

Genetic Algorithm Applications for Phased Arrays

(Invited Paper)

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Abstract—Analytical approaches to phased array optimization started in the mid 1940s and transitioned to numerical techniques that can find a local minimum. Computers spurred the development of many different local optimization algorithms that worked well for a few variables and a cost function with a single minimum. In the 1990s, the genetic algorithm (GA) emerged as a competent optimization algorithm for a wide range of complex cost functions. This paper reviews phased array optimization and lays the foundation for the use of the GA. An extensive reference list is provided and some future research areas are discussed.

Index Terms—Phased arrays, genetic algorithms, arrays, optimization

I. INTRODUCTION

The introduction of genetic algorithms (GAs) to engineering produced a revolution in the design of complex systems. Over the past ten years, GAs moved from arcane toys of computer scientists to mainstay numerical optimization algorithms. Their popularity in phased array antenna design is apparent by the large number of papers published in this area as of the submission of this paper (see [1] to [139]).

This paper begins with a historical development of phased array optimization and a demonstration of a few cost functions. Next, some GA details are presented with a list of advantages over traditional optimization techniques. Finally, a review of GA applications to phased arrays is given and wrapped up with some ideas of where the future lies. A major contribution of this paper is the extensive reference list and categorizing of the references.

II. PHASED ARRAY OPTIMIZATION

The cost function for most phased array optimization is based on the array factor that includes the relative position and weightings of all the elements. An arbitrary array of N elements in three-dimensional space has an array factor given by

$$AF = \sum_{n=1}^N w_n e^{jk[x_n \sin \theta \cos \phi + y_n \sin \theta \sin \phi + z_n \cos \theta]} \quad (1)$$

where

$w_n = a_n e^{j\delta_n}$ = complex weight at element n

a_n = amplitude weight at element n

δ_n = phase weight at element n

(x_n, y_n, z_n) = location of element n

θ = elevation angle

ϕ = azimuth angle

The array designer controls the array factor via the amplitude weights, the phase weights, and/or the element locations in order to meet performance specifications, such as sidelobe levels, beamwidth, nulls, and bandwidth.

Initially, analytical optimization methods were used to find low sidelobe array amplitude weights. The first optimum antenna array distribution was the binomial distribution proposed by Stone [140]. As is now well known, the amplitude weights of the elements in the array correspond to the binomial coefficients, and the resulting array factor has no sidelobes. Dolph mapped the Chebychev polynomial onto the array factor polynomial to get all the sidelobes at an equal level [141]. The Dolph-Chebychev amplitude distribution is optimum in that specifying the maximum sidelobe level results in the smallest beam width, or specifying the beam width, results in the lowest possible maximum sidelobe level. Nine years later, Taylor developed a method to optimize the sidelobe levels and beam width of a line source [142]. Bayliss used a method similar to Taylor's amplitude taper but applied to a monopulse difference pattern [143]. The Taylor and Bayliss tapers are routinely used for low sidelobe arrays. Elliot extended Taylor's work to new horizons including Taylor based tapers with asymmetric sidelobe levels, arbitrary sidelobe level designs, and null free patterns [144].

Analytical approaches to finding optimum array amplitude weights are still used today. They work well, because the unknown array weights are coefficients of a

complex Fourier series. If the unknowns are the element spacings or element phases, then they appear in the complex exponent and are not easily found. Checking all combinations of values of the array variables is not realistic unless the number of variables is small. Optimizing one variable at a time does not work nearly as well as following the gradient vector downhill. The steepest descent method, invented in the 1800's, is based on this concept and is still widely used today. Newton's method uses second derivative information in the form of the Hessian matrix to find the minimum. Although more powerful than steepest descent, calculating the second derivative of the cost function may be too difficult.

In order to avoid the calculation of derivatives, Nelder and Mead introduced the downhill simplex method in 1965 [145]. This technique has become widely used by commercial computing software. A simplex has $n+1$ sides in n -dimensional space. Each iteration generates a new vertex for the simplex. If the new point is better than the worst vertex, then the new point replaces the worst vertex. In this way, the diameter of the simplex gets smaller until it reaches a specified tolerance.

Also during the mid 1960s, successive line minimization methods were developed. A successive line minimization algorithm begins at a random point, chooses a direction to move, then moves in that direction until the cost function begins to increase. The procedure is then repeated in a new direction. A conjugate direction is a new direction that does not interfere with the minimization of the prior direction. The conjugate directions are chosen so that the change in the gradient of the cost function remains perpendicular to the previous direction. Powell devised an efficient way to specify the conjugate directions [146]. If there is additional information on the gradient of the cost function, the conjugate gradient method can be applied. This method simply uses this gradient information to choose the conjugate directions. An even better set of directions can be chosen if the matrix of second partial derivatives, the Hessian matrix, is known. The BFGS algorithm [146]. finds a way to approximate this matrix and employs it in determining the appropriate directions of movement. This algorithm is "quasi-Newton" in that it is equivalent to Newton's method for prescribing the next best point to use for the iteration, yet it doesn't use an exact Hessian matrix. Quadratic programming assumes the cost function is quadratic (variables are squared) and the constraints are linear. This technique is based upon Lagrange multipliers and requires derivatives or approximations to derivatives [147].

Numerical optimization has been used to find nonuniform element spacings, complex weights, and

phase tapers that resulted in desired antenna patterns. Some examples of nonuniform spacing synthesis include dynamic programming [148], Nelder Mead downhill simplex algorithm [149], steepest descent [150], and simulated annealing [151]. Numerical methods were used to iteratively shape the main beam while constraining sidelobe levels for planar arrays [152], [153], and [154]. Linear programming [155] and the Fletcher-Powell method [156] were applied to optimizing the footprint pattern of a satellite planar array antenna. Quadratic programming was used to optimize aperture tapers for various planar array configurations [157] and [158]. Numerical optimization was used to find phase tapers that maximized the array directivity [159], and a steepest descent algorithm used to find the optimum phase taper to minimize sidelobe levels [160].

The numerical optimization algorithms mentioned so far find a minimum in a valley of the cost function closest to the starting point. In other words, the convergence of the algorithm assumes the cost function is quadratic or bowl shaped with a single minimum. The next section gives a few examples of phased array cost functions that need non-local optimization techniques to find the best minimum.

III. PHASED ARRAY COST FUNCTIONS

The cost function for a phased array antenna can be quite complex, so the array factor is often optimized rather than a full wave computational electromagnetics model. Using point sources allows for the modeling of a large number of elements but ignores polarization, mutual coupling, environmental scattering, and other effects. Often, optimizing the array factor provides sufficient design information.

As an example, consider finding the minimum maximum sidelobe level by either adjusting the amplitude weights, element spacing, or phase weights of a linear array that lies along the x -axis and has dipoles parallel to the y -axis (Fig. 1). The spacing, amplitude weights, and phase weights are symmetric with respect to the center of the array. In order to visualize the cost surface, only two variables can be used. Figure 2 is the cost function when the amplitude weights are the optimization variables with limits $0.1 \leq a_{2,3} \leq 1.0$, and $\delta_{1,2}$ and $x_1 = 0.25 \lambda$, $x_2 = 0.75 \lambda$, and $x_2 = 1.25 \lambda$. The cost surfaces for the dipole model and array factor look very similar implying that element location is more important than coupling. Fig. 3 is the cost function when $a_{2,3} = 1.0$ and $\delta_{1,2} = 0$, and the element spacings are bound by $x_1 = 0.25 \lambda$, $x_2 = 0.25 \lambda + \Delta_2$, and $x_3 = 0.25 \lambda + \Delta_2 + \Delta_3$. As with the amplitude weights, the cost surfaces for the dipole model and array

factor look very similar. Fig. 4 is the cost function when $a_{2,3} = 1.0$, $0 \leq \delta_{1,2} \leq \pi$, and $x_1 = 0.25 \lambda$, $x_2 = 0.75 \lambda$, and $x_3 = 1.25 \lambda$. Again, the cost functions for the dipoles and point sources are similar.

All the cost functions in these figures have ridges, narrow valleys, and dramatic variations in slope. The cost surface variations will slow the convergence of most minimization algorithms. Speed of convergence is highly dependent upon the starting point on the cost surface. For the six element case, the minimization algorithms mentioned so far will find the true minimum most of the time. On the other hand, adding more array variables dramatically increases the complexity of the cost surface and renders many "local" optimizers powerless to find a good minimum.

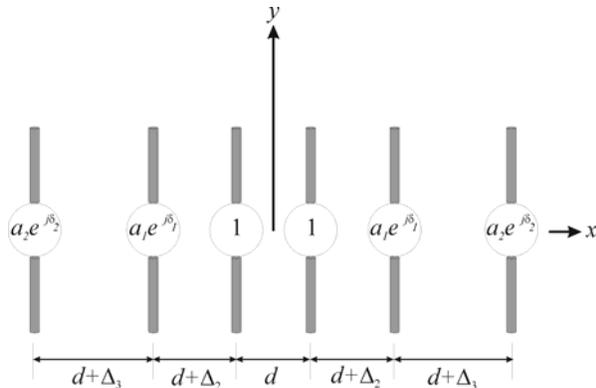


Fig. 1. Diagram of the array that generates the cost functions.

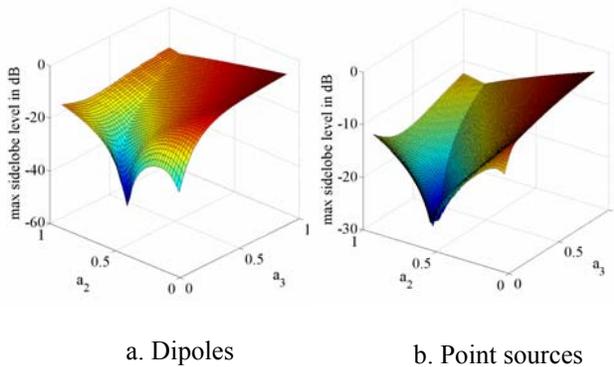
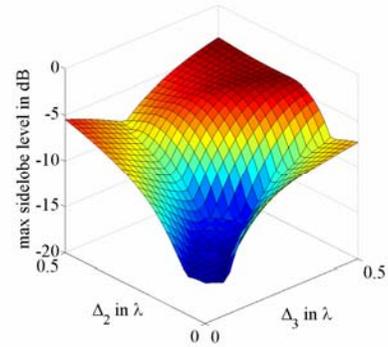
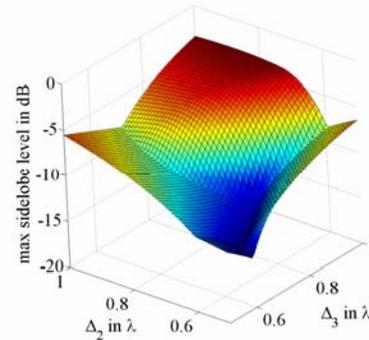


Fig. 2. Cost surface associated with varying the amplitude weights of the six element array.

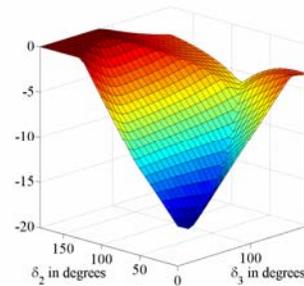


a. Dipoles

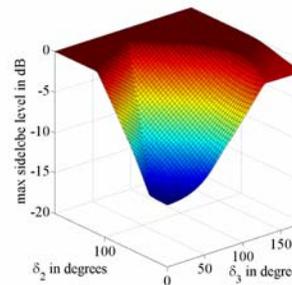


b. Point sources

Fig. 3. Cost surface associated with varying the spacing of the six element array.



a. Dipoles



b. Point sources

Fig. 4. Cost surface associated with varying the phase weights of the six element array.

chronological order from [1] to [139]. The author tried to include all papers dealing with GA applications to phased arrays. Apologies are made to those authors

whose papers were missed. Rather than trying to summarize the research done, readers can look at the references listed under a given topic.

Table 1. References to GA applications for phased arrays are categorized.

Topic	Reference
Array synthesis	[4][11][14][18][21][24][25][26][27][30][37][44][52][53][56][70][71][74][84][85][87][101][109][114][123][126][128][138]
Nulling	[3][19][23]
adaptive/smart arrays	[10][20][22][34][35][40][42][46][50][57][66][75][86][96][105][106][112][117][127][130][134]
Subarray	[7][76][98][118]
Element failures	[38][48][68][115][116]
Mutual coupling	[41][47][51][54][63][64][65][72][80][81][83][94][102][111][124][136]
Multiple beams	[9][49]
Shaped beam synthesis	[28][29][36][39][60][93][103][113][133]
Phase taper	[8][78]
GA combined and other methods	[12][15][16][73][88][89][100][107][115][121][122][129]
Conformal arrays	[42][81][95][98]
GA parameters	[45][55][58][61][90][99][108][132]
Ring arrays	[33][43][125]
Aperiodic arrays	[6][31][32][59][79][92][110][119][120][137]
Direction of arrival arrays	[69][102]
Beam scanning	[97][104][135]
Planar arrays	[62][90][130][131]
Multiple objective optimization	[17][77][98]
Thinned arrays	[1][2][5][13][67][72][81][139]

VI. GA FRONTIERS

The biggest hurdle for GAs is the time needed to find a good optimum solution. At this point, we have a powerful optimization algorithm that can create new designs, but computers and software models that are too slow for the cost function evaluations. Evaluating the cost function quickly and accurately can be done in three ways:

1. *Faster GAs.* Finding the optimum parameters such as population size and mutation rate can make orders of magnitude difference in the number of function calls needed to find an acceptable solution. Adaptively changing the parameters may be very helpful. Hybrid approaches that combine the GA and other approaches, especially local optimizers need more exploration. GAs can produce better and faster results with human input during the operation of the algorithm. Humans can even be used to create a subjective cost associated with a phased array design. Operator bias in the cost function may be as valuable as the mathematical equations in the model.
2. *Faster cost functions.* Using fast, approximate function evaluations in early generations and

converting to slow, more exact function evaluations in later generations has some promise. Efficient hybrid methods can make large problems tractable. Eliminating unimportant variables helps optimization algorithms converge faster. Some costs are more sensitive than others. Some costs require a finer grid for sufficient accuracy than other costs. Adaptively adjusting the grid size can result in significant time savings.

3. *Faster computers.* Clock speed and memory are bottlenecks for large complex optimization problems. GAs are ideally suited for parallel processing, since the cost function for each chromosome can be evaluated simultaneously.

A phased array consists of more than just the antenna elements. Optimization of the feed structure, the active components, the component costs, etc. are possible with a GA. The design of wideband feed networks to match wideband elements is an important step in developing wideband phased arrays. The GA has certainly advanced the design of complex phased arrays. As can be seen in the references, GAs are becoming more accepted as a design tool.

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