RCS of Two Lossy Dielectric Loaded Semi-Elliptic Channels in a Conducting Plane

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Abstract

Analytical solution to the problem of scattering of a plane electromagnetic wave by two lossy dielectric loaded semielliptic channels in a conducting plane is investigated using an iterative procedure to account for the interaction fields between the channels. The incident, scattered and transmitted fields in every region are expressed in terms of complex Mathieu functions. The translation addition theorem is used to compute the higher order scattered fields. Numerical results are presented for the far scattered field for different axial ratios, electrical separation distances, angles of incidence and loss of dielectric materials.

1. Introduction

The electromagnetic scattering from grooves, channels and cracks has many practical applications. The solution may be used to study the scattering by rough surfaces, nondestructive testing of materials, and to check the numerical accuracy of approximate and numerical methods of similar geometries.

Lately, there have been many analytical studies available in the literature on the scattering by hollow and dielectric loaded semi-circular channels [1-5]. Most of these studies are based on the exact dual-series eigenfunction solution. On the other hand, some numerical solutions based on the coupled integral equations for the induced currents were obtained by Senior et. al. [6-7].

Up to date, the analytical solutions available in the literature are for the case of scattering by single semi-elliptic channels loaded by a lossy or lossless dielectric material in a conducting ground plane [8-10]. In this paper, we extend the solution of scattering by a single lossy dielectric loaded semi-elliptic channel in a ground plane to the case of scattering by two adjacent lossy dielectric loaded semielliptic channels in a conducting ground plane.

2. Formulation of the scattering problem

Consider the case of a linearly polarized electromagnetic TM plane wave incident on a two lossy dielectric loaded semi-elliptic channels in a conducting ground plane at an angle ϕ_i with respect to the x axis, as shown in Figure 1. The major axes of the channels are denoted by a_1 and a_2 while the minor axes are denoted by b_1 and b_2 . The ground plane is assumed to be perfectly conducting. The time dependence $e^{j\omega t}$ is assumed and omitted throughout. The electric field component of the TM polarized plane wave of amplitude E_0 is given by

$$E_z^i = E_o e^{jk\rho\cos(\phi - \phi_i)} \tag{1}$$

where k is the wave number in free space. The incident electric field may be expressed in terms of Mathieu functions around the origins o_1 and o_2 as follows

$$E_{1z}^{i} = \sum_{m=0}^{\infty} A_{1em} R_{em}^{(1)}(c_{1},\xi_{1})S_{em}(c_{1},\eta_{1})$$

$$+ \sum_{m=1}^{\infty} A_{1om}R_{om}^{(1)}(c_{1},\xi_{1})S_{om}(c_{1},\eta_{1})$$

$$E_{2z}^{i} = \sum_{m=0}^{\infty} A_{2em}R_{em}^{(1)}(c_{2},\xi_{2})S_{em}(c_{2},\eta_{2})$$

$$+ \sum_{m=1}^{\infty} A_{2om}R_{om}^{(1)}(c_{2},\xi_{2})S_{om}(c_{2},\eta_{2})$$
(3)

where η_1 and η_2 are the intrinsic impedance of ellipse 1 and 2, respectively, and

$$A_{\lim_{om}} = E_o j^m \frac{\sqrt{8\pi}}{N_{em}(c_1)} S_{em}(c_1, \cos\phi_i)$$
(4)

$$A_{2em}_{om} = E_o j^m \frac{\sqrt{8\pi}}{N_{em}(c_2)} S_{em}(c_2, \cos\phi_i) \quad e^{-jkd\cos(\phi_i)}$$
(5)

$$N_{em}(c_1) = \int_{0}^{2\pi} [S_{em}(c_1,\eta_1)]^2 dv$$
(6)

$$N_{em}(c_2) = \int_{0}^{2\pi} [S_{em}(c_2, \eta_2)]^2 dv$$
(7)

and $c_1 = k F_1$, $c_2 = k F_2$, F_1 and F_2 are the semi-focal length of channels one and two, S_{em} and S_{om} are the even and odd angular Mathieu functions of order *m*, respectively, $R_{em}^{(1)}$ and $R_{om}^{(1)}$ are the even and odd radial Mathieu functions of the first kind, N_{em} and N_{om} are the even and odd normalized functions [11], and *d* is the separation distance between the centers of the two channels. The scattered electric fields outside the two semi-elliptic channels are decomposed to two parts: reflected and diffracted fields. These fields should only be written in terms of odd Mathieu functions since the incident and scattered fields should vanish at the conducting plane, i.e. at $\eta = 0$ and $\eta = \pi$. This leads to

$$E_{1z}^{ref} = -\sum_{m=1}^{\infty} A_{1om} R_{om}^{(1)}(c_1, \xi_1) S_{om}(c_1, \eta_1) , \text{ with}$$

$$\phi_{i=2\pi-\phi_i} \quad \text{in equations (4)}$$
(8)

$$E_{2z}^{ref} = -\sum_{m=1}^{\infty} A_{2om} R_{om}^{(1)}(c_2, \xi_2) S_{om}(c_2, \eta_2), \text{ with}$$

$$\phi_{i=2\pi-\phi_i} \quad \text{in equation (5)}$$
(9)

$$E_{1z}^{diff} = \sum_{m=1}^{\infty} B_{1om} R_{om}^{(4)}(c_1, \xi_1) S_{om}(c_1, \eta_1)$$
(10)

$$E_{2z}^{diff} = \sum_{m=1}^{\infty} B_{2om} R_{om}^{(4)}(c_2, \xi_2) S_{om}(c_2, \eta_2)$$
(11)

where B_{1om} and B_{2om} are the unknown odd scattered field expansion coefficients and $R_{om}^{(4)}$ is the odd radial Mathieu function of the fourth kind. The transmitted electric fields inside the two semi-elliptic channels can also be written in terms of Mathieu functions as

$$E_{1z}^{t} = \sum_{m=0}^{\infty} C_{1em} R_{em}^{(1)}(c_{11},\xi_1) S_{em}(c_{11},\eta_1) + \sum_{m=1}^{\infty} C_{1om} R_{om}^{(1)}(c_{11},\xi_1) S_{om}(c_{11},\eta_1)$$
(12)

$$E_{2z}^{t} = \sum_{m=0}^{\infty} C_{2em} R_{em}^{(1)}(c_{22}, \xi_{2}) S_{em}(c_{22}, \eta_{2}) + \sum_{m=1}^{\infty} C_{2om} R_{om}^{(1)}(c_{22}, \xi_{2}) S_{om}(c_{22}, \eta_{2})$$
(13)

where $c_{11} = k_1 F_1$, $c_{22} = k_2 F_2$, $k_1 = \omega \sqrt{\mu_1 \varepsilon_1}$, $k_2 = \omega \sqrt{\mu_2 \varepsilon_2}$, $\varepsilon_1 = \varepsilon_1 - j \varepsilon_1^{"}$, $\varepsilon_2 = \varepsilon_2 - j \varepsilon_2^{'}$ while C_{1em} , C_{2em} and C_{1om} , C_{2om} are the even and odd unknown transmitted field expansion coefficients. The magnetic fields inside and outside the two loaded semi-elliptic channels can be obtained using Maxwell's equations.

3. First order scattered fields

The first order scattered fields result from the separate excitation of each semi-channel by the incident plane wave alone. The first order field expansion coefficients can be determined using the boundary conditions which require the total tangential electric field component inside the channels to vanish at the conducting parts, i.e. at $\xi = \xi_1$, $\xi = \xi_2$ and $\pi < \eta < 2\pi$, while the total tangential electric and magnetic field components to be continuous across the imaginary apertures at $\xi = \xi_1$, $\xi = \xi_2$ and $0 < \eta < \pi$. Using the partial orthogonality properties of the angular Mathieu functions, the first order odd scattered and even transmitted field coefficients can be written in matrix form as follows [10]

$$\begin{bmatrix} Q_1 & Q_2 \\ Q_3 & Q_4 \end{bmatrix} \begin{bmatrix} C_{1e}^1 \\ B_{1o}^1 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$
(14)

$$\begin{bmatrix} Q_5 & Q_6 \\ Q_7 & Q_8 \end{bmatrix} \begin{bmatrix} C_{2e}^1 \\ B_{2o}^1 \end{bmatrix} = \begin{bmatrix} V_3 \\ V_4 \end{bmatrix}$$
(15)

where

$$Q_1 = R_{em}^{(1)}(c_{11},\xi_1)L_{1mn} - R_{em}^{(1)}(c_{11},\xi_1)\Delta_{1n}F_{1mn}$$
(16)

$$Q_2 = R_{om}^{(4)} (c_1, \xi_1) M_{1mn} \,\Delta_{1n} \tag{17}$$

$$Q_{3} = \left\{ R_{em}^{(1)}(c_{11},\xi_{1}) - R_{em}^{(1)'}(c_{11},\xi_{1})\Delta_{1n} \right\} F_{1mn}$$
(18)

$$Q_4 = -\left\{ R_{Om}^{(4)}(c_1,\xi_1) \ -R_{Om}^{(4)'}(c_1,\xi_1)\Delta_{1n} \right\} M_{1mn}$$
(19)

$$V_1 = -\sum_{m=1}^{\infty} 2A_{1om} R_{om}^{(1)'}(c_1, \xi_1) M_{1mn} \Delta_{1n}$$
(20)

$$V_{2} = \sum_{m=1}^{\infty} 2 \left\{ R_{om}^{(1)}(c_{1},\xi_{1}) - R_{om}^{(1)}(c_{1},\xi_{1})\Delta_{1n} \right\} A_{1om} M_{1mn}$$
(21)

$$M_{1mn} = \int_{0}^{\pi} S_{om}(c_{11}, \eta_1) S_{on}(c_1, \eta_1) dv$$
(22)

$$F_{1mn} = \int_{0}^{\pi} S_{em}(c_{11}, \eta_1) S_{on}(c_{11}, \eta_1) dv$$
(23)

$$L_{1mn} = \int_{\pi}^{2\pi} S_{em}(c_{11},\eta) S_{on}(c_{11},\eta_1) dv = -F_{1mn}$$
(24)

$$\Delta_{1n} = \frac{R_{on}^{(1)}(c_{11},\xi_1)}{R_{on}^{(1)}(c_{11},\xi_1)}$$
(25)

and B_{1o}^1 , B_{2o}^1 , and C_{1e}^1 , C_{2e}^1 are the first order odd scattered and even transmitted filed vector matrices. Matrices Q₅, Q₆, Q₇, Q₈, V₃ and V₄ correspond to the second semi-elliptic channel can be written similarly. Equations (14) and (15) may be solved by matrix inversion to obtain the first order scattered field coefficients for given electrical size of semi-elliptic channels, electrical separation, angle of incidence, and lossy dielectric material.

4. Higher order scattered fields

The second order scattered field results from the excitation of each semi-elliptic channel by the scattered field from the other semi-elliptic channel due to the initial incident field. To enforce the boundary conditions, the first order scattered field from the second semi-elliptic channel must be expressed in terms of the coordinate system of the first semi-elliptic channel and vice versa using the addition theorem of Mathieu functions [12], i.e.,

$$R_{om}^{(4)}(c_{2},\xi_{2})S_{om}(c_{2},\eta_{2}) = \sum_{l=1}^{\infty} WO_{olm}^{2 \to 1} R_{ol}^{(1)}(c_{1},\xi_{1})S_{ol}(c_{1},\eta_{1})$$
(26)

where $WO_{olm}^{2\to 1}$ is given by [12]. Again, the boundary conditions require the tangential electric field component inside the channels to vanish at $\xi = \xi_1$, $\xi = \xi_2$ and $\pi < \eta < 2\pi$, while the total tangential electric and magnetic fields components to be continuous across the imaginary apertures at $\xi = \xi_1$, $\xi = \xi_2$ and $0 < \eta < \pi$. Using the partial orthogonality properties of the angular Mathieu functions along with equation (26), we obtain the second order scattered field coefficients for semi-elliptic channel one in matrix form as

$$\begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix} \begin{bmatrix} C_{1e}^2 \\ B_{1o}^2 \end{bmatrix} = \begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix} \begin{bmatrix} B_{2o}^1 \\ B_{2o}^1 \end{bmatrix}$$
(27)

where

$$P_1 = -2 R_{em}^{(1)}(c_{11}, \xi_1) F_{1mn}$$
⁽²⁸⁾

$$P_{2} = R_{om}^{(4)}(c_{1},\xi_{1})M_{1mn}$$
⁽²⁹⁾

$$P_{3} = -2 \left[R_{em}^{(1)'}(c_{11},\xi_{1}) + R_{em}^{(1)'}(c_{11},\xi_{1})(1/\Delta_{n}) \right] F_{1mn}$$

$$P_4 = R_{om}^{(4)} (c_1, \xi_1) M_{1mn}$$
(31)

$$G_{1} = -\sum_{l=1}^{\infty} R_{ol}^{(1)}(c_{1},\xi_{1}) W O_{olm}^{2 \to 1} M_{1ln}$$
(32)

$$G_2 = -\sum_{l=1}^{\infty} R_{ol}^{(1)'}(c_1,\xi_1) W O_{olm}^{2 \to 1} M_{1ln}$$
(33)

$$B_{2o}^{1} = \begin{bmatrix} B_{2om}^{1} \end{bmatrix}$$
(34)

and B_{1o}^2 and C_{1e}^2 are the second order scattered and transmitted field vector matrices for channel one. The vector field matrices C_{2e}^2 , B_{2o}^2 that correspond to the second channel can be obtained similarly.

To obtain a general solution, we solve similarly for the higher order scattered fields, which are sensitive to the electrical sizes and separation distances, angles of incidence and dielectric materials. The general expression for the k^{th} order scattered field coefficients of channel one may be written as

$$\begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix} \begin{bmatrix} C_{1e}^k \\ B_{1o}^k \end{bmatrix} = \begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix} \begin{bmatrix} B_{2o}^{k-1} \\ B_{2o}^{k-1} \end{bmatrix}.$$
(35)

It should be noted that the matrices in equation (35) are computed once (i.e., k=2) for the electrical size, dielectric material, and electrical separation considered and used for the subsequent iterations (i.e., k=3,4,...).

Once the scattered field coefficients are determined, the total far field from the two semi-elliptic channels due to the k^{th} order scattered fields can be determined.

(30)

5. Numerical Results

The scattered near and far fields can be calculated once the scattered field expansion coefficients are computed. The scattered far field expression may be written as follows

$$E_z^s = \left(\frac{j}{k\rho}\right)^{0.5} e^{-jk\rho} P(\phi) \tag{36}$$

where

$$P(\phi) = \sum_{k=1,2,..} \left\{ \sum_{m=1}^{\infty} j^m B_{1om}^k S_{om}(c_1, \cos \phi) + e^{-jkd\cos(\phi)} \sum_{m=1}^{\infty} j^m B_{2om}^k S_{om}(c_2, \cos \phi) \right\}.$$
(37)

In order to solve for the unknown scattered field coefficients, the infinite series are first truncated to include only the first N terms, where N in general is a suitable truncation number proportional to the channel electrical sizes, separation distances and the dielectric loading materials. In the computation, the value of N has been chosen to impose a convergence condition that provides solution accuracy with at least four significant figures [14]. It is found that increasing the electrical sizes of the channel will increase the total truncation number of N terms. Also, to set a criterion for terminating the iteration process, the scattered field after each iteration is calculated and divided by the total field scattered from the pervious iterations, and the process is terminated when the ratio is smaller than 10^{-4} .

The accuracy of the numerical results is checked against the special case of two semi-circular channels loaded with a lossless dielectric material [13]. Fig. 2 shows the normalized backscattered field versus the electrical size ka_{1,2} for two identical loaded semi-circular channels with an incident angle $\phi_i = 90^\circ$, axial ratio $a_{1,2}/b_{1,2}=1.0$ and d=8.0 $a_{1,2}$. The electrical sizes are taken from 0.5 to 5.0. The solid line represents the solution of [13], which is in excellent agreement with our calculation represented by circles. Also, the dashed line represents the lossy dielectric circular channels from which we can see that the resonances start to disappear [14]. Figure 3 is similar to Fig. 2 except for semielliptic channels with axial ratio $a_{1,2}/b_{1,2}=2.0$. It can be seen that the location of the resonances has been changed when it is compared with the circular channels. It is worth mentioning that the numerical results given by [13] were only for loaded channels and no hollow cases. Figure 4 is similar to Fig. 3 except for the incident angle changed to 45 degrees. It can be seen that the number of resonances is increased significantly when the incident angle is changed from 90 to 45 degrees.

Fig. 5 shows the normalized backscattered field versus the electrical separation distance kd for two dielectric loaded

identical semi-elliptic channels with ka_{1,2}=2.0, a_{1,2}/b_{1,2}=1.5 and $\phi_i = 60^\circ$. The electrical separation is taken from 5.0 to 16.5. Fig. 6 shows the echo pattern width versus the scattering angle ϕ for two dielectric loaded identical channels with ka_{1,2}=1.5, a_{1,2}/b_{1,2}=2.0, kd=5.0 and $\phi_i = 90^\circ$.

Fig. 7 is similar to 6 except for $\phi_i = 60^\circ$. Figure 8 shows the normalized backscattered field versus the incident angle ϕ_i for two dielectric loaded two channels with ka_{1,2}=2.0, a_{1,2}/b_{1,2}=1.5 and kd=5.0.

6. Conclusions

Analytical solution and numerical results of the electromagnetic scattering by a two lossy dielectric loaded semi-elliptic channels in a ground plane is obtained for the case of TM (transverse magnetic) polarization. The validity and accuracy of the obtained numerical results were verified against the special case of two lossless semicircular channels. It is worth mentioning that the number of higher order scattered fields used in the computation of numerical results was ranged from k=2 to 4. The agreement was excellent in all cases. It was shown that the presence of lossy and lossless dielectric materials in the channels has significantly changed the scattered field patterns when it was compared with the hollow case. The present work will be extended to the case of an infinite array of semi-elliptic channels in a ground plane since this would be useful for the study of scattering by rough surface.

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Figure 1. Scattering geometry of two lossy dielectric loaded semi-elliptic channels in a ground plane.



Figure 2. Backscattered field versus electrical size ka_{1,2} for two dielectric loaded identical semi-circular channels with $a_{1,2}/b_{1,2}=1.0$ and $\phi_i = 90^o$.



Figure 4. Backscattered field versus electrical size ka_{1,2} for two dielectric loaded identical semi-elliptic channels with $a_{1,2}/b_{1,2}=2.0$ and $\phi_i = 45^o$.



Figure 3. Backscattered field versus electrical size ka_{1,2} for two dielectric loaded identical semi-elliptic channels with $a_{1,2}/b_{1,2}=2.0$ and $\phi_i = 90^o$.



Figure 5. Backscattered field versus electrical separation distance kd for two dielectric loaded identical semi-elliptic channels with ka_{1,2}=2.0, a_{1,2}/b_{1,2}=1.5 and $\phi_i = 60^o$.



Figure 6. Echo pattern width versus the scattering angle ϕ for two dielectric loaded identical semi-elliptic channels with ka_{1,2}=1.5, a_{1,2}/b_{1,2}=2.0, kd=5.0 and $\phi_i = 90^\circ$.



Figure 7. Echo pattern width versus the scattering angle ϕ for two dielectric loaded identical semi-elliptic channels with ka_{1,2}=1.5, a_{1,2}/b_{1,2}=2.0, kd=5.0 and $\phi_i = 60^{\circ}$.



Figure 8. Backscattered field versus the incident angle ϕ_i for two dielectric loaded identical semi-elliptic channels with ka_{1,2}=2.0, a_{1,2}/b_{1,2}=1.5.

References

- B. K. Sachdeva, and R. A. Hurd, "Scattering by a dielectric-loaded trough in a conducting plane," J. Appl. Phys., vol. 48, no.4, pp. 1473-1476, 1977.
- M. K. Hinders, and A. D. Yaghjian, "Dual-series solution to scattering from a semicircular channel in a ground plane," IEEE Microwave Guided Wave Lett., vol. 1, no. 9, pp. 239-242, 1991.
- T. J. Park, H. J. Eom, W.-M. Boerner, and Y. Yamaguchi," TE-plane wave scattering from a dielectric-loaded semi-circular trough in a conducting plane," J. Electromagnetic Waves Applicat., vol. 7, pp. 235-245, 1993.
- H. A. Ragheb, "Electromagnetic scattering from a coaxial dielectric circular cylinder loading a semi channel gap in a ground plane," IEEE Trans. Microwave Theory Tech., vol. 43, no. 6, pp. 1303-1309, 1995.
- T. Shen, D. Dou, and Z. Sun, "Gaussian beam scattering from a semicircular channel in a conducting plane," J. Electromagnetic Waves Applicat., vol. 16, pp. 67-85, 1997.
- T. B. Senior and J. L. Volakis, "Scattering by gaps and cracks," IEEE Trans. Antennas Propag. Vol. 37, pp. 744-750, 1989.
- T. B. Senior, K. Sarabandi, and J. Natzke, "Scattering by narrow gap," IEEE Trans. Antennas Propag. Vol. 38, pp. 1102-1110, 1990.
- W. J. Byun, J. W. Yu, and N. H. Myung, "TM scattering from hollow and dielectric –filled semi elliptic channel with arbitrary eccentricity in a perfectly conducting plane," IEEE Trans. Microwave Theory Tech., vol. 46, no. 9, pp. 1336-1339, 1998.
- A-K. Hamid, "Scattering from dielectric coated elliptic cylinder loading a semi-elliptic channel in a ground plane," J. of Electromagnetic Waves and Applications, Vol. 19, No. 2, pp. 257-269, 2005.
- A-K. Hamid, "Echo pattern width of a lossy dielectric loaded semi-elliptic channel in a ground plane," Electromagnetics. Submitted.
- P. M. Morse and H. Feshbach, Methods of theoretical physics, vols. I and II. New York: McGraw-Hill, 1953.
- A-K. Hamid and M. I. Hussein, "Iterative solution to the electromagnetic plane wave scattering by two parallel conducting elliptic cylinders," J. of Electromagnetic Waves and Applications, vol. 17, no. 6, pp. 813-828, 2003.
- J-W. Yu, W. J. Byun and N. H. Myung, "Multiple scattering from two dielectric-filled semi-circular channels in a conducting plane: TM case," IEEE Trans. Antennas Propag. vol. 50, pp. 1250-1253, 2002.

14. A-K. Hamid, M. I. Hussein, H. Ragheb, and M. Hamid, "Mathieu functions of complex arguments and their applications to the scattering by lossy elliptic cylinders," Applied Computational Electromagnetics Society Journal, vol. 17, no. 3, pp. 209-217, 2002.

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