# On the Theoretical Analysis of Radiation Pattern and Gain of Printed Monopole Antennas

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*Abstract* — This paper reports a theoretical approach to analyze radiation pattern and gain of Printed Monopole Antennas (PMA). Theoretical analysis of PMAs is performed by modeling PMA as an asymmetrical dipole antenna. The effect of patch and ground plane is considered separately then combined. The far field expressions for rectangular and circular PMAs are derived and verified with available experimental results from published work and High Frequency Structure Simulator (HFSS) simulated results. The analytical, simulated and available measured results are in close agreement. The theoretical gain for rectangular and circular monopole antennas are also computed and compared with HFSS simulation results.

*Index Terms* — Asymmetrical dipole antenna, gain, Printed Monopole Antennas (PMA), radiation pattern.

#### I. INTRODUCTION

Printed monopole antennas (PMA) are prominent candidate for broadband and ultra-wide band applications, having features of large impedance bandwidth and omnidirectional radiation patterns. Some of the simulation and experimental works on PMAs are available in the literature [1]-[5]. However, theoretical analysis of radiation characteristics of PMAs is not adequately dealt in the literature. Microstrip line fed printed monopole antenna can be considered as an asymmetrically driven dipole antenna, in which the patch and the ground plane form two arms of the dipole [6]. The spectral domain field components of an infinitesimal current source on an ungrounded dielectric layer can be found in [7], but it doesn't account radiation pattern and gain calculation. However in [7], numerical approach is adapted to calculate input impedance and reflection coefficient for rectangular and F shaped PMA. But the present literature is focused on developing analytical approach to calculate radiation pattern and gain of rectangular and circular PMA taking into account the current distribution on the patch as well the effect of the ground plane below the feed line. To the best of the knowledge of the authors, theoretical treatment of PMA along with closed form expressions for the far field radiation patterns of rectangular and circular PMAs has not been reported in literature. The theoretical results of radiation patterns for rectangular and circular PMAs fed by  $50\Omega$  microstrip line are compared with available experimental data given in [3] and [4], and simulation results obtained using HFSS. In addition to this, the calculated theoretical gain is also verified by HFSS simulation results.

## **II. THEORY**

# A. Radiated field of an HED lying on an ungrounded substrate

The radiated fields of a PMA can be formulated using Green's function of an HED lying on an ungrounded substrate and from the knowledge of current distribution on the patch as well as the ground plane below the feed line. So, an HED lying on a lossless and ungrounded dielectric layer is considered first. To derive spectral domain electric and magnetic field Green's function, an HED is assumed to be lying on a lossless dielectric layer located at  $(x_0, y_0)$  shown in Fig. 1. The x-directed current is defined as  $J_x = \hat{x}\delta(x-x_0)(y-y_0)$  and the effect of  $J_x$  is considered by applying boundary conditions.

The transverse components of the electric field  $\tilde{E}_x$ and  $\tilde{E}_y$  at (x = h) are given by [7]:

$$\tilde{E}_{x} = \frac{j}{\alpha \epsilon_{0} k_{\rho}^{2}} \left[ \frac{k_{x}^{2} u_{2}}{D_{TM}} + \frac{k_{0}^{2} k_{y}^{2}}{D_{TE}} \right] \tilde{J}_{x}, \qquad (1)$$

$$\tilde{E}_{y} = \frac{j}{\omega\epsilon_{0}k_{\rho}^{2}} \left[ \frac{k_{y}k_{x}u_{2}}{D_{TM}} - \frac{k_{0}^{2}k_{y}k_{x}}{D_{TE}} \right] \tilde{J}_{x}, \qquad (2)$$

where

$$D_{TM} = 1 + \frac{u_2 \epsilon_r \left( u_1 + u_0 \epsilon_r tanh(u_1 h) \right)}{u_1 \left( u_0 \epsilon_r + u_1 tanh(u_1 h) \right)},$$
  

$$D_{TE} = u_2 + \frac{\left( u_0 + u_1 tanh(u_1 h) \right)}{\left( 1 + \frac{u_0}{u_1} tanh(u_1 h) \right)},$$
  

$$u_2^2 = -k_z^2 = k_\rho^2 - k_0^2, \quad u_1^2 = -k_z^2 = k_\rho^2 - \epsilon_r k_0^2,$$
  

$$u_0^2 = -k_z^2 = k_\rho^2 - k_0^2, \quad k_\rho^2 = k_x^2 + k_y^2.$$

The far field radiation pattern of an HED on an ungrounded dielectric layer in region 2 (z > h) can be written as [8]:

$$E_{\theta} = \frac{e^{-jk_0 r}}{2\pi r} \Big[ \cos(\phi) \tilde{E}_x + \sin(\phi) \tilde{E}_y \Big], \qquad (3)$$

$$E_{\phi} = \frac{e^{-jk_0r}}{2\pi r} \Big[ -\sin(\phi)\cos(\theta)\tilde{E}_x + \cos(\phi)\cos(\theta)\tilde{E}_y \Big].$$
(4)

Now, substituting Equations (1), (2) in Equations (3) and (4),  $k_0 sin(\theta) cos(\phi)$ ,  $k_0 sin(\theta) sin(\phi)$ ,  $k_0 cos(\theta)$  in place of  $k_x$ ,  $k_y$  and  $k_z$  we get the final far field expressions. The far field radiation pattern of an HED on an ungrounded dielectric layer is expressed as:

$$E_{\theta} = \alpha_1 \frac{n(\theta)\cos(\theta) \{\epsilon_r \cos(\theta) + jn(\theta)\tan(\beta_1 h)\}}{2\epsilon_r n(\theta)\cos(\theta) + j\tan(\beta_1 h) \{n^2(\theta) + \epsilon_r^2 \cos^2(\theta)\}},$$
(5)

$$E_{\phi} = \alpha_2 \frac{\left\{n(\theta)\sec(\theta) + j\tan(\beta_1 h)\right\}}{2n(\theta)\sec(\theta) + j\tan(\beta_1 h)\left\{n^2(\theta)\sec^2(\theta) + 1\right\}},$$
(6)

where

$$\alpha_{1} = -\cos\left(\phi\right) \left(\frac{j\omega\mu_{0}}{4\pi r}\right) e^{-jk_{0}r}, \alpha_{2} = \sin\left(\phi\right) \left(\frac{j\omega\mu_{0}}{4\pi r}\right) e^{-jk_{0}r}$$
$$\beta_{1} = k_{0}n(\theta), \ n(\theta) = \sqrt{\epsilon_{r} - \sin^{2}\left(\theta\right)}.$$

The theoretical gain (G) of an HED on an ungrounded dielectric layer in a given direction  $(\theta, \phi)$  can be expressed as [9]:

$$G = \frac{4\left(\sin^{2}\phi \mid E_{\theta} \mid^{2} + \cos^{2}\phi \mid E_{\phi} \mid^{2}\right)}{\int_{0}^{\frac{\pi}{2}} (\sin\theta) \left[\mid E_{\theta} \mid^{2} + \mid E_{\phi} \mid^{2}\right] d\theta}$$
(7)

It may be noted that the above Green's function for far field depend on both substrate thickness and dielectric constant  $(\epsilon_r)$ . Thus, the variation of thickness and dielectric material and their effects on the field as well as in the gain can be theoretically observed.

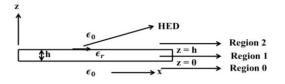


Fig. 1. Geometry of an HED along x-axis on the interface of dielectric and free space.

# **B. Radiated fields of PMA**

The above expressions in Equations (5) and (6) give far fields of a HED on a dielectric substrate. The current supported by the feed in printed monopole antenna shown in Fig. 2 can be expressed in terms of incident traveling wave  $(e^{-jk_0(x+f_g)})$  and reflected wave  $(\Gamma e^{jk_0(x+f_g)})$ due to impedance discontinuity at the junction of feed and the patch. Note that  $x + f_g$  is the total length of the feed including feed gap  $(f_g)$  and  $\Gamma$  is the current reflection coefficient. Thus, the net current given to PMA through the feed line can be given by [10]:

$$J(x, y) = \hat{a}_{x} I_{0} (e^{-jk_{0}(x+f_{g})} + \Gamma e^{jk_{0}(x+f_{g})}) .$$
(8)

Hence,

$$\tilde{J}(k_x,k_y) = \int_{-L/2}^{L/2} \int_{-W/2}^{W/2} J(x,y) e^{-j(k_x x + k_y y)} dx dy .$$
(9)

So, after replacing  $k_x$  and  $k_y$  by  $k_0 sin(\theta) cos(\phi)$ ,  $k_0 sin(\theta) sin(\phi)$  and for  $\Gamma = -1$ , Equation (9) can be written as:

$$\tilde{J}(\theta,\phi) = 4\sin c \left(0.5k_0 W \sin(\theta) \sin(\phi)\right) \left[\sigma_1 - \sigma_2\right], \quad (10)$$
  
where

$$\begin{split} \sigma_{\mathrm{i}} &= 2\sin\left(0.5k_{\mathrm{o}}L\sin(\theta)\cos(\phi)\right)\cos\left(0.5k_{\mathrm{o}}L\right)\left\{k_{\mathrm{o}}\cos\left(f_{\mathrm{s}}k_{\mathrm{o}}\right) + jk_{\mathrm{o}}\sin(\theta)\cos(\phi)\sin\left(f_{\mathrm{s}}k_{\mathrm{o}}\right)\right\},\\ \sigma_{\mathrm{2}} &= 2\cos\left(0.5k_{\mathrm{o}}L\sin(\theta)\cos(\phi)\right)\sin\left(0.5k_{\mathrm{o}}L\right)\left\{k_{\mathrm{o}}\sin(\theta)\cos(\phi)\cos\left(f_{\mathrm{s}}k_{\mathrm{o}}\right) + jk_{\mathrm{o}}\sin\left(f_{\mathrm{s}}k_{\mathrm{o}}\right)\right\}\right\},\end{split}$$

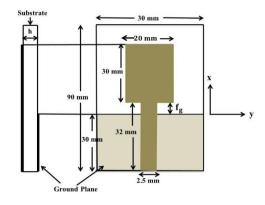


Fig. 2. Geometry of rectangular printed monopole antenna on dielectric substrate ( $\epsilon_r = 4.3$ ,  $tan\delta = 0.02$ ) of thickness h=1.52 mm in [3].

It may be noted that current distribution in Equation (10) includes a quadrature term. In case of printed monopole antennas, the ground plane also contributes

to the radiation field. The ground plane acts as an asymmetric image of the monopole to form an asymmetrically driven dipole antenna. The current distribution in the ground plane can be given as:

$$\tilde{J}_{g}(\theta,\phi) = 4\sin c \left(0.5k_{0}W_{g}\sin(\theta)\sin(\phi)\right) \left[\sigma_{11} - \sigma_{12}\right], \quad (11)$$
  
where

$$\sigma_{11} = 2k_0 \sin\left(0.5k_0L_e\sin(\theta)\cos(\phi)\right)\cos\left(0.5k_0L_e\sin(\theta)\cos(\theta)\right)$$

$$\begin{split} \sigma_{11} &= 2k_0 \sin\left(0.5k_0 L_g \sin(\theta)\cos(\phi)\right) \cos\left(0.5k_0 L_g\right),\\ \sigma_{12} &= 2k_0 \cos\left(0.5k_0 L_g \sin(\theta)\cos(\phi)\right) \sin\left(0.5k_0 L_g\right) \sin(\theta)\cos(\phi) \,. \end{split}$$
 $L_{e}$ ,  $W_{e}$  represent the length and width of the ground plane of PMA. Thus, the overall radiation pattern for the

rectangular printed monopole antenna, including the effect of the partial ground plane as shown in Fig. 2 can be written as:

$$E_{\theta pr} = \left(\frac{j\omega\mu_0}{4\pi r}\right) e^{-jk_0r} (\tilde{J}(\theta,\phi) + \tilde{J}_g(\theta,\phi)) E_\theta, \quad (12)$$

$$E_{\phi pr} = \left(\frac{j\omega\mu_0}{4\pi r}\right) e^{-jk_0 r} (\tilde{J}(\theta,\phi) + \tilde{J}_g(\theta,\phi)) E_{\phi}.$$
 (13)

The gain for the case of a rectangular printed monopole antenna can be calculated using Equation (7). The closed form expressions for the far field radiation patterns of circular printed monopole antenna shown in Fig. 3 can be written as:

$$E_{\theta_{pc}} = \left(\frac{e^{-jk_0r}}{r}\right) (\tilde{J}(\theta,\phi) + \tilde{J}_g(\theta,\phi)) E_{\theta}, \qquad (14)$$

$$E_{\phi_{pc}} = \left(\frac{e^{-jk_0r}}{r}\right) (\tilde{J}(\theta,\phi) + \tilde{J}_g(\theta,\phi)) E_{\phi}.$$
(15)

For circular PMA,  $\tilde{J}(\theta, \phi)$  can be given as:

$$\tilde{J}(\theta,\phi) = \int_{0}^{2\pi} \int_{0}^{a} J(a\cos(\theta), a\sin(\theta)) e^{-j(k_x a\cos(\theta) + k_y a\sin(\theta))} a dad\theta,$$

where  $k_x = k_0 sin(\theta) cos(\phi)$ ,  $k_y = k_0 sin(\theta) sin(\phi)$  and a is the radius of the circle. Similar to rectangular PMA, the gain of circular printed monopole antenna can be found using Equation (7).

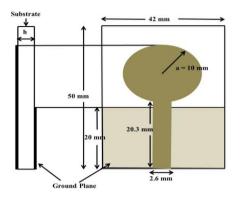


Fig. 3. Geometry of circular printed monopole antenna on dielectric substrate ( $\epsilon_r = 4.7, tan\delta = 0.02$ ) of thickness h=1.5 mm in [4].

#### **III. RESULTS**

This section presents the results computed using the analytical expressions derived for the rectangular PMA and circular PMA and compares the same with the measured results available in literature as well with the results obtained through HFSS simulations. The radiation patterns of rectangular printed monopole antenna are shown in Fig. 4 and Fig. 5, whereas the gain plot is shown in Fig. 6. The gain plot shows that the gain is decreasing in the given frequency band. In other words, it can be concluded that the antenna is more directive for lower frequencies in comparison to higher frequencies.

The radiation patterns of circular printed monopole antenna are shown in Fig. 7 and Fig. 8, whereas the gain plot is shown in Fig. 9. From Fig. 9 it can be observed that gain is almost constant for the given band.

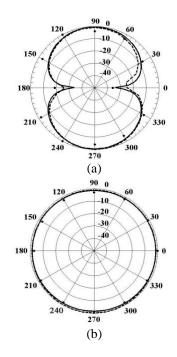
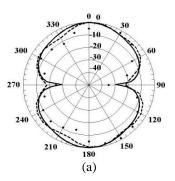


Fig. 4. Radiation patterns: (a) E-plane and (b) H-plane of the rectangular printed monopole antenna shown in Fig. 2 at 2.45 GHz (theory (-), simulation using HFSS (--), and measured (.) [3]).



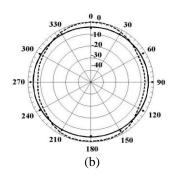


Fig. 5. Radiation patterns: (a) E-plane and (b) H-plane of the rectangular printed monopole antenna shown in Fig. 2 at 5.2 GHz (theory (—), simulation using HFSS (- –), and measured (.) [3]).

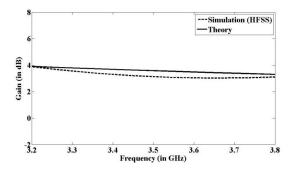


Fig. 6. Gain of rectangular printed monopole antenna on dielectric substrate ( $\epsilon_r = 4.3$ ,  $tan\delta = 0.02$ ) of thickness h=1.52 mm.

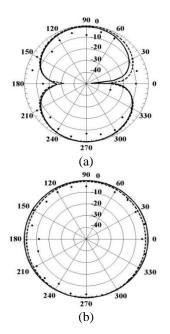


Fig. 7. Radiation patterns: (a) E-plane and (b) H-plane of the circular printed monopole antenna shown in Fig. 3 at 3 GHz (theory (—), simulation using HFSS (– –), and measured (.) [4]).

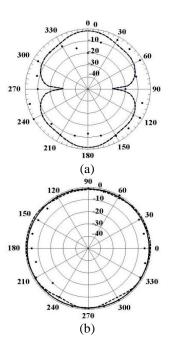


Fig. 8. Radiation patterns: (a) E-plane and (b) H-plane of the circular printed monopole antenna shown in Fig. 3 at 6.5 GHz (theory (—), simulation using HFSS (– –), and measured (.) [4]).

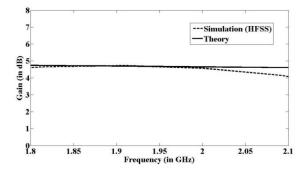


Fig. 9. Gain of circular printed monopole antenna on dielectric substrate ( $\epsilon_r = 4.7, tan\delta = 0.02$ ) of thickness h=1.5 mm.

#### **IV. CONCLUSION**

In this paper, the transverse field components in spectral domain are derived for a horizontal electric dipole on a lossless dielectric layer, which is not backed by a conducting ground plane, are used to calculate the radiation patterns for rectangular and circular printed monopole antenna. Since the ground plane also affects the radiation characteristics of PMAs, taking the ground plane as an asymmetric image of the monopole the overall far field components of PMA are derived. However, the modeling has some limitations because the concept can be implemented for regular shape of the patch and the ground plane only. But the theoretical results in the present cases for the radiation pattern are in good agreement with HFSS and available experimental results. Further, the theoretical gains of both PMAs are also verified using HFSS simulations.

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