A STOCHASTIC ALGORITHM FOR THE EXTRACTION OF PARTIAL INDUCTANCES IN IC INTERCONNECT STRUCTURES

K. Chatterjee
Department of Electrical and Computer Engineering
The Cooper Union for the Advancement of Science and Art
New York, NY 10003-7120
Laboratory for Electromagnetic and Electronic Systems
Massachusetts Institute of Technology
Cambridge, MA 02139
Phone: (212) 353-4333
Fax: (212) 353-4341
Email: chatte@cooper.edu, kausik@mit.edu

ABSTRACT

With recent increases in operating frequencies, the modeling and extraction of on-chip inductance is becoming an increasingly significant consideration. The inductance models include the "loop inductance" models and the "partial inductance" models. In this paper, we develop a stochastic solution methodology for the extraction of partial inductances in IC interconnect structures. An important advantage of this approach is that it requires no discretization meshing of either the volume or the surface of the problem domain. As a result, it has very low memory requirements compared to the more conventional deterministic techniques. Another advantage of this approach is that it is inherently parallelizable and a linear increase in speed is expected with the increase in the number of processors. Excellent agreement has been obtained with analytical benchmark solutions.

Keywords: IC Interconnect modeling, Partial Inductance, Stochastic algorithm, Monte Carlo.

INTRODUCTION

As a consequence of scaling in sizes, the interconnect model used in the chip industry has undergone several changes. Presently, low resistance nets are described by purely capacitive models, while high resistance nets are described by relatively more accurate RC models. However, with operating frequencies reaching the multi-GHz range, the role of on-chip inductance is becoming increasingly important, as the inductive impedance is directly proportional to the frequency of operation. The inclusion of inductance in the interconnect model is particularly necessary in clock distribution networks, signal and power lines, which have wide wires and hence low resistance. The detrimental effects of inductive impedance on system performance include increase in signal delay times and signal overshoot which can cause breakdown of the gate-oxide layer. The introduction of low resistance copper interconnects has further increased the significance of inductance in IC design and accurate modeling and extraction of inductance is necessary.

The principal complexity in the extraction of inductance is that one needs to have the knowledge of currents in advance. However, the current distribution in today's complicated interconnect structures depends on the device and interconnect resistances, inductances and capacitances. Therefore, the modeling of the current distribution is a difficult proposition. The conventional approaches to inductance extraction involve loop inductance models [1], which make various simplifying assumptions in determining the current distribution. In these loop inductance models, typically the capacitive effects are omitted during resistance and inductance extraction. A RLC model is then constructed by adding lumped or distributed interconnect capacitance to the extracted resistance and inductance.

A radically different approach [2] to the modeling of inductance has been suggested in literature, which precludes the need to determine the current distribution in advance. This approach is based on the Partial Element Equivalent Circuit (PEEC) [3] method. In this approach, the interconnect lines are divided into wire segments and self and mutual inductances are extracted for these "partial elements". These extracted inductances are then glued together with various resistances and capacitances to form an effective RLC circuit model. It has been demonstrated [2] that this PEEC-based approach is more accurate than the loop inductance models in that the latter overestimates the signal delay time and the undershoot. The primary reason behind this lies in the fact the PEEC-based models take into account the mutual inductances between the different "partial elements" of a particular loop, while the loop inductance models take into account only the mutual inductance between different loops. The subject of this paper is a stochastic extraction of the self and mutual inductance of these partial elements.

IC interconnect structures are rectilinear in nature. At low frequencies, when the wire segments are parallel to each other, exact analytical expressions [4] exist for the self and mutual inductances of the wire segments, assuming uniform current distribution across the wire cross sections. However, there is absence of such expressions for arbitrary wire-geometry, and even in the case of parallel rectangular wire segments, these analytical expressions for mutual inductance are numerically unstable for wire segments separated by a large distance.

In this work, we develop a novel stochastic algorithm [5] for the extraction of the self and mutual partial inductances. This algorithm is characterized by the absence of discretization meshing of either the volume or the surface of the problem domain. Hence, for today's complicated interconnect structures, the memory requirements are expected to be significantly less than discretization based algorithms. Another advantage of this proposed stochastic algorithm is that it should be completely parallelizable and the speed of computation is expected to increase linearly with the increase in the number of processors. The fundamentals of the algorithm were briefly presented in Ref. [6], along with its applications to frequency-independent inductance extraction. In this work, we present the details of the algorithm, along with its applications to both frequency-independent and frequency-dependent problems.

INTEGRAL FORMULATION FOR INDUCTANCE

The most general formulation [7] for self and mutual inductance in conductor systems follows from a definition of inductance based on magnetic energy. The magnetic energy stored in a two-conductor system, where the two conductors are designated as *i* and *j* is given as

$$W = \frac{1}{2} L_i I_i^2 + \frac{1}{2} L_j I_j^2 + M_{ij} I_i I_j. \qquad (1)$$

Above, W represents the magnetic energy stored in the two-conductor system; L_i and L_j represent the self-inductances of the *i*-th and the *j*-th conductor; I_i and I_j represent the respective currents, while M_{ij} represents the mutual inductance between the conductors. The total magnetic energy can also be written in an integral formulation involving the current densities in the two conductors and equating that to the expression for current density in equation (1), the following expressions for self and mutual inductances are obtained [7]:

$$\begin{aligned} L_i &= \frac{\mu_o}{4\pi I_i^2} \int_{v_i} d^3 x_i \int_{v_i} d^3 x_i' \frac{\mathbf{J}(\mathbf{x_i}) \cdot \mathbf{J}(\mathbf{x_i'})}{\left|\mathbf{x_i} - \mathbf{x_i'}\right|}, \\ M_{ij} &= \frac{\mu_o}{4\pi I_i I_j} \int_{v_i} d^3 x_i \int_{v_j} d^3 x_j \frac{\mathbf{J}(\mathbf{x_i}) \cdot \mathbf{J}(\mathbf{x_j})}{\left|\mathbf{x_i} - \mathbf{x_j}\right|}. \end{aligned} \qquad (2)$$

Above, the self inductance is formulated as a six-dimensional integral over the position coordinates of the *i*-th conductor, while the mutual inductance is formulated as a six-dimensional integral over the position coordinates over the *i*-th and the *j*-th conductor; **x,** v, **J** with an appropriate suffix represent the position coordinate, volume and current density; d^3x with an appropriate suffix represents an infinitesimally small volume element and μ_0 is the magnetic permeability of free space.

The current density **J**, is given as a solution of Maxwell-Helmholtz equation [8] and in the frequency-domain is written as

$$\nabla^2 \mathbf{J} - \gamma^2 \mathbf{J} = \mathbf{0}, \qquad (3)$$

where, γ is the propagation constant of the medium given by $\gamma^2 = -\mu\varepsilon\omega^2 + i\mu\sigma\omega$; μ, ε, σ and ω are the permeability, permittivity, conductivity and frequency respectively. The current density, $\mathbf{J}(\mathbf{r})$ at a given point **r**, subject to appropriate Dirichlet boundary conditions, can be written as a surface integral over the surface of the problem domain [9]:

$$\mathbf{J}(\mathbf{r}) = \oint \mathbf{J}(\mathbf{r}^{'})\left[\nabla_{\mathbf{r}^{'}} G(\mathbf{r},\mathbf{r}^{'}) \cdot \hat{\mathbf{n}}\right] ds^{'}. \qquad (4)$$

Above, $G(\mathbf{r},\mathbf{r}^{'})$ represents the volumetric Green's function to equation (3), and is a solution to equation [9]

$$\nabla^2 G(\mathbf{r},\mathbf{r}^{'}) - \gamma^2 G(\mathbf{r},\mathbf{r}^{'}) = \delta(\mathbf{r} - \mathbf{r}^{'}). \qquad (5)$$

Above $\delta(\mathbf{r}-\mathbf{r}^{'})$ is a dirac-delta function centered at $\mathbf{r}^{'}$ and homogeneous Dirichlet boundary conditions are imposed in calculating the volumetric Green's function. The integral formulation given in (4) can be substituted in equation (2) for the calculation of self and mutual inductances. As a result, the task of inductance extraction involves evaluating a multi-dimensional integral, which in this work has been done stochastically. At low frequencies, the current densities in equation (2) can be assumed to be constant. Hence the stochastic evaluation of self and mutual inductances is in effect a Monte Carlo integration [10] of the integrals given in equation (2) over the position coordinates of the respective conductors. We will now describe briefly the fundamentals of the Monte Carlo integration technique used in this work, known as the Sample Mean Monte Carlo [10].

Let us consider a function $f(x)$ defined over the interval $a \le x \le b$. Our desire is to estimate the integral

$$\mathbf{I} = \int_a^b dx\, f(x). \qquad (6)$$

In the event, the integral is improper, absolute convergence [11] is assumed. We select an arbitrary probability density function $p(x)$. A random variable ξ is defined corresponding to a probability density function $p(x)$. We now introduce another random variable κ defined as

$$\kappa = \frac{f(\xi)}{p(\xi)}. \qquad (7)$$

Then, the expectation value of the random variable κ, written as $M(\kappa)$, is an estimate of the integral $\mathbf{I}$, which can be rewritten as

$$\mathbf{I} = M(\kappa) = \int_a^b dx \left[\frac{f(x)}{p(x)}\right] p(x). \qquad (8)$$

The integral can be evaluated by sampling the quantity $f(x)/p(x)$ within box brackets according to the probability density function $p(x)$ with the help of a random-number generator [12] and averaging over a statistically large number of such samples. It can be noted that the Monte Carlo integration technique is ideally adapted to the estimation of multi-dimensional integrals such as the ones in equation (2), as only the integrand needs to be sampled irrespective of the dimensionality of the integral. Also, the integration technique is inherently parallelizable, as the stochastically independent samples can be sampled in different processors with very little inter-processor communication.

For the extraction of frequency-dependent inductance, an expression for the volumetric Green's function to the Helmholtz equation in (3) needs to be obtained in heterogeneous problem domains. However, there is an absence of an analytical expression for the volumetric Green's function in materials of arbitrary heterogeneity. Keeping that in mind, we have developed an approximate expression for the solution of equation (5) based on iterative perturbation theory. The details of this work have been published [13, 14] and we will discuss it briefly within the context of two-dimensional problems.

The Green's function $G(\mathbf{r},\mathbf{r}^{'})$ is estimated over a circular problem domain and is assumed to be zero on the boundary of the circular domain, as the frequency-dependent problem studied in this work is a Dirichlet problem.

Let us define the zeroth-order approximation $G^{(0)}$ for G, subject to Dirichlet boundary conditions. Therefore,

$$\nabla^2 G^{(0)} = \delta(\mathbf{r} - \mathbf{r_o}). \qquad (9)$$

Above, $\mathbf{r}(\rho, \theta)$ is the point where the zeroth-order approximation is calculated given a delta function centered at $\mathbf{r_o}(\rho_o, \theta_o)$. Using (5) for iteration, we can then generate a first-order approximation $G^{(1)}$ in terms of $G^{(0)}$:

$$\nabla^2 G^{(1)} = \delta(\mathbf{r} - \mathbf{r_o}) + \gamma^2 G^{(0)}. \qquad (10)$$

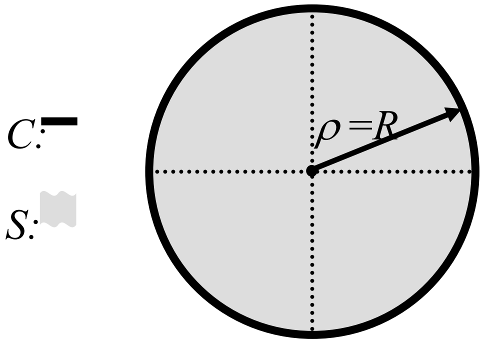

Figure 1. A circle of arbitrary radius R over which the Green's function given by the solution of equation (5) is estimated.

The solution to Poisson equation (9) is well known; it has the form, in polar coordinates [9]

$$G^{(0)} = \frac{1}{4\pi} \ln\left[R^2 \frac{A}{B} \right],$$
$$A = \rho^2 + \rho_o^2 - 2\rho\rho_o \cos(\theta - \theta_o), \quad (11)$$
$$B = \rho^2\rho_o^2 + R^4 - 2\rho\rho_o R^2 \cos(\theta - \theta_o).$$

Now, we are in a position to evaluate $G^{(1)}$ from equation (10). Using the expression for $G^{(0)}$ from equation (11) and with the right hand side of equation (10) as the Poisson source term, we find an expression for the first-order approximation to the solution of equation (5) given by

$$G^{(1)}(\mathbf{r} \mid \mathbf{r_o}) = \iint_S d^2 r_S G^{(0)}(\mathbf{r} \mid \mathbf{r}_S) \left[\delta(\mathbf{r}_S - \mathbf{r_o}) + \gamma^2(\mathbf{r}_S) G^{(0)}(\mathbf{r}_S \mid \mathbf{r_o}) \right] = G^{(0)}(\mathbf{r} \mid \mathbf{r_o}) + \iint_S d^2 r_S \, \gamma^2(\mathbf{r}_S) \, G^{(0)}(\mathbf{r} \mid \mathbf{r}_S) G^{(0)}(\mathbf{r}_S \mid \mathbf{r_o}). \quad (12)$$

Note that $G^{(1)}$ given by equation (12), is an approximate expression for G as given by the solution of equation (5). The integration variable in equation (12) represents an infinitesimal area element on the circular-domain surface S in Fig. 1. It can be noted that homogeneous Dirichlet conditions are satisfied by the approximate Green's function expression in Eq. (12).

We next use this approximate Green's function $G^{(1)}$ to develop a general solution to equation (3) within our circular domain in Fig. 1. In our problem, the two-dimensional problem domain is in *x-y* plane, and a time-harmonic current at a given frequency is impressed in the *z*-direction. Based on [9] and equations (4) and (12), the current density at the center of the circular domain is given by a line integral about the domain circumference C as

$$J_z(\text{center}) \approx \int_0^{2\pi} d\theta \, J_z(R,\theta) \left[\frac{1}{2\pi} + \frac{1}{4\pi^2} \sum_q W_q(\theta) \right], \quad (13)$$

where

$$W_q(\theta) = \gamma_q^2 \int_0^R d\eta \int_{(q-1)\pi/2}^{q\pi/2} d\xi \, \frac{C}{D},$$
$$C = \eta \ln(\eta / R)(R^2 - \eta^2), \quad (14)$$
$$D = R^2 + \eta^2 - 2R\eta \cos(\theta - \xi).$$

For simplicity, above, we take γ^2 to be piecewise constant with respective values γ_q in θ-quadrants q = 1, 2, 3, and 4. The quantities within square brackets in equation (13) are 2D versions, respectively, of surface and volume Green's functions encountered in 3D problem domains. The function W_q represents perturbative corrections arising from the $\gamma^2\mathbf{J}$ term in the original Maxwell-Helmholtz equation (3). In equations (13) and (14), η and ξ are variables of integration. η takes values between 0 and R, while θ assumes values between $(q-1)\pi/2$ to $q\pi/2$ for a particular quadrant. Equations (13) and (14) constitute the starting point for defining a random-walk [13, 14] algorithm for solving (3) in 2D domains with arbitrary piecewise-constant spatial variation in γ, subject to arbitrary Dirichlet boundary conditions.

The total current, I_z, through the cross section can be calculated by integrating the current density given in equation (3) over the problem domain (*ds* being an infinitesimal area unit) and can be written as

$$I_z = \iint_S ds \, J_z. \quad (15)$$

The integral expression for the current density from equation (13) is substituted in equation (15) to obtain a multi-dimensional integral expression for total current through the conductor surface.

The internal impedance per unit length is defined as [8]

$$Z_i = \frac{E_z(dc\,value)}{I_z}. \quad (16)$$

At this point, the crucial thing to note is that for estimating frequency-dependent impedance, we need not estimate electric field or current density at any point within the problem domain. The problem of impedance extraction is reduced to estimating the overall multi-dimensional integral expression for current obtained from (15) using the floating random-walk method [13, 14] and then using (16) to evaluate the internal impedance per unit length. We will now discuss the details of the floating random-walk method.

The floating random-walk algorithm is a Monte Carlo evaluation of an infinite series of multi-dimensional integrals. In our chosen benchmark problem, a time-harmonic current density in the *z*-direction at a single frequency is impressed on a circular conductor in the *x-y* plane. Our goal is to calculate the current through the conductor as given by equation (15). The starting point of a random-walk is based on a pre-determined probability distribution.

The random-walks propagate as "hops" of different sizes from circle centers to circumferences, consistent with a statistical interpretation [13, 14] of equation (15). Maximally sized circles, subject to limitations imposed by iterative perturbation theory, are used with hop-location probability rules again consistent with equation (15).

We define, with each hop, a numerical weight factor derived from equation (15) in conjunction with equations (12), (13) and (14). The product of these weight factors over a walk, multiplied by the solution at the problem boundary—where the walk must terminate—gives a statistical estimate for I_z. We can thus obtain an accurate statistical estimate for I_z by averaging over a statistically large number of random-walks. Mathematically, we can write such an estimate as

$$I_z \approx \frac{1}{N}\sum_{n=1}^{N} I_z^{(n)}, \quad (17)$$

where N is the number of walks and $I^{(n)}$ is the contribution from the nth-walk. The error in the result has two components:

1) A deterministic error arising from the truncation of the iterative perturbation based Green's function in equation (12) and can be controlled by controlling the radius of the hop.

2) A statistical 1-σ error, σ, given by [15]

$$\sigma = \frac{\sigma_E}{\sqrt{N}}, \quad (18)$$

where σ_E is the standard deviation of the estimates from different random-walks and N is the number of random-walks. As a result, the statistical error can be controlled by controlling the number of random-walks.

It can be seen that computational resources need not be wasted in evaluating the currents or fields at any point in the problem domain. Instead one just needs to evaluate the multi-dimensional integrals given by equation (15). It can again be noted that similar to the situation of the frequency-independent problem, the approach is completely parallelizable, as the integrand can be sampled in different processors at the same time. We now give the details of the benchmark problems that have been handled in this work.

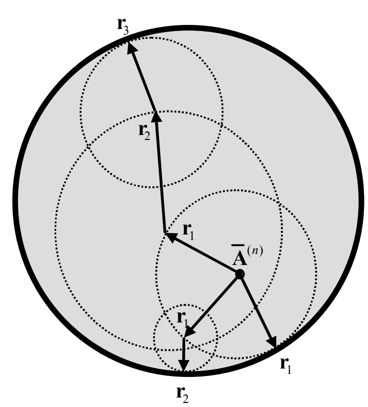

Figure 2. A schematic diagram of a circular cross section is shown. One-, two-, and three- hop random-walks are represented.

RESULTS

Frequency-independent benchmark problems: The algorithm has been benchmarked against several mutual inductance extraction problems in one, two and three dimensions. The numerical results have been matched with analytical solutions given in Ref. [4] and excellent agreement has been obtained. The results are given in Table 1 and the benchmark problem geometries are presented in Figures (3) to (7). In these benchmark problems, A = B = C = D = R = T = 5 μm and Q = S = U = V = 1 μm. For each of these problems, we have taken 5000 sample points and the error from the analytical solution has been restricted to a fraction of one percent in each case. The computational time for each one of these benchmark problems has been seen to be a fraction of a second in MATLAB 6.1 on a 1.8 GHz Intel Pentium IV personal computer. The exact and statistical errors are computed and they are seen to be in close agreement.

Table 1. Analytical and numerical results for the benchmark problems. Columns: A = Benchmark problems, B = Analytical results (pH), C = Numerical results (pH), D = Exact errors normalized to the analytical results, E = Statistical errors normalized to the analytical results.

A	B	C	D	E
1.	0.35844	0.35859	0.0004	0.0005
2.	0.28797	0.28830	0.0011	0.0010
3.	0.28698	0.28630	0.0023	0.0025
4.	0.21326	0.21323	0.0001	0.0002
5.	0.28800	0.28878	0.0027	0.0024

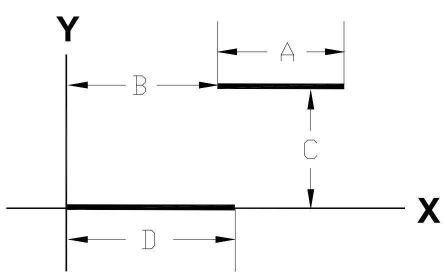

Figure 3. Two parallel filaments of negligible width and thickness. The current is in the x-direction.

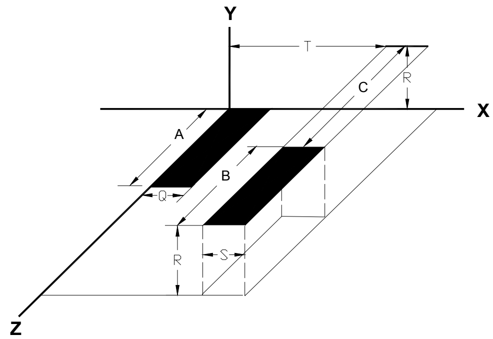

Figure 4. Two parallel tapes of negligible thickness. The current is in the z-direction.

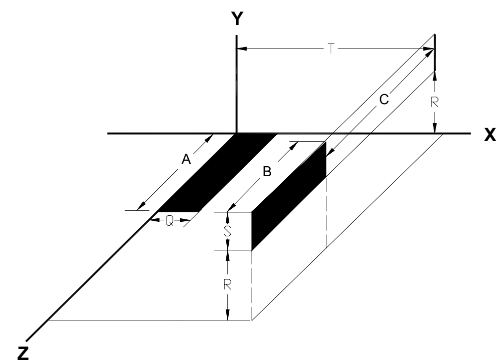

Figure 5. Two tapes of negligible thickness, whose axis are parallel and widths are perpendicular. The current is in the z-direction.

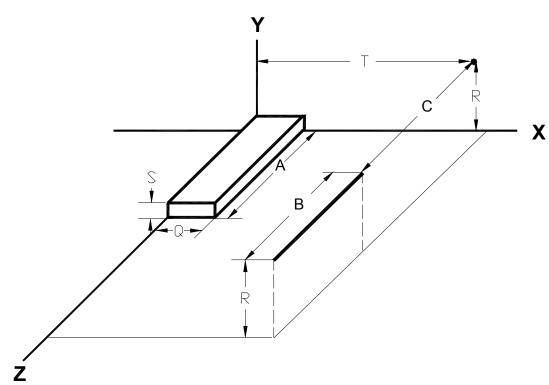

Figure 6. A thin filament of negligible width and thickness is placed parallel to a rectangular bar. The current is in the z-direction.

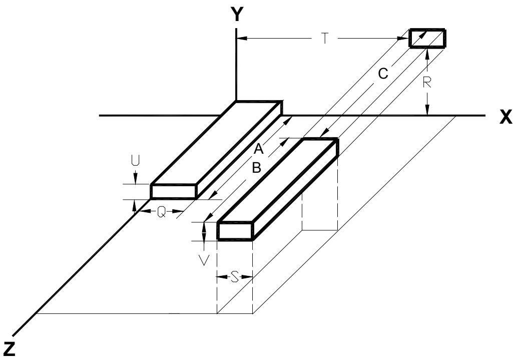

Figure 7. Two rectangular bars placed parallel to each other. The current is in the *z*-direction.

Frequency-dependent benchmark problem: For the frequency-dependent problem, a *z*-directed time-harmonic and spatially invariant current is impressed on a single circular conductor. The frequency-dependent impedance contains a resistive term and an inductive term. Table (2) shows the results for the frequency-dependent self impedance of a cross section of radius 1.0 μm at frequencies ranging from 1 GHz to 25 GHz. The resistivity for conducting material is given by $\rho = 1.8\,\mu\Omega - \text{cm}$ and the magnetic permeability is assumed to be that of free space. For extracting impedance, a total of only 1000 random-walks were performed at each frequency. It can be seen from Table (2), that the error in the estimate of frequency-dependent resistance and inductive impedance is around 1 percent and the absolute error is comparable to the statistical error. The resistance and inductive impedances are plotted as a function of frequency in figures (8) and (9) respectively. As in the case of the frequency-independent problem, numerical computations have been performed in MatLab 6.1 on a 1.8 GHz Intel Pentium IV personal computer, and the computation time at 25 GHz frequency is of the order of a few seconds.

Table 2. Numerical results for the frequency-dependent self-impedance of a conducting circular cross section. Columns: A = Frequency (GHz), B = Skin depth as a fraction of radius, C = Analytical result (ohm/meter), D = Random-walk result (ohm/meter), E = Exact error (ohm/meter), F = Statistical error (ohm/meter).

A	B	C	D	E	F
1	2.14	5735+314i	5738+312i	3-2i	1+1i
5	0.96	5870+1552i	5917+1534i	47-18i	40+15i
10	0.68	6262+2997i	6315+2962i	53-35i	55+30i
15	0.55	6827+4268i	6888+4225i	61-43i	59+41i
20	0.48	7482+5347i	7549+5297i	67-50i	70+45i
25	0.43	8159+6252i	8234+6193i	75-59i	72+51i

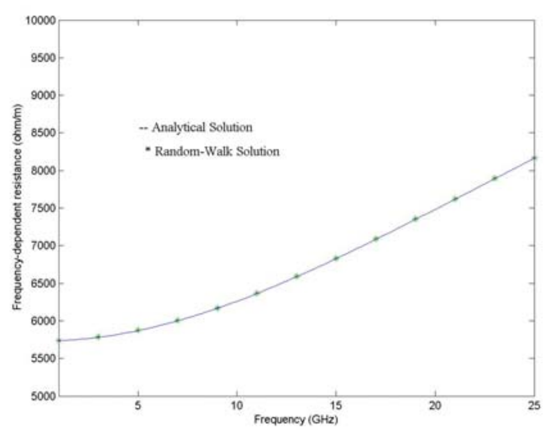

Figure 8. Frequency-dependent resistance per unit length for a conductor of 1 μm radius with a resistivity of $1.8\,\mu\Omega - \text{cm}$ and magnetic permeability of free space.

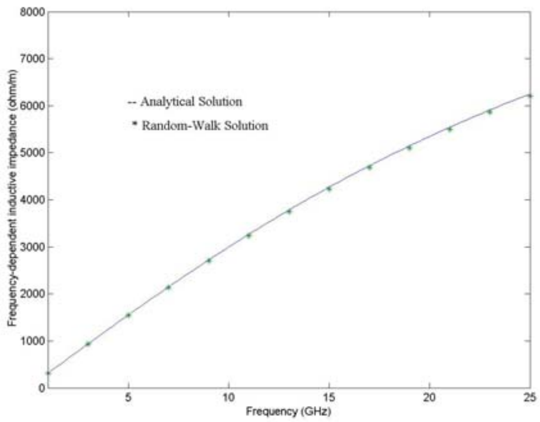

Figure 9. Frequency-dependent inductive impedance per unit length for a conductor of 1 μm radius with a resistivity of $1.8\mu\Omega-\text{cm}$ and magnetic permeability of free space.

CONCLUSION & FUTURE WORK

Summarizing, a stochastic algorithm for the extraction of partial inductances in IC interconnect structures has been developed. The algorithm has been validated with the help of frequency-independent and frequency-dependent benchmarks. The extension of this algorithm to more complicated frequency-dependent benchmark problems will form the basis of future work. Stochastic solution of the PEEC-based RLC circuit matrix will also be emphasized. It is believed that with additional development, this algorithm can be developed into an IC CAD tool for inductance extraction.

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Kausik Chatterjee was born in India in 1969. In 1992, he received a Bachelor of Engineering degree in Electrical Engineering from Jadavpur University, Calcutta, India. Subsequently, in 1995, he received a Master of Technology degree in Nuclear Engineering from Indian Institute of Technology, Kanpur, India, and in 2002, he received his Ph.D degree in Electrical Engineering from Rensselaer Polytechnic Institute, Troy, New York. In 2002, he joined the faculty at California State University, Fresno as an Assistant Professor of Electrical and Computer Engineering, a position he held till 2005. He had been a visiting scientist at MIT Laboratory for Electromagnetic and Electronic Systems (2004-2005), a National Research Council Faculty Associate at Air Force Research Laboratory, Wright-Patterson Air Force Base (2004) and NASA, Langley Research Center (2005) and a Junior Engineer at Durgapur Steel Plant, India (1992-1993). He is currently an Assistant Professor in the Department of Electrical and Computer Engineering at Cooper Union and an Adjunct Scientist at the MIT Laboratory for Electromagnetic and Electronic Systems. His research interests include the development of stochastic algorithms for important deterministic equations in nature and a theory for high temperature superconductors.