# RCS of Two Lossy Dielectric Loaded Semi-Elliptic Channels in a Conducting Plane 

A-K. Hamid<br>Department of Electrical and Computer Eng.<br>University of Sharjah<br>P.O. Box 27272, Sharjah, U.A.E<br>email: akhamid@Sharjah.ac.ae


#### Abstract

Analytical solution to the problem of scattering of a plane electromagnetic wave by two lossy dielectric loaded semielliptic channels in a conducting plane is investigated using an iterative procedure to account for the interaction fields between the channels. The incident, scattered and transmitted fields in every region are expressed in terms of complex Mathieu functions. The translation addition theorem is used to compute the higher order scattered fields. Numerical results are presented for the far scattered field for different axial ratios, electrical separation distances, angles of incidence and loss of dielectric materials.


## 1. Introduction

The electromagnetic scattering from grooves, channels and cracks has many practical applications. The solution may be used to study the scattering by rough surfaces, nondestructive testing of materials, and to check the numerical accuracy of approximate and numerical methods of similar geometries.

Lately, there have been many analytical studies available in the literature on the scattering by hollow and dielectric loaded semi-circular channels [1-5]. Most of these studies are based on the exact dual-series eigenfunction solution. On the other hand, some numerical solutions based on the coupled integral equations for the induced currents were obtained by Senior et. al. [6-7].
Up to date, the analytical solutions available in the literature are for the case of scattering by single semi-elliptic channels loaded by a lossy or lossless dielectric material in a conducting ground plane [8-10]. In this paper, we extend the solution of scattering by a single lossy dielectric loaded semi-elliptic channel in a ground plane to the case of scattering by two adjacent lossy dielectric loaded semielliptic channels in a conducting ground plane.

## 2. Formulation of the scattering problem

Consider the case of a linearly polarized electromagnetic TM plane wave incident on a two lossy dielectric loaded semi-elliptic channels in a conducting ground plane at an angle $\phi_{i}$ with respect to the $x$ axis, as shown in Figure 1. The major axes of the channels are denoted by $a_{1}$ and $a_{2}$ while the minor axes are denoted by $b_{1}$ and $b_{2}$. The ground plane is assumed to be perfectly conducting. The time dependence $e^{j o t}$ is assumed and omitted throughout. The electric field component of the TM polarized plane wave of amplitude $E_{0}$ is given by

$$
\begin{equation*}
E_{z}^{i}=E_{o} e^{j k \rho \cos \left(\phi-\phi_{i}\right)} \tag{1}
\end{equation*}
$$

where $k$ is the wave number in free space. The incident electric field may be expressed in terms of Mathieu functions around the origins $\mathrm{o}_{1}$ and $\mathrm{o}_{2}$ as follows

$$
\begin{align*}
& E_{1 z}^{i}=\sum_{m=0}^{\infty} A_{1 e m} R_{e m}^{(1)}\left(c_{1}, \xi_{1}\right) S_{e m}\left(c_{1}, \eta_{1}\right)  \tag{2}\\
& +\sum_{m=1}^{\infty} A_{1 o m} R_{o m}^{(1)}\left(c_{1}, \xi_{1}\right) S_{o m}\left(c_{1}, \eta_{1}\right) \\
& E_{2 z}^{i}=\sum_{m=0}^{\infty} A_{2 e m} R_{e m}^{(1)}\left(c_{2}, \xi_{2}\right) S_{e m}\left(c_{2}, \eta_{2}\right)  \tag{3}\\
& +\sum_{m=1}^{\infty} A_{2 o m} R_{o m}^{(1)}\left(c_{2}, \xi_{2}\right) S_{o m}\left(c_{2}, \eta_{2}\right)
\end{align*}
$$

where $\eta_{1}$ and $\eta_{2}$ are the intrinsic impedance of ellipse 1 and 2 , respectively, and

$$
\begin{align*}
& A_{\substack{\text { lem } \\
o m}}=E_{o} j^{m} \frac{\sqrt{8 \pi}}{N_{\substack{e m \\
o m}}\left(c_{1}\right)} S_{\substack{e m \\
o m}}\left(c_{1}, \cos \phi_{i}\right)  \tag{4}\\
& A_{2 e m}=E_{o} j^{m} \frac{\sqrt{8 \pi}}{N_{e m}\left(c_{2}\right)} S_{e m}\left(c_{2}, \cos \phi_{i}\right) e^{-j k d \cos \left(\phi_{i}\right)}  \tag{5}\\
& o m \\
& o m
\end{align*}
$$

$$
\begin{align*}
& N_{e m}\left(c_{1}\right)=\int_{0 m}^{2 \pi}\left[S_{e m}\left(c_{1}, \eta_{1}\right)\right]^{2} d v  \tag{6}\\
& N_{o m}  \tag{7}\\
& N_{e m}\left(c_{2}\right)=\int_{0}^{2 \pi}\left[S_{o m}\left(c_{2}, \eta_{2}\right)\right]^{2} d v
\end{align*}
$$

and $c_{1}=k F_{1}, c_{2}=k F_{2}, F_{1}$ and $F_{2}$ are the semi-focal length of channels one and two, $S_{e m}$ and $S_{o m}$ are the even and odd angular Mathieu functions of order $m$, respectively, $R_{e m}^{(1)}$ and $R_{o m}^{(1)}$ are the even and odd radial Mathieu functions of the first kind, $N_{e m}$ and $N_{o m}$ are the even and odd normalized functions [11], and $d$ is the separation distance between the centers of the two channels. The scattered electric fields outside the two semi-elliptic channels are decomposed to two parts: reflected and diffracted fields. These fields should only be written in terms of odd Mathieu functions since the incident and scattered fields should vanish at the conducting plane, i.e. at $\eta=0$ and $\eta=\pi$. This leads to
$E_{1 z}^{r e f}=-\sum_{m=1}^{\infty} A_{\text {lom }} R_{o m}^{(1)}\left(c_{1}, \xi_{1}\right) S_{o m}\left(c_{1}, \eta_{1}\right)$, with
$\phi_{i=2 \pi-\phi_{i}} \quad$ in equations (4)
$E_{2 z}^{r e f}=-\sum_{m=1}^{\infty} A_{2 o m} R_{o m}^{(1)}\left(c_{2}, \xi_{2}\right) S_{o m}\left(c_{2}, \eta_{2}\right)$, with
$\phi_{i=2 \pi-\phi_{i}} \quad$ in equation (5)
$E_{1 z}^{\text {diff }}=\sum_{m=1}^{\infty} B_{1 o m} R_{o m}^{(4)}\left(c_{1}, \xi_{1}\right) S_{o m}\left(c_{1}, \eta_{1}\right)$
$E_{2 z}^{\text {diff }}=\sum_{m=1}^{\infty} B_{2 o m} R_{o m}^{(4)}\left(c_{2}, \xi_{2}\right) S_{o m}\left(c_{2}, \eta_{2}\right)$
where $B_{1 o m}$ and $B_{2 o m}$ are the unknown odd scattered field expansion coefficients and $R_{o m}^{(4)}$ is the odd radial Mathieu function of the fourth kind. The transmitted electric fields inside the two semi-elliptic channels can also be written in terms of Mathieu functions as

$$
\begin{gather*}
E_{1 z}^{t}=\sum_{m=0}^{\infty} C_{1 e m} R_{e m}^{(1)}\left(c_{11}, \xi_{1}\right) S_{e m}\left(c_{11}, \eta_{1}\right)  \tag{12}\\
+\sum_{m=1}^{\infty} C_{1 o m} R_{o m}^{(1)}\left(c_{11}, \xi_{1}\right) S_{o m}\left(c_{11}, \eta_{1}\right)
\end{gather*}
$$

$$
\begin{align*}
& E_{2 z}^{t}=\sum_{m=0}^{\infty} C_{2 e m} R_{e m}^{(1)}\left(c_{22}, \xi_{2}\right) S_{e m}\left(c_{22}, \eta_{2}\right)  \tag{13}\\
& +\sum_{m=1}^{\infty} C_{2 o m} R_{o m}^{(1)}\left(c_{22}, \xi_{2}\right) S_{o m}\left(c_{22}, \eta_{2}\right)
\end{align*}
$$

where $\quad c_{11}=k_{1} F_{1}, \quad c_{22}=k_{2} F_{2}, \quad k_{1}=\omega \sqrt{\mu_{1} \varepsilon_{1}}$, $k_{2}=\omega \sqrt{\mu_{2} \varepsilon_{2}}, \varepsilon_{1}=\varepsilon_{1}^{\prime}-j \varepsilon_{1}^{\prime \prime}, \varepsilon 2=\varepsilon_{2}^{\prime}-j \varepsilon_{2}^{\prime}$ while $C_{\text {em }}$, $C_{2 e m}$ and $C_{1 o m}, C_{2 o m}$ are the even and odd unknown transmitted field expansion coefficients. The magnetic fields inside and outside the two loaded semi-elliptic channels can be obtained using Maxwell's equations.

## 3. First order scattered fields

The first order scattered fields result from the separate excitation of each semi-channel by the incident plane wave alone. The first order field expansion coefficients can be determined using the boundary conditions which require the total tangential electric field component inside the channels to vanish at the conducting parts, i.e. at $\xi=\xi_{1}, \xi=\xi_{2}$ and $\pi<\eta<2 \pi$, while the total tangential electric and magnetic field components to be continuous across the imaginary apertures at $\xi=\xi_{1}, \xi=\xi_{2}$ and $0<\eta<\pi$. Using the partial orthogonality properties of the angular Mathieu functions, the first order odd scattered and even transmitted field coefficients can be written in matrix form as follows [10]

$$
\begin{align*}
& {\left[\begin{array}{ll}
Q_{1} & Q_{2} \\
Q_{3} & Q_{4}
\end{array}\right]\left[\begin{array}{l}
C_{1 e}^{1} \\
B_{1 o}^{1}
\end{array}\right]=\left[\begin{array}{l}
V_{1} \\
V_{2}
\end{array}\right]}  \tag{9}\\
& {\left[\begin{array}{ll}
Q_{5} & Q_{6} \\
Q_{7} & Q_{8}
\end{array}\right]\left[\begin{array}{l}
C_{2 e}^{1} \\
B_{2 o}^{1}
\end{array}\right]=\left[\begin{array}{l}
V_{3} \\
V_{4}
\end{array}\right]} \tag{10}
\end{align*}
$$

where

$$
\begin{align*}
& Q_{1}=R_{e m}^{(1)}\left(c_{11}, \xi_{1}\right) L_{1 m n}-R_{e m}^{(1)^{\prime}}\left(c_{11}, \xi_{1}\right) \Delta_{1 n} F_{1 m n}  \tag{16}\\
& Q_{2}=R_{o m}^{(4)^{\prime}}\left(c_{1}, \xi_{1}\right) M_{1 m n} \Delta_{1 n}  \tag{17}\\
& Q_{3}=\left\{R_{e m}^{(1)}\left(c_{11}, \xi_{1}\right)-R_{e m}^{(1)^{\prime}}\left(c_{11}, \xi_{1}\right) \Delta_{1 n}\right\} F_{1 m n} \\
& Q_{4}=-\left\{R_{O m}^{(4)}\left(c_{1}, \xi_{1}\right)-R_{o m}^{(4)^{\prime}}\left(c_{1}, \xi_{1}\right) \Delta_{1 n}\right\} M_{1 m n}  \tag{19}\\
& V_{1}=-\sum_{m=1}^{\infty} 2 A_{1 o m} R_{o m}^{(1)^{\prime}}\left(c_{1}, \xi_{1}\right) M_{1 m n} \Delta_{1 n} \tag{20}
\end{align*}
$$

$$
\begin{align*}
& V_{2}=\sum_{m=1}^{\infty} 2\left\{R_{o m}^{(1)}\left(c_{1}, \xi_{1}\right)-R_{o m}^{(1)^{\prime}}\left(c_{1}, \xi_{1}\right) \Delta_{1 n}\right\} A_{1 o m} M_{1 m n}  \tag{21}\\
& M_{1 m n}=\int_{0}^{\pi} S_{o m}\left(c_{11}, \eta_{1}\right) S_{o n}\left(c_{1}, \eta_{1}\right) d v  \tag{22}\\
& F_{1 m n}=\int_{0}^{\pi} S_{e m}\left(c_{11}, \eta_{1}\right) S_{o n}\left(c_{11}, \eta_{1}\right) d v  \tag{23}\\
& L_{1 m n}=\int_{\pi}^{2 \pi} S_{e m}\left(c_{11}, \eta\right) S_{o n}\left(c_{11}, \eta_{1}\right) d v=-F_{1 m n}  \tag{24}\\
& \Delta_{1 n}=\frac{R_{o n}^{(1)}\left(c_{11}, \xi_{1}\right)}{R_{o n}^{(1)}\left(c_{11}, \xi_{1}\right)} \tag{25}
\end{align*}
$$

and $B_{1 o}^{1}, B_{2 o}^{1}$, and $C_{1 e}^{1}, C_{2 e}^{1}$ are the first order odd scattered and even transmitted filed vector matrices. Matrices $\mathrm{Q}_{5}, \mathrm{Q}_{6}, \mathrm{Q}_{7}, \mathrm{Q}_{8}, \mathrm{~V}_{3}$ and $\mathrm{V}_{4}$ correspond to the second semi-elliptic channel can be written similarly. Equations (14) and (15) may be solved by matrix inversion to obtain the first order scattered field coefficients for given electrical size of semi-elliptic channels, electrical separation, angle of incidence, and lossy dielectric material.

## 4. Higher order scattered fields

The second order scattered field results from the excitation of each semi-elliptic channel by the scattered field from the other semi-elliptic channel due to the initial incident field. To enforce the boundary conditions, the first order scattered field from the second semi-elliptic channel must be expressed in terms of the coordinate system of the first semi-elliptic channel and vice versa using the addition theorem of Mathieu functions [12], i.e.,
$R_{\text {om }}^{(4)}\left(c_{2}, \xi_{2}\right) S_{o m}\left(c_{2}, \eta_{2}\right)=$
$\sum_{l=1}^{\infty} W O_{o l m}^{2 \rightarrow 1} R_{o l}^{(1)}\left(c_{1}, \xi_{1}\right) S_{o l}\left(c_{1}, \eta_{1}\right)$
where $W O_{o l m}^{2 \rightarrow 1}$ is given by [12]. Again, the boundary conditions require the tangential electric field component inside the channels to vanish at $\xi=\xi_{1}, \quad \xi=\xi_{2}$ and $\pi<\eta<2 \pi$, while the total tangential electric and magnetic fields components to be continuous across the imaginary apertures at $\xi=\xi_{1}, \quad \xi=\xi_{2}$ and $0<\eta<\pi$. Using the partial orthogonality properties of the angular Mathieu functions along with equation (26), we obtain the second order scattered field coefficients for semi-elliptic channel one in matrix form as

$$
\left[\begin{array}{ll}
P_{1} & P_{2}  \tag{27}\\
P_{3} & P_{4}
\end{array}\right]\left[\begin{array}{c}
C_{1 e}^{2} \\
B_{1 o}^{2}
\end{array}\right]=\left[\begin{array}{cc}
G_{1} & 0 \\
0 & G_{2}
\end{array}\right]\left[\begin{array}{c}
B_{2 o}^{1} \\
B_{2 o}^{1}
\end{array}\right]
$$

where
$P_{1}=-2 R_{e m}^{(1)}\left(c_{11}, \xi_{1}\right) F_{1 m n}$
$P_{2}=R_{o m}^{(4)}\left(c_{1}, \xi_{1}\right) M_{1 m n}$
$P_{3}=-2\left[R_{\text {em }}^{(1)^{\prime}}\left(c_{\left.11, \xi_{1}\right)+R_{e m}^{(1)}}^{\left.\left(c_{11}, \xi_{1}\right)\left(1 / \Delta_{n}\right)\right]} F_{1 m n}\right.\right.$
$P_{4}=R_{o m}^{(4)^{\prime}}\left(c_{1}, \xi_{1}\right) M_{1 m n}$
$G_{1}=-\sum_{l=1}^{\infty} R_{o l}^{(1)}\left(c_{1}, \xi 1\right) W O_{o l m}^{2 \rightarrow 1} M_{1 l n}$
$G_{2}=-\sum_{l=1}^{\infty} R_{o l}^{(1)^{\prime}}\left(c_{1}, \xi_{1}\right) W O_{o l m}^{2 \rightarrow 1} M_{1 l n}$
$B_{2 o}^{1}=\left\lfloor B_{2 o m}^{1}\right\rfloor$
and $B_{1 o}^{2}$ and $C_{1 e}^{2}$ are the second order scattered and transmitted field vector matrices for channel one. The vector field matrices $C_{2 e}^{2}, B_{2 o}^{2}$ that correspond to the second channel can be obtained similarly.

To obtain a general solution, we solve similarly for the higher order scattered fields, which are sensitive to the electrical sizes and separation distances, angles of incidence and dielectric materials. The general expression for the $k^{t h}$ order scattered field coefficients of channel one may be written as
$\left[\begin{array}{ll}P_{1} & P_{2} \\ P_{3} & P_{4}\end{array}\right]\left[\begin{array}{c}C_{1 e}^{k} \\ B_{1 o}^{k}\end{array}\right]=\left[\begin{array}{cc}G_{1} & 0 \\ 0 & G_{2}\end{array}\right]\left[\begin{array}{c}B_{2 o}^{k-1} \\ B_{2 o}^{k-1}\end{array}\right]$.
It should be noted that the matrices in equation (35) are computed once (i.e., $k=2$ ) for the electrical size, dielectric material, and electrical separation considered and used for the subsequent iterations (i.e., $k=3,4, \ldots$ ).

Once the scattered field coefficients are determined, the total far field from the two semi-elliptic channels due to the $k^{t h}$ order scattered fields can be determined.

## 5. Numerical Results

The scattered near and far fields can be calculated once the scattered field expansion coefficients are computed. The scattered far field expression may be written as follows
$E_{z}^{S}=\left(\frac{j}{k \rho}\right)^{0.5} e^{-j k \rho_{P(\phi)}}$
where

$$
\begin{align*}
& P(\phi)=\sum_{k=1,2, . .}\left\{\sum_{m=1}^{\infty} j^{m} B_{1 o m}^{k} S_{o m}\left(c_{1}, \cos \phi\right)\right.  \tag{37}\\
& \left.+e^{-j k d \cos (\phi)} \sum_{m=1}^{\infty} j^{m} B_{2 o m}^{k} S_{o m}\left(c_{2}, \cos \phi\right)\right\}
\end{align*}
$$

In order to solve for the unknown scattered field coefficients, the infinite series are first truncated to include only the first $N$ terms, where $N$ in general is a suitable truncation number proportional to the channel electrical sizes, separation distances and the dielectric loading materials. In the computation, the value of $N$ has been chosen to impose a convergence condition that provides solution accuracy with at least four significant figures [14]. It is found that increasing the electrical sizes of the channel will increase the total truncation number of $N$ terms. Also, to set a criterion for terminating the iteration process, the scattered field after each iteration is calculated and divided by the total field scattered from the pervious iterations, and the process is terminated when the ratio is smaller than $10^{-4}$.

The accuracy of the numerical results is checked against the special case of two semi-circular channels loaded with a lossless dielectric material [13]. Fig. 2 shows the normalized backscattered field versus the electrical size $\mathrm{ka}_{1,2}$ for two identical loaded semi-circular channels with an incident angle $\phi_{i}=90^{\circ}$, axial ratio $\mathrm{a}_{1,2} / \mathrm{b}_{1,2}=1.0$ and $\mathrm{d}=8.0$ $a_{1,2}$. The electrical sizes are taken from 0.5 to 5.0 . The solid line represents the solution of [13], which is in excellent agreement with our calculation represented by circles. Also, the dashed line represents the lossy dielectric circular channels from which we can see that the resonances start to disappear [14]. Figure 3 is similar to Fig. 2 except for semielliptic channels with axial ratio $a_{1,2} / b_{1,2}=2.0$. It can be seen that the location of the resonances has been changed when it is compared with the circular channels. It is worth mentioning that the numerical results given by [13] were only for loaded channels and no hollow cases. Figure 4 is similar to Fig. 3 except for the incident angle changed to 45 degrees. It can be seen that the number of resonances is increased significantly when the incident angle is changed from 90 to 45 degrees.

Fig. 5 shows the normalized backscattered field versus the electrical separation distance kd for two dielectric loaded
identical semi-elliptic channels with $\mathrm{ka}_{1,2}=2.0, \mathrm{a}_{1,2} / \mathrm{b}_{1,2}=1.5$ and $\phi_{i}=60^{\circ}$. The electrical separation is taken from 5.0 to 16.5. Fig. 6 shows the echo pattern width versus the scattering angle $\phi$ for two dielectric loaded identical
channels with $\mathrm{ka}_{1,2}=1.5, \mathrm{a}_{1,2} / \mathrm{b}_{1,2}=2.0, \mathrm{kd}=5.0$ and $\phi_{i}=90^{\circ}$.
Fig. 7 is similar to 6 except for $\phi_{i}=60^{\circ}$. Figure 8 shows the normalized backscattered field versus the incident angle $\phi_{i}$ for two dielectric loaded two channels with $\mathrm{ka}_{1,2}=2.0$, $\mathrm{a}_{1,2} / \mathrm{b}_{1,2}=1.5$ and $\mathrm{kd}=5.0$.

## 6. Conclusions

Analytical solution and numerical results of the electromagnetic scattering by a two lossy dielectric loaded semi-elliptic channels in a ground plane is obtained for the case of TM (transverse magnetic) polarization. The validity and accuracy of the obtained numerical results were verified against the special case of two lossless semicircular channels. It is worth mentioning that the number of higher order scattered fields used in the computation of numerical results was ranged from $\mathrm{k}=2$ to 4 . The agreement was excellent in all cases. It was shown that the presence of lossy and lossless dielectric materials in the channels has significantly changed the scattered field patterns when it was compared with the hollow case. The present work will be extended to the case of an infinite array of semi-elliptic channels in a ground plane since this would be useful for the study of scattering by rough surface.

## ACKNOWLEDGEMENT

The author wishes to acknowledge the support provided by the University of Sharjah, U.A.E.


Figure 1. Scattering geometry of two lossy dielectric loaded semi-elliptic channels in a ground plane.


Figure 2. Backscattered field versus electrical size $\mathrm{ka}_{1,2}$ for two dielectric loaded identical semi-circular channels with $\mathrm{a}_{1,2} / \mathrm{b}_{1,2}=1.0$ and $\phi_{i}=90^{\circ}$.


Figure 3. Backscattered field versus electrical size $\mathrm{ka}_{1,2}$ for two dielectric loaded identical semi-elliptic channels with $\mathrm{a}_{1,2} / \mathrm{b}_{1,2}=2.0$ and $\phi_{i}=90^{\circ}$.


Figure 4. Backscattered field versus electrical size $\mathrm{ka}_{1,2}$ for two dielectric loaded identical semi-elliptic channels with $\mathrm{a}_{1,2} / \mathrm{b}_{1,2}=2.0$ and $\phi_{i}=45^{\circ}$.


Figure 5. Backscattered field versus electrical separation distance kd for two dielectric loaded identical semi-elliptic channels with $\mathrm{ka}_{1,2}=2.0, \mathrm{a}_{1,2} / \mathrm{b}_{1,2}=1.5$ and $\phi_{i}=60^{\circ}$.


Figure 6. Echo pattern width versus the scattering angle $\phi$ for two dielectric loaded identical semi-elliptic channels with $\mathrm{ka}_{1,2}=1.5, \mathrm{a}_{1,2} / \mathrm{b}_{1,2}=2.0, \mathrm{kd}=5.0$ and $\phi_{i}=90^{\circ}$.


Figure 7. Echo pattern width versus the scattering angle $\phi$ for two dielectric loaded identical semi-elliptic channels with $\mathrm{ka}_{1,2}=1.5, \mathrm{a}_{1,2} / \mathrm{b}_{1,2}=2.0, \mathrm{kd}=5.0$ and $\phi_{i}=60^{\circ}$.


Figure 8. Backscattered field versus the incident angle $\phi_{i}$ for two dielectric loaded identical semi-elliptic channels with $\mathrm{ka}_{1,2}=2.0, \mathrm{a}_{1,2} / \mathrm{b}_{1,2}=1.5$.

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A.-K. Hamid was born in Tulkarm, West Bank, on Sept. 9, 1963. He received the B.Sc. degree in Electrical Engineering from West Virginia Tech, West Virginia, U.S.A. in 1985. He received the M.Sc. and Ph.D. degrees from the University of Manitoba, Winnipeg, Manitoba, Canada in 1988 and 1991, respectively, all in Electrical Engineering. From 1991 to 1993, he was with Quantic

Laboratories Inc., Winnipeg, Manitoba, Canada, developing two and three dimensional electromagnetic field solvers using boundary integral method. From 1994 to 2000 he was with the Faculty of Electrical Engineering at King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia. Since Sept. 2000 he has been an Associate Prof. in the Electrical/Electronics and Computer Engineering Department at the University of Sharjah, Sharjah, United Arab Emirates. His research interest includes EM wave scattering from two and three dimensional bodies, propagation along waveguides with discontinuities, FDTD simulation of cellular phones, and inverse scattering using neural networks.

