Measurement Uncertainty of Antenna Efficiency Measured Using the Two-Antenna Method in a Reverberation Chamber

Wei Xue [1], Yuxin Ren [2], Xiaoming Chen [1], Zhengpeng Wang [3], Yingsong Li [4,5], and Yi Huang [6]

[1] School of Information and Communications Engineering
Xi'an Jiaotong University, Xi'an, 710049, China

[2] China Academy of Information and Communications Technology, Beijing, 100191, China
renyuxin@caict.ac.cn

[3] Electronics and Information Engineering
Beihang University, Beijing, 100191, China

[4] College of Information and Communication Engineering
Harbin Engineering University, Harbin, 150001, China

[5] Key Laboratory of Microwave Remote Sensing
National Space Science Center, Chinese Academy of Sciences, Beijing, 100190, China

[6] Department of Electrical Engineering and Electronics
University of Liverpool, Liverpool, L69 3GJ, UK.

Abstract **─** With decades of development, the reverberation chamber (RC) has been proven to be a popular facility to determine antenna efficiency. One-, two- and three- antenna methods have been proposed to measure antenna efficiency without the need of a reference antenna. Due to the stochastic nature of RC-based measurements, the statistical analysis of the uncertainty is indispensable. Recently, the statistical uncertainty models for the one- and three-antenna methods were derived, however, the statistical model for the two-antenna method is still unknown to date. In this paper, the statistical uncertainty model of the two-antenna method is proposed. The approximated relative uncertainty is also given. The derived statistical uncertainty is verified by both simulations and measurements. It is experimentally verified that the statistical model can cope with hybrid stirring and assess the measurement uncertainty with and without frequency stirring in an efficient and convenient way.

Index Terms **─** Antenna efficiency, measurement uncertainty, reverberation chamber, two-antenna method, hybrid stirring.

I. INTRODUCTION

The reverberation chamber (RC) is an electrically large and highly conductive enclosure, which is initially proposed for electromagnetic compatibility (EMC) testing in 1968 [1]. The electromagnetic (EM) fields within the RC can be regarded as a superposition of resonant cavity modes with different weights [2]. Various stirring techniques (e.g., mechanical stirring, source stirring, and frequency stirring) [3]-[5] are usually adopted to stir (redistribute) the EM modes, resulting in different combinations of the weights. From the viewpoint of statistics, the EM field averaged over all stirring states can be considered as uniform, isotropic, and homogeneous. Owning to RC's particular properties (e.g., cost-effectiveness, good repeatability, and high efficiency), its application has extended from EMC testing to other areas, including over-the-air (OTA) testing (e.g., total radiated power, total isotropic sensitivity, throughput, and adjacent channel leakage power ratio) [6]-[9], antenna measurements (e.g., free-space *S*-parameter [2], [9], efficiency [1], [10]-[13], radiation pattern [14]-[16], and diversity gain [17]) , material characterization (e.g., absorption cross section, permittivity, and shielding effectiveness) [1], [2], etc. In this work, we focus on the antenna efficiency measurements.

In order to determine the antenna efficiency in a more convenient and accurate way, a few RC-based methods have been proposed. Most of these methods need a reference antenna with known efficiency [1]. A reference antenna covering the desired band of interest may not be always available in practice, thus, three non-reference antenna methods, i.e., one-, two-, and three-

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antenna methods, were proposed in [10] to overcome this problem. Knowing the enhanced backscattering effect in the RC [1], [10], meanwhile, utilizing the relationship between the quality factors in time-domain (Q_{TD}) and frequency-domain (Q_{FD}), the antenna efficiency can be determined without the need of a reference antenna. Due to the stochastic nature of RC-based measurements, the statistical analysis of measurement uncertainty is necessary and beneficial. The statistical uncertainty models of the standard reference antenna method and two non-reference antenna methods (i.e., one- and three-antenna methods) were presented in [18] and [19], respectively. However, the statistical uncertainty model of the two-antenna method is still unknown to date. It is worth stressing that these three non-reference antenna methods have different prerequisites and expressions, further different measurement uncertainties. Therefore, to achieve a rigorous uncertainty assessment, the statistical analysis must be performed independently for each non-reference antenna method.

In this paper, the statistical uncertainty model of the two-antenna method is proposed. The approximated relative uncertainty (when the number of independent samples is large) is also given. Simulations and measurements are performed to validate the proposed uncertainty model. Moreover, hybrid stirring (i.e., mechanical stirring and frequency stirring) is considered based on the proposed uncertainty model. Analytical and empirical uncertainties with and without frequency stirring are analyzed. Good agreements are observed.

The rest of this paper is organized as follows: Section II gives a brief introduction to the two-antenna method and exhibits the derivation of the statistical uncertainty. Simulations and measurements are conducted in Section III and IV, respectively. Hybrid stirring and comparisons of three non-reference antenna methods are also discussed in Section IV. Section V draws the conclusion.

II. THEORETICAL ANALYSIS

A common setup for antenna efficiency measurement using the two-antenna method is depicted in Fig. 1. Two antennas under test (i.e., Antenna 1 and Antenna 2) are both connected to the vector network analyzer (VNA). The mechanical stirrers are driven and controlled by the motor controller. At each stirring state, all the S-parameters are collected and stored by the VNA.

According to [10], once the measurement procedure is completed, the antenna efficiencies can be determined by:

$$\eta_1 = \sqrt{\frac{C_{RC}\langle |S_{11,s}|^2\rangle}{e_b Q_{\mathrm{TD}}}},\ \eta_2 = \sqrt{\frac{C_{RC}\langle |S_{22,s}|^2\rangle}{e_b Q_{\mathrm{TD}}}}\ , \quad (1)$$

where $C_{RC} = 16\pi^2 V/\lambda^3$, V is the inner volume of the RC and λ is the wavelength, $Q_{\mathrm{TD}} = \omega\tau_{\mathrm{RC}}$ with ω being the angular frequency and τ_{RC} being the chamber decay time. $S_{ii,s} = S_{ii} - \langle S_{ii}\rangle$ (i = 1, 2) denotes the stirred part of S_{ii}, and $\langle\bullet\rangle$ represents the ensemble average over all stirring states. e_b represents the enhanced backscatter coefficient, which can be calculated using [10]:

$$e_b = \frac{\sqrt{\langle |S_{11,s}|^2\rangle\langle |S_{22,s}|^2\rangle}}{\langle |S_{21,s}|^2\rangle}. \quad (2)$$

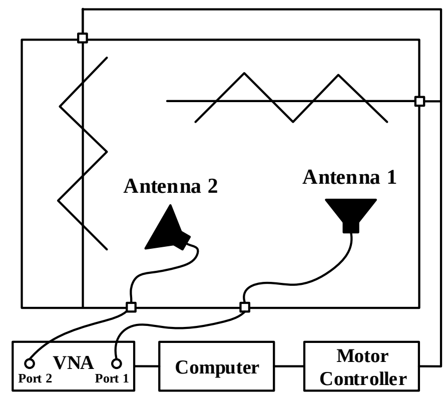


Fig. 1. Setup for antenna efficiency measurement using the two-antenna method.

Note that the measured antenna efficiencies of Antenna 1 and Antenna 2 have similar expressions, therefore, the statistical uncertainties should also be similar. For the sake of conciseness, we focus on η_1 here. To exhibit the analysis of the uncertainty clearly and intuitively, we denote $X = \langle |S_{11,s}|^2\rangle$ and $Y = e_b$. In this way, (1) can be rewritten as:

$$\eta_1 = \sqrt{\frac{C_{RC}}{Q_{\mathrm{TD}}}}\sqrt{\frac{X}{Y}}\ . \quad (3)$$

It should be stressed that X and Y are random variables (RVs) following the distributions below [1], [19], [20]:

$$f(x) = \frac{N^N}{x_0^N \Gamma(N-1)} x^{N-1}\exp(-Nx/x_0)\ , \quad (4)$$

$$f(y) = \frac{y_0^N}{y^{N+1}\Gamma(N)^3}\left[{}_2F_1(\frac{3N}{2},\frac{3N}{2};\frac{1}{2};\frac{y_0^2}{4y^2})\Gamma(\frac{3N}{2})^2 - {}_2F_1(\frac{3N+1}{2},\frac{3N+1}{2};\frac{3}{2};\frac{y_0^2}{4y^2})\frac{y_0}{y}\Gamma(\frac{3N+1}{2})^2 \right], \quad (5)$$

where $x_0 = \langle |S_{11,s}|^2\rangle = 2Q_{\mathrm{TD}}/C_{RC}$ and $y_0 = 2$ denote x and y in an ideal RC, respectively. ${}_pF_q(a_1,...,a_p;b_1,...,b_q;x)$ is the generalized hypergeometric function [21], Γ is the

Gamma function. N represents the number of independent mechanical stirring samples. The standard deviations (STDs) of X and Y can be calculated using [19], [20]:

$$\mathrm{STD}(x) = \frac{x_0}{\sqrt{N}}, \tag{6}$$

$$\mathrm{STD}(y) = \frac{y_0}{N-1}\sqrt{N^2\frac{N-1}{N-2} - \frac{\Gamma(N+1/2)^4}{\Gamma(N)^4}}. \tag{7}$$

According to the law of propagation of uncertainty [22], the uncertainty of η_1, i.e., $u(\eta_1)$, can be estimated as:

$$u(\eta_1)=\sqrt{\left(\frac{\partial\eta_1}{\partial x}u(x)\right)^2 + \left(\frac{\partial\eta_1}{\partial y}u(y)\right)^2}. \tag{8}$$

Substituting (3), (6), and (7) into (8), we have:

$$\frac{u(\eta_1)}{\eta_1} = \sqrt{\frac{1}{4N} + \frac{1}{4(N-1)^2}\left(N^2\frac{N-1}{N-2} - \frac{\Gamma(N+1/2)^4}{\Gamma(N)^4}\right)}. \tag{9}$$

When N is large, we have:

$$\lim_{N\to\infty}\frac{\Gamma(N+t)}{\Gamma(N)N^t} = 1. \tag{10}$$

Further, Eq. (9) can be approximated as:

$$\lim_{N\to\infty} u_{\mathrm{rel}}(\eta_1) = \lim_{N\to\infty}\frac{u(\eta_1)}{\eta_1} = \frac{1}{\sqrt{2N}}. \tag{11}$$

It should be stressed that Eq. (9) is the analytical expression of the relative uncertainty, which is applicable for any value of N. By comparison, Eq. (11) is an approximation of the relative uncertainty, which is more concise yet only suitable for the cases of large N. As will be shown and discussed in Section III, the gap between Eqs. (9) and (11) diminishes rapidly as N increases, and becomes negligible when N is large.

III. SIMULATIONS

In order to verify the derived uncertainty model, Monte Carlo simulations are employed. For simplicity and without loss of generality, the true antenna efficiency is assumed to be 100% ($\eta_1 = 100\%$) in this section. For each number of independent samples N and each random variable, we randomly generated 1000 × N samples following the exponential distributions with different distribution parameters, i.e., $|S_{11,s}|^2, |S_{22,s}|^2 \sim Exp(C_{RC}/2Q)$ and $|S_{21,s}|^2 \sim Exp(C_{RC}/Q)$. Note that $Y = e_b$ can be obtained based on the generated samples by utilizing Eq. (2).

Figure 2 shows the simulated, analytical, and approximated relative STDs as functions of N. The STD from the Lindeberg-Levy central limit theory ($1/\sqrt{N}$) is also presented in the figure for comparison. The analytical relative STD does not exist when N is equal to 1 and 2. For a unified comparison of different STDs, all the STDs are plotted when $N \geq 3$.

It can be seen from Fig. 2 that both analytical and approximated relative STDs are in accordance with the simulated ones. It should be noted that the approximated relative STD is obtained under the assumption of large N. Surprisingly, the approximated relative STD outperforms the analytical one even when N is small. As N grows larger, the gaps between the simulated, analytical and approximated relative STDs decrease rapidly. Moreover, these three relative STDs become indistinguishable as N further increases. Therefore, the approximated one is preferred in practice for its concision. The same as the analytical and approximated relative STDs, $1/\sqrt{N}$ also provides an overestimation. However, discrepancies can still be observed when $N = 1000$. This indicates the factor of $1/\sqrt{2}$ cannot be omitted for a rigorous STD assessment of measured efficiencies using the two-antenna method.

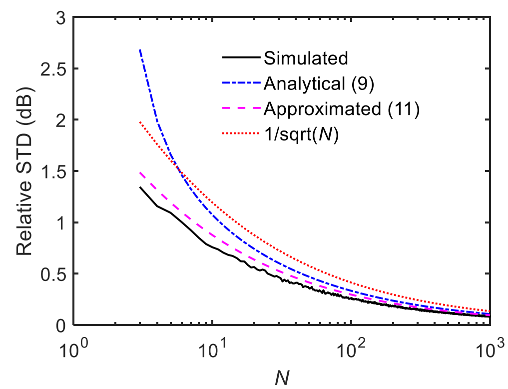


Fig. 2. Simulated, analytical and approximated STDs as functions of N. The curve of $1/\sqrt{N}$ is also presented as a reference.

IV. MEASUREMENTS AND RESULTS

Extensive measurements are also performed to verify the derived statistical model in an RC (as shown in Fig. 3). The inner size of the used RC is 1.50 × 1.44 × 0.92 m^3. The RC contains two mechanical stirrers (one vertical and one horizontal) and one platform. Two double-ridged horn antennas (Antennas 1 and 2) are adopted as antennas under test. Antenna 1 is mounted on a trestle with adjustable height and orientation, while Antenna 2 is located in the corner of the RC and oriented to the horizontal stirrer.

Key parameters for assessment of the relative uncertainty of antenna efficiency measured using the two-antenna method are shown in Table 1. Considering that the lowest usable frequency (LUF) of the RC is about 0.87 GHz and the operating frequency range of the two horn antennas is 0.8 GHz - 4.6 GHz, in order to have a better field uniformity and ensure the antennas perform well, the testing frequency range is selected as 2 GHz - 3 GHz (which is the center operating frequency range of the antennas). Another frequency range can be used. Yet the main purpose to experimentally validate the uncertainty model, while the specific frequency range is less important. The platform rotates with 36° per step. At each platform rotation state, two mechanical stirrers rotate with 72° per step simultaneously. It has been verified that these 50 samples are independent to each other [19]. At each stirring state, all the *S*-parameters are sampled and stored by the VNA with a frequency interval of 1 MHz. In order to characterize the statistics of the measured antenna efficiencies, the nine-case assessment method [23]-[24] is adopted here. Specifically, the trestle used to support Antenna 1 is adjusted to three heights, and at each height Antenna 1 is oriented to three directions. To ensure the independence of these nine measurements, the distance between any two adjacent heights is set to 15 cm (which is larger than half-wavelength at the lowest testing frequency) and the selected three orientations are orthogonal to each other.

Once the whole measurement procedure is completed, nine sets of antenna efficiencies can be determined using Eq. (1), and the empirical relative uncertainty can be further calculated. Note that $N = 50$ is known. Therefore, the analytical and approximated relative uncertainties can be directly calculated using Eqs. (9) and (11), respectively.

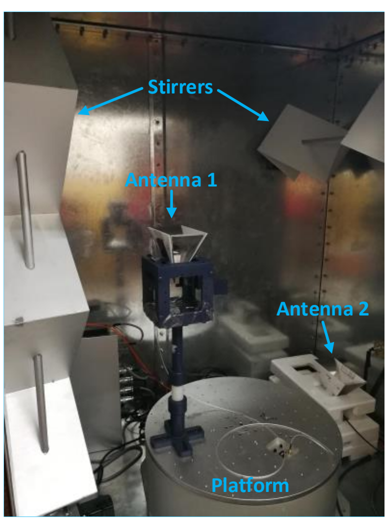


Fig. 3. Photograph of the experimental RC setup for antenna efficiency measurement using the two-antenna method.

Table 1: Key parameters for assessment of the relative uncertainty of antenna efficiency measured using the two-antenna method

Parameter		Value
Frequency range		2 GHz – 3 GHz
Number of frequency points		1001
IF bandwidth	Calibration	100 Hz
	Measurements	1 kHz
VNA output power level		-10 dBm
Stirrer and platform step size ($S \times P$)		72° × 36°
Number of stirring positions		5 × 10
Number of locations of Antenna 1		9

A. Analysis of measurement uncertainty

The empirical, analytical and approximated relative STDs are shown in Fig. 4. Noting that $N = 50$, the analytical and approximated relative STDs are very close to each other. Good agreements between the empirical, analytical and approximated relative STDs can be observed; and the approximated relative STD seems to provide a better estimation. By comparison, there exist obvious discrepancies between $1/\sqrt{N}$ and the empirical STD, which is in accordance with the theoretical analysis.

It should be stressed that both the analytical and approximated relative STDs provide good estimation of the relative STD. Therefore, it is more convenient and efficient to use the approximated STDs in practice. Moreover, the approximated STDs provide a better estimation regardless of *N*.

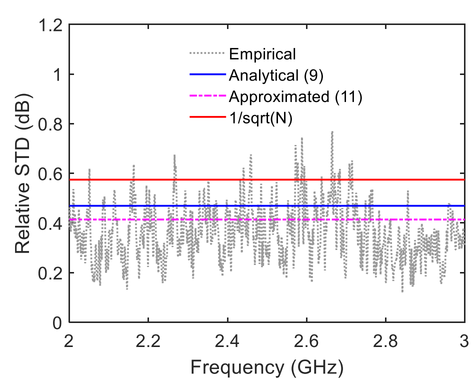


Fig. 4. Empirical, analytical and approximated relative STDs when $N = 50$. The curve of $1/\sqrt{N}$ is also presented as a reference.

B. Hybrid stirring

Multiple stirring techniques are commonly adopted to achieve a better uniformity of the EM field within the RC. Therefore, hybrid stirring is considered here to increase the usability of the statistical uncertainty model.

The coherence bandwidth (B_C) is defined as the frequency shift (Δf) when the magnitude of the complex autocorrelation function ($|\rho(\Delta f)|$) drops from 1 to 0.5 [23]. The complex autocorrelation function can be calculated using:

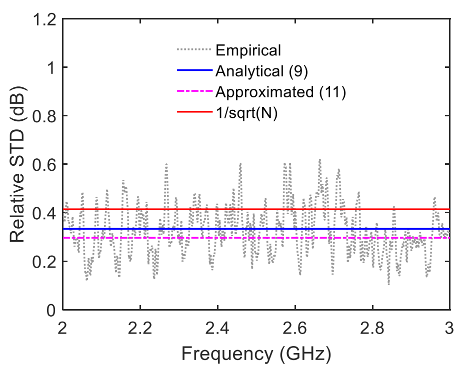
$$\rho_f(\Delta f) = \frac{\langle S_{21}^*(f)S_{21}(f+\Delta f)\rangle_N}{\langle |S_{21}(f)|^2\rangle_N}, \tag{12}$$

where * denotes the conjugation operator.

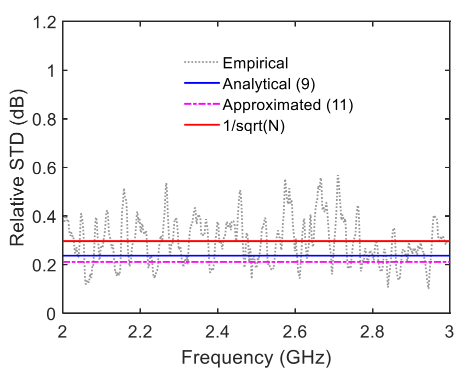
It is known that measurements performed at frequencies separated by more than one coherence bandwidth can be treated as independent. Thus, the number of independent frequency samples can be determined by [23]:

$$N_F = \frac{\mathrm{BW}}{\langle B_C\rangle}, \tag{13}$$

where BW is the bandwidth of frequency stirring, $\langle B_C\rangle$ denotes the coherence bandwidth averaged over BW. We have calculated $|\rho_f(\Delta f)|$ over the whole testing bandwidth, and found $\langle B_C\rangle \approx 2.5$ MHz.



(a)



(b)

Fig. 5. Empirical and analytical relative STDs with: (a) 5 MHz and (b) 10 MHz frequency stirring when $N = 50$.

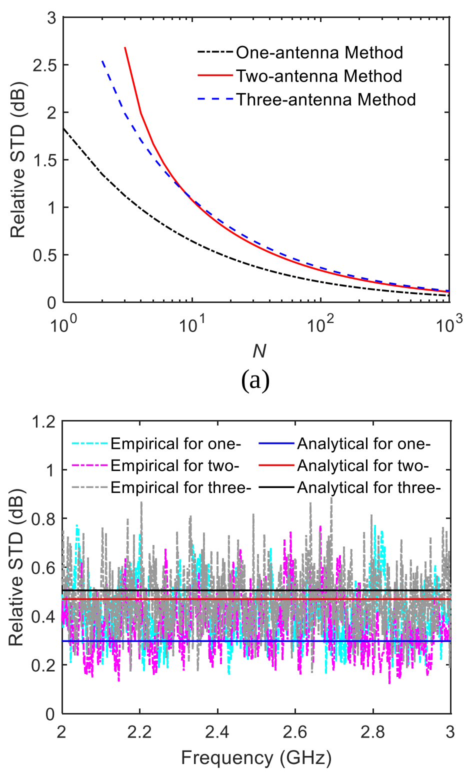
Taking N_F into consideration, the number of independent samples should be $N \times N_F$ (where N represents the number of independent mechanical stirring samples). In other words, the parameter N in the statistical uncertainty model should be modified as $N \times N_F$ when frequency stirring is utilized.

For hybrid stirring, frequency stirring with two different bandwidths, i.e., a 5 MHz frequency stirring ($N_F = 2$) and a 10 MHz frequency stirring ($N_F = 4$), are considered. Corresponding empirical and analytical relative STDs are shown in Fig. 5. As expected, all STDs decrease when the frequency stirring bandwidth increases from 5 MHz to 10 MHz. It can be seen that the analytical relative STDs are in accordance with the empirical ones for both frequency stirring situations. Moreover, in accordance with previous findings, $1/\sqrt{N}$ possesses relatively large estimation error. Nevertheless, the statistical model is applicable to not only single mechanical stirring, but also hybrid stirring.

C. Comparisons of three non-reference antenna methods

In order to exhibit a comprehensive and intuitively comparisons of three non-reference antenna methods, more simulations and measurements are performed. For the one- and three-antenna methods, the corresponding relative uncertainties can be easily derived based on the statistical models [19]. Figure 6 (a) shows the analytical relative STDs of three non-reference antenna methods when $N \in [1, 10^3]$. It can be seen that the one-antenna method possesses the lowest uncertainty regardless of N, while the other two methods provide comparable performance. Intuitively speaking, this is easy to understand. Suppose that we are measuring η_1 and all the prerequisites of corresponding methods are satisfied, the analytical uncertainty comes from: S_{11} for one-antenna method, S_{11}, S_{22} and S_{21} for two-antenna method, S_{21}, S_{21} and S_{32} for three-antenna method. Obviously, the one-antenna method is associated with only one random variable, while two- and three-antenna methods are associated with three random variables. However, it is worth stressing that the uncertainty may be different in practical measurements.

Figure 6 (b) shows η_1 measured using three non-reference antenna methods when $N = 50$, corresponding analytical values are also plotted for a convenient comparison. As can be seen, the empirical relative STDs of one- and three-antenna methods show good agreements with the corresponding analytical ones. Meanwhile, the empirical relative STD of two-antenna method is comparable with that of three-antenna method, which is in accordance with analytical analysis. However, the empirical STD of one-antenna method is far larger than the analytical one. This is mainly due to the inconsistence between the measured and assumed e_b [10], [19]. For two-antenna method, the uniformity of e_b may not be well in accordance with assumption, however, the uncertainty of e_b is already considered in the theoretical analysis. As a result, the empirical and analytical STDs show good agreement. Nevertheless, the uncertainty models of two- and three-antenna methods seem to be more stable than that of one-antenna method.



(a)

(b)

Fig. 6. Comparisons of relative STDs of one-, two-, and three-antenna methods: (a) Analytical relative STDs when $N \in [1, 10^3]$, and (b) analytical and empirical STDs when $N = 50$.

V. CONCLUSIONS

In this paper, the statistical uncertainty model of the two-antenna method was proposed. Both Monte Carlo simulations and RC measurements were performed to verify the proposed statistical model. Good agreements between the analytical and empirical relative uncertainties were observed. Hybrid stirring was also investigated. Corresponding measurement results were in accordance with the theoretical analysis, indicating that the statistical model could also be applicable to analysis of measurement uncertainty when frequency stirring was adopted. Moreover, it was theoretically and experimentally verified that the approximated relative uncertainty was $1/\sqrt{2N}$ (instead of $1/\sqrt{N}$). In practical measurements, the approximated relative uncertainty is an efficient and convenient way to assess the measurement uncertainty.

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