# Mode Expansion with Moment Method (ME-MM) to Analyze Dielectric Resonator Loaded Cavities

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Abstract—An efficient hybrid Mode Expansion-(ME) Moment Method (MM) or ME-MM is proposed to simulate the eigenvalue problem of multi-layer dielectric resonators (DRs) within cylindrical and rectangular cavities. Resonant frequencies and field distributions for several DRs are presented. The method's efficiency and accuracy are validated by comparison with commercial software, such as the HFSS, and other numerical methods. Finally, an air gap tunable DR analyzed via shows the ME-MM potential to design tunable DRs and filters.

Index Terms—Mode Expansion, Moment Method, Multi-layer Dielectric Resonators, Resonant Frequency, Field Distribution

# I. INTRODUCTION

Resonators, filters and multiplexers play critical roles in many telecommunication systems, such as satellite and mobile communications [1], [2]. The size of these components is directly related to the wavelength and varies from less than one inch to more than a foot. Strip-line and micro-strip-line structures have been successfully adopted to avoid the bulkiness of waveguide structures. However, when high powerhandling capability and/or low loss are needed, waveguide remain choice devices Since materials of high dielectric constant, high quality factor, and low coefficient of thermal expansion have been developed 70s [3], homogeneously the mid in and inhomogeneously-filled waveguide components have been studied and used in communications, navigation and various types of radar systems. With so much interest in dielectric structures, this paper presents a new methodology, referred to as a hybrid Mode Expansion Moment Method (ME-MM), to analyze dielectric resonator (DRs) loaded cavities. The efficiency and accuracy are compared with HFSS or other numerical methods.

Analysis and modeling of waveguide resonant and transmission structures have been research topics in the past decades, especially for DR loaded cavities. When a DR is placed in an open space, the analysis is usually performed under the assumption that the fields are completely restricted inside the dielectric materials due to its high dielectric constant. As such, the DR edges can be treated as perfect magnetic walls (PMW) and the modes and field distribution can be easily determined by calculating the field variations in each direction [4]-[6]. Obviously, this type of configuration is not practical and the PMW treatment is too approximate. A more accurate model was suggested by taking away the PMW on the two ends of a cylindrical DR, while keeping the PMW on the side of the DR and extending to infinity along the DR axial direction [7], [8]. In this case, the fields outside the DR decay exponentially along the axial direction. To represent these fields, an extra subscript  $\delta$  is added to the normal  $TE_{01}$  and  $TM_{01}$  modes as  $TE_{01\delta}$  and  $TM_{01\delta}$ . Further modifications assuming imperfect magnetic walls on all the surfaces were also suggested, but the analysis is still approximate with axial symmetric modes only [9-11].

Placing the DR between two parallel conducting plates provides a partial configuration. The resonator is formed by cutting a piece of cylindrical dielectric waveguide with a conducting plate at each end. This configuration is important for dielectric material measurement applications [12-14]. Rigorous modal analysis, including the axially symmetrical modes and the non-axial-symmetric modes for this geometry, was given in [13] In any practical applications, conducting enclosures for the DR are unavoidable. This is because fields the DR must be shielded and also for packaging because the DR array interact with circuits components outside.

A popular analytical method is the finite difference method [15], [16] which transforms the D.E. into a system of algebraic equations by simply replacing derivatives with finite differences. The finite element method is another popular approach and when making it more efficient and accurate for neural bars where Green's functions are used, an electromagnetic boundary value problem can be converted in a surface or volume integral equation in terms of the equivalent electric and/or magnetic currents. The integral equation itself is then transformed to a set of linear equations by expanding the unknown currents as a superposition of a set of basis functions and by

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evaluating the inner products to get the equations with a set of testing functions (MM) [17-20].

The most popular configuration is the cylindrical, solid DR or ring DR coaxially loaded in a cylindrical enclosure. Recently, a configuration of cylindrical DRs loaded in a rectangular box was studied [21], [22]. In these cases, conductor loss was minimized by placing the enclosure conductor walls at some distance away from the DR. However, if the cylinder size is the major concern, those distances may be partially or completely eliminated.

The mode matching method is often employed to further characterize the guided modes in a waveguide and to find the scattering properties of a waveguide discontinuity. The configuration is typically divided into regions that the fields in each region can be typically expressed as a summation of its eigenmode functions. By matching the boundary conditions and using the orthogonal properties of the eigenmode functions, a set of linear equations then generated for the coefficients of the eigenmode functions. The resonant frequencies are then found by equating the determinant of the equation matrix to zero [23]. In this paper, a new method combining the Mode Matching and Moment Method (ME-MM) is proposed. The key advantage of the approach is that the matrix dimensions are determined by the number of basis functions used on the inter-surface rather than the number of modes in the expansion mode.

### II. CONFIGURATION AND ANALYSIS

A typical configuration of the dielectric resonator is shown in Fig. 1. It depicts a dielectric cylinder of radius  $r_2$  and height  $l_2$ , supported by a concentric circular dielectric ring of radius  $r_1$  and height  $l_1$ . The entire configuration may be separated to two sections:

Region I Post Region P:

 $\begin{cases} \rho \le r_2 & \text{cylindrical case} \\ \rho \le a & \text{rectangular case} \end{cases}$ 

<u>Region II</u> Waveguide Region W:  $\int \rho \ge r_2$  cylindrical case

 $\rho \ge a, \ 0 \le z \le L, \ -a \le x \le a$  rectangular case

The post region P (region II) is further subdivided into sub-regions as follows:

• cylindrical case

$$\begin{cases} \text{Region } P_1 : \rho \le r_1 \\ \text{Region } P_2 : r_1 \le \rho \le r_2 \end{cases}$$

- b) rectangular case
  - $\begin{cases} \text{Region } P_1 : \rho \le r_1 \\ \text{Region } P_2 : r_1 \le \rho \le r_2 \\ \text{Region } P_3 : r_2 \le \rho \le a . \end{cases}$

Here,  $b \ge a$  for the rectangular case with 2b representing the long-side length (Fig.1b).

The linear system for the fields and eigenvalues in the resonator is constructed by introducing a modal function representation of the fields with linear multi-layer parallel plate waveguides.

At  $z = L_k$  (k = 1, 2 ...) and  $\rho = r_i$  (i = 1, 2 ...), the tangential field components are then enforced to be continuous along the axial and radial directions. After mode matching, this gives rise to the linear system of equation in the modal field coefficients in the outer region [22],

$$\begin{bmatrix} W_{11} & W_{12} & W_{13} & W_{14} \\ W_{21} & W_{22} & W_{23} & W_{24} \end{bmatrix} \begin{bmatrix} R_n^e \\ T_n^e \\ R_n^h \\ T_n^h \end{bmatrix} = 0.$$
(1)



Fig.1. Configurations of Dielectric resonators: (a) Cylindrical enclosure, (b) Rectangular enclosure.

The submatrices  $W_{ij}$  and their element expressions are listed in [22]. Their dimensions are  $J \times J$  with Jrepresenting the number of roots that the characteristic equation in the cylindrical post region. n is the index number along  $\varphi$  direction. And  $R_n^e, T_n^e, R_n^h$ , and  $T_n^h$ are the field coefficients in the  $P_I$  region. The superscripts e and h represent TM and TEeigenmodes, respectively.

The fields in the  $P_I$  region (I=2 for the cylindrical configuration, and I=3 for rectangular configuration in Fig.1) can be expressed as:

$$\vec{E}_{l}^{P_{I}} = \sum_{n} \sum_{j} \left[ R_{nj}^{e} B_{n}^{(1)} (\xi_{j}^{le} a) + T_{nj}^{e} B_{n}^{(2)} (\xi_{j}^{le} a) \right] \vec{e}_{lnj}^{le} + \sum_{n} \sum_{j} \left[ R_{nj}^{h} B_{n}^{\prime(1)} (\xi_{j}^{lh} a) + T_{nj}^{h} B_{n}^{\prime(2)} (\xi_{j}^{lh} a) \right] \left| \xi_{j}^{lh} \right| \vec{e}_{lnj}^{lh}$$
(2)

$$\begin{aligned} \overline{H}_{t}^{P_{l}} &= \sum_{n} \sum_{j} \left[ R_{nj}^{h} B_{n}^{(1)} (\xi_{j}^{lh} a) + T_{nj}^{h} B_{n}^{(2)} (\xi_{j}^{lh} a) \right] \overline{h}_{tnj}^{lh} \\ &+ \sum_{n} \sum_{j} \left[ R_{nj}^{e} B_{n}^{\prime(1)} (\xi_{j}^{le} a) + T_{nj}^{e} B_{n}^{\prime(2)} (\xi_{j}^{le} a) \right] \left| \xi_{j}^{le} \right| \overline{h}_{tnj}^{le} \end{aligned} (3)$$

where,

$$\hat{z}_{inj}^{le} = \hat{z} \begin{cases} \sin n\varphi \\ \cos n\varphi \end{cases} e_{zj}^{le} (\gamma_j^{le} z) + \hat{\varphi} \begin{cases} \cos n\varphi \\ -\sin n\varphi \end{cases} \frac{n}{r \left(\xi_j^{le}\right)^2} e_{zj}^{le'} (\gamma_j^{le} z)$$

$$(4)$$

$$\bar{e}_{inj}^{Ih} = \hat{\varphi} \begin{cases} \cos n\varphi \\ -\sin n\varphi \end{cases} \frac{1}{\left(\xi_{i}^{Ih}\right)^{2}} h_{zi}^{Ih} (\gamma_{j}^{Ih} z)$$
(5)

$$j\omega\mu\bar{h}_{mj}^{le} = \hat{\varphi} \begin{cases} \sin n\varphi \\ \cos n\varphi \end{cases} \frac{k_i^2(z)}{\left(\xi_j^{le}\right)^2} e_{zj}^{le}(\gamma_j^{le}z) \tag{6}$$

$$j\omega\mu\bar{h}_{nj}^{lh} = \hat{z} \begin{cases} \cos n\varphi \\ -\sin n\varphi \end{cases} h_{zj}^{lh} (\gamma_j^{lh}z) + \hat{\varphi} \begin{cases} -\sin n\varphi \\ -\cos n\varphi \end{cases} \frac{n}{r\left(\xi_j^{lh}\right)^2} h_{zj}^{lh'} (\gamma_j^{lh}z) \quad .$$

As usual  $e_{mj}^{le}(\gamma_j^{le}z), e_{mj}^{lh}(\gamma_j^{le}z), h_{mj}^{le}(\gamma_j^{lh}z)$ , and  $h_{mj}^{lh}(\gamma_j^{lh}z)$ are the eigen-functions in the parallel-plate waveguides [25]. The values for  $\xi_j^{le}, \xi_j^{lh}, \gamma_j^{le}$ , and  $\gamma_j^{lh}$ in each region can be easily obtained by solving its characteristic equation subject to  $\xi^2 = \gamma^2 + k^2$ . We also remark that  $B_n^{(k)}$  is the  $k^{st}$  Bessel function or modified Bessel function given by:

$$B_n^{(1)}(x) = \begin{cases} J_n(x); x^2 \ge 0\\ I_n(|x|); x^2 \le 0 \end{cases}$$
(8)

$$B_n^{(2)}(x) = \begin{cases} Y_n(x); x^2 \ge 0\\ K_n(|x|); x^2 \le 0 \end{cases}.$$
(9)

The field expressions in the waveguide region can also be represented with a modal superposition. For the cylindrical configuration, the field expressions are the same as in (2) and (3) with different coefficients. For rectangular structure, the fields in the waveguide region are

$$E_{t}^{W_{p}} = \sum_{m} \sum_{i} \left[ A_{mi}^{W_{p}} e^{-\gamma_{mj}y} + B_{mi}^{W_{p}} e^{\gamma_{mj}y} \right] \hat{e}_{tmi}^{p}$$
(10)

$$H_{t}^{W_{p}} = \sum_{m} \sum_{i} \left[ A_{mi}^{W_{p}} e^{-\gamma_{mj}y} - B_{mi}^{W_{p}} e^{\gamma_{mj}y} \right] \hat{h}_{tmi}^{p} \quad .$$
(11)

On the inter-surface between the post and the outer waveguide region, we expand the tangential electrical field  $\bar{\varepsilon}_t(\varphi, z)$  as

$$\varepsilon_z^S(\varphi, z) = \sum_{n} \sum_i \left( U_{ni}^z \cos n\varphi + V_{ni}^z \sin n\varphi \right) e_z^s(k_{zi} z) \quad (12)$$

$$\varepsilon_{\varphi}^{S}(\varphi, z) = \sum_{n} \sum_{i} \left( U_{ni}^{\varphi} \cos n\varphi + U_{ni}^{\varphi} \sin n\varphi \right) e_{\varphi}^{s}(k_{zi}z) \quad (13)$$

where the basis functions  $e_z^s(k_{zi}z)$  and  $e_{\varphi}^s(k_{zi}z)$  can take different forms, including triangular, sinusoidal, or other sub-domain representations. Here we choose the sinusoidal full domain basis functions for  $e_z^s(k_{zi}z)$  and  $e_{\varphi}^s(k_{zi}z)$  since they closely represent the actual field distributions. Applying the boundary condition of the tangential field components

$$\left. \vec{E}_t \right|_{P_t} = \vec{E}_t \Big|_W = \vec{E}_t^S \tag{14}$$

$$\left. \vec{H}_t \right|_{P_t} = \left. \vec{H}_t \right|_{W} = \left. \vec{H}_t^S \right. \tag{15}$$

And getting the resulting equations via Galerkin's method coupled with Mode Matching yields

$$\begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \\ G_{31} & G_{32} & G_{33} & G_{34} \\ G_{41} & G_{42} & G_{43} & G_{44} \end{bmatrix} \begin{bmatrix} U^{\varphi} \\ V^{\varphi} \\ U^{z} \\ V^{z} \end{bmatrix} = 0.$$
(16)

We should note that in (16), here  $\overline{\overline{G}}_{ij}$ 's are sub-matrices whose dimensions are determined by the number of basis functions  $(N_{\varphi}, N_z)$ . Detailed element expressions for  $\overline{\overline{G}}$  can be found in [25]. The resonant frequency of the DR loaded cavity is obtained by solving  $|\overline{\overline{G}}| = 0$ . Substituting the resonant frequency back into (2), (3), (10) and (11), the field distributions in different regions are readily obtained.

For the rectangular geometry (Fig.1b), similar procedures are applied and given in [25]. It is important to point out the analytical integration cannot be carried out from the mutual inner products, because of the different coordinate systems at the post region  $P(\rho, \varphi, z)$  and the waveguide region W(x, y, z). To reduce the CPU time spent on the numerical integrals, Bessel-Fourier series are used to translate the numerical integrations into simple summations as follows:

$$\sin\left[\frac{m\pi}{2a}(\rho\sin\varphi+a)\right]e^{\mp\gamma_{mi}\rho\cos\varphi}$$

$$=\sum_{k=-\infty}^{\infty}\sin\left(\frac{m\pi}{2}+k\varphi\right)F(\mp k,\rho,\gamma_{mi})$$

$$\cos\left[\frac{m\pi}{2a}(\rho\sin\varphi+a)\right]e^{\mp\gamma_{mi}\rho\cos\varphi}$$

$$=\sum_{k=-\infty}^{\infty}\cos\left(\frac{m\pi}{2}+k\varphi\right)F(\mp k,\rho,\gamma_{mi})$$
(17)
(17)
(17)
(18)

where

$$F = \begin{cases} J_{k} \left[ T(\rho) \right] \exp \left[ \mp jk \tan^{-1} \left( \frac{2a \left| \gamma_{mi} \right|}{m\pi} \right) \right], \\ \gamma_{mi}^{2} \leq 0 \\ J_{k} \left[ T(\rho) \right] \left[ \left( \frac{m\pi + 2a\gamma_{mi}}{m\pi - 2a\gamma_{mi}} \right) \right]^{\frac{\pi}{2}}, \\ \gamma_{mi}^{2} \geq 0 \& m\pi \geq 2a \left| \gamma_{mi} \right| \\ \left( -1 \right)^{\frac{k}{2}} I_{k} \left[ \left| T(\rho) \right| \right] \left[ \left( \frac{m\pi + 2a\gamma_{mi}}{m\pi - 2a\gamma_{mi}} \right) \right]^{\frac{\pi}{2}}, \\ \gamma_{mi}^{2} \geq 0 \& m\pi \leq 2a \left| \gamma_{mi} \right| \end{cases}$$

$$(19)$$

$$T(\rho) = \frac{\rho}{2\pi} \sqrt{\left(m\pi\right)^2 - 4a^2 \gamma_{mi}^2}$$
(20)

$$\gamma_{mi}^{2} + k^{2} = \left(\frac{m\pi/2a}{2a}\right)^{2} + \left(\frac{i\pi/L}{2}\right)^{2}$$
(21)

$$k^2 = \omega^2 \mu_0 \varepsilon . \tag{22}$$

Using the above transformation technique, rectangular-configuration is the same as that for the analysis for the circular-configuration. In the following section, several numerical examples for DRs inside a cylindrical or rectangular enclosure are presented.

# **III. NUMERICAL RESULTS**

A FORTRAN program was developed to compute the resonant frequencies and field patterns of the cylindrical multi-layers DRs loaded inside a cylindrical or a rectangular waveguide junction.

Table 1 shows the convergence and accuracy of the ME-MM in comparison with HFSS for a twolayer DR (Fig.1 (a)). We can readily see that the ME-MM is convergent and very fast (only six basis functions needed).

Table 2 gives the calculation time and computer memory (the number of tetrahedrons in HFSS and basis functions number in ME-MM) in the HFSS and the new method for two different cases:

# Table 1 Resonant Frequency Convergence Testing

 $2a = 1.6cm; L = 3.0cm; 2r_1 = 1.6cm; 2r_2 = 1.6cm;$  $l_1 = 0.55cm; l_2 = 0.8cm; \varepsilon_{-1} = 10.; \varepsilon_{-2} = 35.7$ 

$$l_1 = 0.55cm; l_2 = 0.8cm; \varepsilon_{r_1} = 10.; \varepsilon_{r_2} = 0$$
  
 $N = 0$  (TE case)

$N_{\varphi} = 0$ (1 - $0$			
$N_z$	$f_r$ (GHz)	HFSS result	Error (%)
4	3.3162		0.29
6	3.3208	3 3258	0.15
8	3.3210	5.5250	0.14
12	3.3212		0.14



Fig. 2. Cross-sectional View of a DR loaded cavity.

Case A:

$$\begin{aligned} &2a = 1.6cm; L = 3.0cm; 2r_1 = 1.6cm; 2r_2 = 1.6cm\\ &l_1 = 0.55cm; l_2 = 0.8cm; \varepsilon_{r1} = 10.; \varepsilon_{r2} = 35.7\\ &\text{Case B:} \quad \begin{aligned} &2a = 1"; L = 1"; 2r_1 = 0"; 2r_2 = 0.689"\\ &l_1 = 0.275"; l_2 = 0.23"; \varepsilon_{r1} = 1.; \varepsilon_{r2} = 38. \end{aligned}$$

From this table, it is seen that in order to reach the same accuracy, HFSS needs at least 100 times more memory and 50 times more CPU time as compared to the ME-MM.

Fig. 2 shows the dimensions and material parameters of another DR configuration in a cavity without substrate. The resonant frequencies for different modes using FEM [29], modal matching [30] and the new method are listed in Table 3. It is clear that ME-MM obtains results closer to the more accurate modal matching method. For the rectangular configurations, the resonant frequencies for two simple cases of dielectric rods within the cavities' are calculated and compared with HFSS in Table 4.

Table 2 Computer Resource Comparison

		Calculating Time/No. of Tetrahedrons or Basis Function)	Resonant Frequency (GHz)
Case A: TE	HFSS	896'16" /(10492)	3.3258
	ME-MM	1'05"/(6)	3.3208
Case A: TM	HFSS	896'16" /(10492)	3.733
	ME-MM	1'27" /(16)	3.763
Case B: HE	HFSS	149'46" /(2894)	4.1264
	ME-MM	3'05" /(12)	4.1545

	Ref [29]	Ref [30]	ME-MM
	(GHz)	(GHz)	(GHz)
TE01	3.435	3.428	3.433
TE02	5.493	5.462	5.322
TM01	4.601	4.551	4.537
HE11	4.271	4.224	4.227
HE12	4.373	4.326	4.316

Table 3 Resonant-Frequency Comparison

Table 4 Resonant Frequencies for Rectangular Configurations

(cm)	Calculating	HFSS	$\Delta =$ error
	Result	(GHz)	(%)
	(GHz)		
Case (A)			
a=3; b=3;			
l=4; r0=1;	1.320	1.301	1.54
$\varepsilon_r = 35$			
Case(B)			
a=3; b=4;			
l=4;r0=1;	1.210	1.220	0.82
$\varepsilon_r = 35$			



Fig. 3. Tunable DR with air gap,  $\varepsilon_A = \varepsilon_D = 10, \varepsilon_B = \varepsilon_C = 35.7$ .

Finally a tunable air-gap DR (resonant variable to the air gap d) model in Fig. 3 is investigated. As in Table 5, the resonant frequency increases 1.6% when the air gap changes from 0.3 cm to 0.45 cm. Also, we observe that the CPU time using ME-MM is at least 30 times faster and also with much less memory needs than HFSS.

The tangential field distributions of  $E_{\varphi} \sim r$  and  $H_z \sim z$  are shown in Fig. 4, and again it is obvious that the boundary conditions are matched perfectly by ME-MM with very few unknowns, and have much reasonable trend in comparison to HFSS curves. Accurate field distributions are essential for DR filters design. Typically they determine the excitation position, and coupling-windows choosing for multi-cavity DR filter [24].



	HFSS	HFSS d=0.3cm ME-MM d=0.3cm ME-MM d=0.35cm HFSS d=0.45cm	3.14346
	ME-MM		3.1247
те	HFSS		3.17595
IE	ME-MM		3.1440
	HFSS		3.21151
	ME-MM		3.1757

Table 5 Tunable Resonance Frequency by Air Gap

### IV. CONCLUSION

A new analysis method (ME-MM) was proposed to analyze the multi-layer DR loaded cavities. Accurate resonant frequencies and field distributions for several different dielectric resonators were evaluated by ME-MM. As compared with HFSS and other numerical methods, our ME-MM saves substantial CPU time and memory, without loss of accuracy. Accurate field distributions can also be obtained by the proposed method essential to provide sufficient details for DR filter design. An airsubstantial tunable DR was analyzed using the ME-MM, showing the capability of this methodology to design tunable cavity-loaded DR filters.

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