f_2 \sqrt{\epsilon_r} + W_0 \sqrt{\epsilon_e},$$
  

$$F_1 P \sqrt{\epsilon_r} + W \sqrt{\epsilon_e} + l \varphi_0 =$$
  

$$f_1 \sqrt{\epsilon_r} + W_0 \sqrt{\epsilon_e},$$
(1c)

where  $F_iP$  is the physical distance between points  $F_i$  and P. Also,  $l\phi_{0,1}=\phi_{0,1}\times(\lambda_g/2\pi)$  and  $\lambda_g$  is the wavelength.

Normalizing Eqs. (1a-c) by the electrical length of the central focal length,  $\sqrt{\epsilon_r} f_1$ , results in:

$$\frac{F_2P}{f_1} = \beta - \omega - a - b, \qquad (2a)$$

$$\frac{F_3P}{f_1} = \beta - \omega + a - b, \tag{2b}$$

$$\frac{F_1P}{f_1} = 1 - \omega - b, \qquad (2c)$$

where

$$B = \frac{f_2}{f_1}$$
,  $a = \frac{\varphi_1}{f_1}$ ,  $b = \frac{\varphi_0}{f_1}$  and  $\omega = \frac{W - W_0}{f_1}$ 

Based on the geometrical configuration of Rotman lens (Fig. 1),

$$F_2 P^2 = (-f_2 \cos \alpha - X)^2 + (-f_2 \sin \alpha + Y)^2$$
(3a)

$$= f_2^2 + X^2 + Y^2 + 2f_2 X \cos \alpha - 2f_2 Y \sin \alpha,$$
  

$$E B^2 = (-f_1 \cos \alpha - X)^2 + (-f_1 \sin \alpha - X)^2$$

$$= f_2^2 + X^2 + Y^2 + 2f_2 X \cos \alpha + 2f_2 Y \sin \alpha.$$
 (3b)

$$F_1 P^2 = (f_1 + X)^2 + Y^2.$$
 (3c)

After normalizing Eqs. (3a), (3b) and (3c) by  $f_1^2$  and equating them with the square of Eqs. (2a), (2b) and (2c) respectively, the set of three goal equations is obtained. By solving these three equations, the positions of the X and Y coordinates of the phase centers of array ports and transmission line lengths (Wi) are obtained. This method can be used for any practical array curves including linear arrays. For conventional linear array configurations the expressions  $l\phi_0$  and  $l\phi_0+l\phi_1$  are replaced by zero and  $\pm Y_3 \sqrt{\epsilon_i} \sin(\psi_a)$  respectively, in which  $\Psi_{\alpha}$  is the maximum scan angle of the array. But in this paper, we use the modified phase distributions for both linear and conformal arrays. In the next section, the optimization procedures to obtain the phase distributions for a linear array and a specified conformal array are explained.

### III. OPTIMUM PHASE AND AMPLITUDE DISTRIBUTIONS OF THE CONFORMAL AND LINEAR ARRAYS

To obtain the required phase distributions of  $\varphi_1$  and  $\varphi_0$  in Rotman lens design formulations, at first the configuration of the conformal array and its radiating elements, must be determined. Therefore, antenna elements and array configurations are discussed in the following sections.

# A. Conformal and linear array configurations and their radiation elements

Because of the wide operating frequency band in this study (10-14 GHz), double ridged horn antennas are used as the radiating elements of the array antennas. As shown in the Fig. 2, radiation characteristics of this antenna are almost constant and VSWR<2 in the operating frequency band.



Fig. 2. Radiation characteristics of double ridged horn antenna: (a) VSWR, (b) normalized amplitude of electric field radiation patterns, E and H-plane.

Two configurations of array antennas including the linear array and the conformal arc array are synthesized

in this study. To have a precise comparison of the radiation patterns of the synthesized arrays, we use equal number of elements with equal element spacing for both of these arrays. The main problem of the array designs with wide scan angles is the existence of grating lobes, so the array elements are placed as close as possible to each other. Figure 3 (a) shows a linear array of 16 double ridged horn antennas. The conformal array can be formed on any desired curves. Without loss of generality in this paper, the conformal array elements stand on a quarter of a circle. Figure 3 (b) shows the conformal array configuration ( $\gamma$ =45<sup>0</sup>).



Fig. 3. Array antenna configurations of double ridged horn antennas: (a) linear array, (b) arc curve conformal array ( $\gamma$ =45<sup>0</sup>).

Since one of the most important parameters in a finite array is the coupling between array elements, in the next section active element radiation patterns [11] of the arrays are used in the synthesis process of the array antennas.

# **B.** Synthesis of linear and conformal arrays using the PSO algorithm

In this section, the PSO algorithm is used to obtain the optimum phase and amplitude distributions for linear and conformal array elements. The phase distributions are used to construct the Rotman lens design formulations and the amplitude distributions are used to design of the Rotman lens ports. To this end, the phases and amplitudes of the array elements are used to produce the particles of the PSO algorithm. The particles move in a multidimensional search space so that every particle adjusts its position with respect to its adjacent particles while considering their prior experience. In general, the velocity and position of each particle,  $p_k$ , is expressed as follows [12]:

$$v_{k}(t) = v_{k}(t-1) + c_{1}r_{1}[P_{k}(t-1) - x(t-1)] + c_{2}r_{2}[\overline{G}_{k}(t-1) - \overline{x}(t-1)]$$
(4)  
$$\overline{x}(t) = \overline{x}(t-1) + \overline{v}_{k}(t).$$

In Eq. (4),  $\bar{x}$  and  $\bar{v}$  represent the position and the velocity of particles respectively. The positive constants  $c_1$  and  $c_2$  are usually equal,  $c_1 = c_2 = 2$ .  $r_1$  and  $r_2$  are two random values in the range (0,1). The best previous position of k<sub>th</sub> particle (coefficient) is presented as  $\bar{P}_k(t-1)$  and  $\bar{G}_k(t-1)$  denotes the best k<sub>th</sub> particle in the population. Equation (4) is used to update the velocity and position of the coefficients as a function of their previous velocity and positions, then the coefficients move towards a new position. In each update the phases and amplitudes of the array elements are replaced with new ones and the obtained radiation pattern is compared with the desired radiation pattern. To this purpose, the error function or the fitness function is defined as the absolute difference between the target and calculated patterns of the array.

The optimization process is continued until the error function converges to the acceptable value. Figure 4 shows the desired radiation patterns in broadside and the maximum scan angle of  $-65^{\circ}$ . The fitness function used in the PSO procedure is defined as:

Fitness function = 
$$\sum_{i=1}^{5} W_i \sum_{\theta=\theta_{i-1}}^{\theta_i} |F_o - F_d|,$$

F<sub>d</sub> is the desired goal radiation pattern and F<sub>N</sub> is the normalized calculated complex array factor. Also,  $F_N(\theta, \phi) = \sum_{n=1}^{N} A_n e^{j\phi_n} E_n(\theta, \phi)$ , in which  $E_n(\theta, \phi)$  is an embedded complex number that represents active radiation fields for the  $n^{th}$  element and must be multiplied by the amplitude and phase vector  $A_n e^{j\phi_n}$ . Weighting coefficients depicted in the desired patterns are defined in each part of the patterns to scale the error of that part in the optimization procedure.

# C. Phase and amplitude distributions for linear and conformal array Rotman lens designs

Considering the modified formulations in Eq. (1), knowing the phase distributions of  $\varphi_0$  and  $\varphi_1$  is necessary in the Rotman lens design. Also, because the Rotman lens operates based on the TTD properties, it is possible to use the radiation pattern of the array elements at an arbitrary frequency in the operating frequency band (10-14 GHz) for the PSO procedure. Therefore, in this paper we use the active element radiation patterns at center frequency to synthesize the conformal array antenna.

The PSO algorithm has been run with the use of 800 particles and 500 iterations to obtain the optimized amplitude and phase distributions of the arrays. Figure 4 (a) shows the optimized radiation pattern for zero scan angle and Fig. 4 (b) shows the optimized radiation

pattern for  $-65^{\circ}$  scan angle for the linear and conformal arrays.



Fig. 4. Optimized radiation patterns: (a) zero scan angle ( $w_1=w_5=0.4$ ,  $w_2=w_4=2.2$  and  $w_3=1.8$ ), (b)  $-65^0$  scan angle ( $w_1=0.8$ ,  $w_2=1.8$ ,  $w_3=3.4$ ,  $w_4=2.8$  and  $w_5=1.6$ ).

Figure 4 (b) shows the fitted pattern for  $-65^{\circ}$  scan angle. It is shown that the linear array could not make a good fitted pattern. Figures 5 and 6 show the phase and amplitude distributions of array elements for zero and  $-65^{\circ}$  scan angles.



the conformal array the radiating elements are oriented in different directions based on the array curvature. Therefore, the radiation patterns of conformal array elements are different. In the Rotman lens design the amplitude distributions for different scan angles are realized by suitable design of the beam ports corresponding to each scan angle in the radiation pattern. This principle is studied in the next section.

#### **IV. DESIGN OF THE ROTMAN LENSES**

As mentioned earlier, to design the Rotman lenses to feed the linear and conformal array antennas, it is necessary to have the phase distributions  $\varphi_0$  and  $\varphi_1$ . Beside the phase distributions, the amplitude distributions of array elements for these scan angles must be produced by the Rotman lenses. To design the Rotman lens for the arc-shaped array, two points must be made clear. First, the phase distributions of the array elements and second the amplitude distributions of the array elements for different scan angles. By obtaining the optimized phase distributions and applying them to the modified Rotman lens equations, the lens can realize the desired phase distributions. To create the desired amplitude distributions, especially for maximum scan angles, the input ports must be designed and directed correctly. These amplitude distributions depend on beam ports widths, the shape of the inner receiver curve and the angle of input ports orientations. The widths of the beam ports and shape of the inner receiver curve can be realized by controlling the trifocal lens parameters. Table 1 shows the parameters used for designing the proposed Rotman lenses.

$f_1$	Center focal length	262 mm
$f_2$	Off focal length	249.5 mm
NB	Number of beam ports	11
NA	Number of array ports	16
α	Focal angle	24º
$\mathcal{E}_r$	Cavity permittivity	2.2
$W_0$	Center transmission line length	194 mm
$f_0$	Center frequency	12 GHz

Table 1: Rotman lens design parameters

Fig. 6. Amplitude distributions of array elements: (a) zero scan angle, (b)  $-65^{\circ}$  scan angle.

In the conformal array antenna, unlike the linear array antenna, the phase distributions for zero and  $-65^{\circ}$ scan angles, represent the nonlinear behavior of the phase distributions. Also, the amplitude distribution for  $-65^{\circ}$  scan angle shows that the elements which point to the  $-65^{\circ}$  scan angle have greater amplitudes. Difference between the amplitude distributions in conformal and linear arrays is due from the different orientation of array ports and radiating elements. Unlike the linear array, in



Fig. 5. Phase distributions of array elements: (a) zero scan angle, (b)  $-65^{\circ}$  scan angle.

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The optimization results for amplitude distribution of  $-65^{\circ}$  scan angle shows that the elements which correspond to this scan angle have higher amplitudes. This amplitude distribution has an important role in controlling the side lobes of the radiation pattern. To have this amplitude distribution on antenna elements, beam ports have to be oriented towards the desired angle. Figure 7 shows the designed Rotman lenses with 11 beam ports and 16 array ports for linear and conformal arrays respectively. Because of amplitude tapering for the linear array in Fig. 6, each beam port is directed towards the center of the array port contour, like in [13].

Figure 7 (b) shows that the orientation of beam port 1 for the conformal array Rotman lens is not towards the center of the array port contour. In fact, the orientation of beam port 1 is shifted by  $\Gamma$  degrees from the center of the array curve, as shown in Fig. 7 (b). This is done based on the obtained optimized amplitude distributions for the conformal array (as shown by the solid lines in Fig. 6) and causes certain array elements to have higher amplitudes.



Fig. 7. Modified Rotman lens structures: (a) linear array, (b) conformal array.

The purpose of the dummy ports is to minimize the multipath interference [14]. The substrate of the microstrip structure is *Rogers RT/duroid 5880(tm) with*  $\varepsilon_r = 2.2$ ,  $tan\delta = 0.0009$ . Figure 8 shows two amplitude distributions across array ports for the two Rotman lens designs shown in Fig. 7, when beam port 1 is excited.



Fig. 8. Amplitude distribution across array ports when port 1 is excited.

### V. SIMULATION RESULTS OF THE PROPOSED ROTMAN LENSES

Figure 9 shows the radiation patterns of the linear array of Fig. 3 (a) fed by the modified Rotman lens of Fig. 7 (a) at the frequencies of 10, 12 and 14 GHz. To better clarify the grating lobes, the radiation patterns of 7-11 beam ports are not shown.

Figures 9 (a) and 9 (b) show that the gain of the array antenna at the maximum scan angle decreases considerably at the frequencies of 10 and 12 GHz, and Fig. 9 (c) shows that not only the gain decreases, but also the grating lobes increase considerably at 14 GHz. At the maximum scan angle the decrease in the gain is 3.45 dB and 5.3 dB at the frequencies of 12 and 14 GHz respectively, compared to the gain at the zero scan angle.

Figures 10 (a-c) show the radiation patterns of the conformal array of Fig. 3 (b) fed by the modified Rotman lens of Fig. 7 (b) at the frequencies of 10, 12 and 14 GHz. These figures show that the maximum gain reduction of the conformal array at different frequencies is acceptable up to the  $-65^{0}$ scan angle and the grating lobes do not increase at these frequencies.

To better compare the maximum scan angle of linear and conformal arrays, the acceptable radiation patterns for the maximum available scan angles are depicted for both of the array configurations in Fig. 11. The acceptable maximum scan angles for the conformal array and the linear array are  $-65^{\circ}$  and  $-55^{\circ}$  respectively.



Fig. 9. Radiation pattern of the linear array of double ridged horn antennas fed by modified Rotman lens: (a) 10 GHz, (b) 12 GHz, and (c) 14 GHz.



Fig. 10. Radiation pattern of the conformal array of double ridged horn antennas fed by the modified Rotman lens: (a) 10 GHz, (b) 12 GHz, and (c) 14 GHz.



Fig. 11. Optimized radiation patterns for  $-65^{\circ}$  and  $-55^{\circ}$  scan angles for conformal and linear arrays.

#### **VI. CONCLUSION**

An improvement in the scan angle of a wide band array antenna is obtained through feeding a conformal array antenna with a modified design of the Rotman lens. The proposed Rotman lens structure is based on the trifocal design method with new phase distributions in zero and maximum scan angles. The effect of amplitude distributions across array elements are fully considered in the design process. These phase and amplitude distributions are obtained by synthesizing the conformal array using the PSO algorithm. The radiation patterns of the linear and conformal arrays with equal numbers of elements are compared and the enhancement in the scan angle of the conformal array antenna is clearly observed.

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