INTRODUCTION TO THE METHOD OF MOMENTS TECHNIQUE

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Background

This article provides a brief introduction to the Method of Moments (MoM) technique. There are many articles and books written on this topic. This is intended to provide a quick overview for a reader interested in understanding the basic technique. For a more complete description of MoM the reader is referred to the many publications available.

The technique described here is more properly called the Boundry Element Technique (BEM), since the Method of Moments is a broader term and includes many techniques called by othe rnames. While the term "Method of Moments" is commonly used to mean the Boundry Element Method, to avoid confusion, the Method of Moments term is used here.

The Method of Moments technique (also called Moment-method technique) have become very popular in the past 30 years or so. MoM algorithms are generally run on high speed workstation or mainframe computers, but they can be used to model a wide variety of problems without requiring the user to assume a particular current distribution. MoM is used extensively to model radar cross section and antenna applications, and has recently been applied to EMI/EMC problems.

The structure to be modeled is converted into a series of metal plates and wires. In fact, often a solid structure is converted into a wire frame model, eliminating the metal plates. Once the structure is defined, the wires are broken into wire segments (short compared to a wavelength so the assumption of constant current on that segment is valid) and the plates are divided into patches (small compared to a wavelength so the assumption of constant current on that patch is valid). From this structure, a set of linear equations is created. The solution to this set of linear equations is the RF currents on each wire segment and surface patch. Once the RF current is known for each segment and patch, the electric field at any point in space can be determined by solving for each segment/patch and performing the vector summation.

The Basics of MoM

MoM actually refers to a general procedure for solving linear mathematical equations of the form,

$$L(\vec{f}) = \vec{E} \qquad (1)$$

where L is a linear operator f is an unknown response and E is a known excitation. [1][2][3] For electromagnetic modeling, the known excitation is usually an imposed electric or magnetic field, and the unknown response is generally a current distribution. Once the currents are known everywhere on the structure, the electric and magnetic fields can be found at any point in space. The equation relating the currents and fields is known as the electric field integral equation (EFIE) when the know excitation is an electric field or the magnetic field integral equation (MFIE) when the excitation is a magnetic field. The relative usefulness and accuracy of a particular MoM model depends, in part, on the assumptions made in the process of deriving the integral equation and whether EFIE or MFIE is used. For EMI/EMC modeling, the source is usually an applied voltage (modeled as an electric field across a short distance), so the EFIE is typically used.

The first step in the MoM solution process is to describe the unknown response (in this case the current distribution) as a finite sum of basis functions,

$$\vec{f} = \sum_{j=1}^{N} \alpha_j \vec{f}_j \qquad (2)$$

where: f_j = the j^{th} basis function
α_j = unknown coefficient

Using this approximation the entire current distribution can now be solved by finding the values for the N coefficients, α_j.

The second step in the MoM procedure is to define a set of weighting functions, w_i, which may or may not be the same as the basis functions. Defining the inner product as,

$$\langle A, B \rangle \equiv \oint_{s} (A \bullet B)\, ds \qquad (3)$$

where s is the entire surface on which A and B are defined. We can then take the inner product of (1) with each of the chosen weighting functions,

$$\langle \vec{w}_i, L(\vec{f}) \rangle = \langle \vec{w}_i, \vec{E} \rangle \quad i = 1,2,\ldots N \qquad (4)$$

Using the linearity of L and making the substitution in equation (2),

$$\sum_{j=1}^{N} \alpha_j \langle \vec{w}_j, L(\vec{f}_j) \rangle = \langle \vec{w}_i, \vec{E} \rangle \qquad (5)$$

Setting:

$$\vec{Z}_{ij} = \langle \vec{w}_i, L(\vec{f}_j) \rangle$$
$$\vec{I}_j = \alpha_j$$
$$\vec{E}_j = \langle \vec{w}_i, \vec{E} \rangle$$

And converting to matrix notation:

$$[\vec{Z}][\vec{I}] = [\vec{E}] \qquad (6)$$

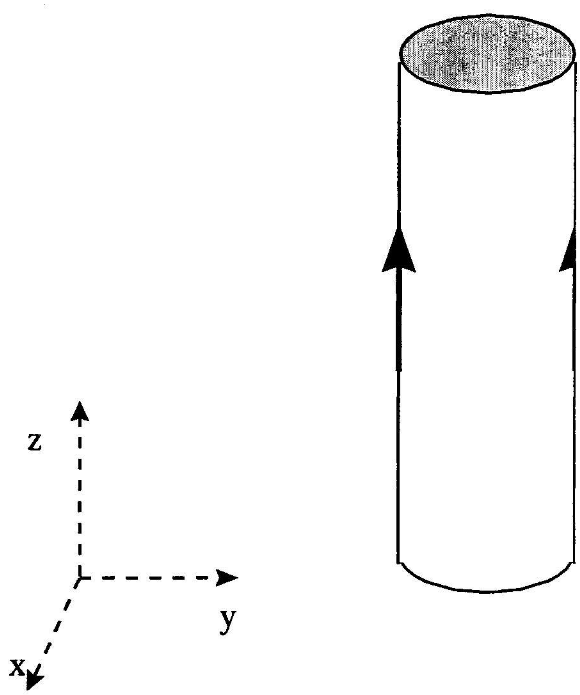

Figure 1 MoM Geometry for Current on a Wire Segment

The only unknown quantity in equation (6) is I, which is a vector containing the N coefficients describing the current distribution. Provided the matrix Z is not singular (that is, the problem has a unique solution), it is possible to solve for I in equation (6) as shown in equation (7).

$$[\vec{I}] = [\vec{Z}]^{-1}[\vec{E}] \qquad (7)$$

Therefore, the current on a particular segment in the model is found from the contribution of the source, and the contribution from all other currents in the model. Equation (7) represents a set of N linear equations, with N unknowns.

Filling the Impedance Matrix

Once the basics of MoM are understood, the next task is to fill the impedance matrix, then the MoM solution simply requires inverting [Z] and a matrix multiplication with the source. Although there are a number of different formulations to find [Z], this work used a thin-wire formulation called Pocklington's integral equation. In general, the electric field from a current is given by

$$\vec{E}(r) = -j\omega\,\vec{A} - j\frac{1}{\omega\mu\varepsilon}\nabla(\nabla \bullet \vec{A}) = -j\frac{1}{\omega\mu\varepsilon}\left(\omega^2\mu\varepsilon\,\vec{A} + \nabla(\nabla \bullet \vec{A})\right) \qquad (8)$$

where:

$$\vec{A}(r) = \mu \iint_S \vec{J}_S(r')\frac{e^{-j\beta R}}{4\pi R}ds' \qquad (9)$$

However, since the observation of the current is only along the surface, and only the z component is needed (see Figure 1) equation (8) is re-written as:

$$E_z(r) = -j\frac{1}{\omega\mu\varepsilon}\left(\beta^2 A_z + \frac{\partial^2 A_z}{\partial z^2}\right) \qquad (10)$$

and with only the z component equation (9) becomes:

$$A_z = \frac{\mu}{4\pi}\int_{-l/2}^{l/2}\int_0^{2\pi} J_z \frac{e^{-j\beta R}}{R} a\, d\phi'\, dz' \qquad (11)$$

since the wire is assumed to be thin (λ>>a), the current density J_z is not a function of the azimuth angle φ, and the current density becomes:

$$J_z = \frac{1}{2\pi a} I_z(z') \qquad (12)$$

where I_z is assumed to be an equivalent filament line current located on the wire segment's surface. Therefore,

$$A_z = \frac{\mu}{4\pi}\int_{-l/2}^{l/2}\left[\frac{1}{2\pi a}\int_0^{2\pi}\frac{e^{-j\beta R}}{R} a\, d\phi'\right] dz' \qquad (13)$$

$$R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$
$$R = \sqrt{\rho^2 + a^2 - 2\rho\, a\cos(\phi-\phi') + (z-z')^2} \qquad (13a)$$

where ρ is the radial distance to the observation point and a is the radius.

Since the wire segment is symmetrical about φ, the observation of the current is not a function of φ, so φ=0 will be substituted into equation (13). Since the observation of the current is on the surface of the wire, ρ=a, and equation (13) becomes,

$$A_z(\rho = a) = \mu \int_{-l/2}^{l/2} I_z(z')\, G(z,z')\, dz' \qquad (14)$$

$$G(z,z') = \frac{1}{2\pi}\int_0^{2\pi}\frac{e^{-j\beta R}}{4\pi R} d\phi' \qquad (14a)$$

$$R(\rho = a) = \sqrt{4a^2 \sin^2\left(\frac{\phi'}{2}\right) + (z-z')^2} \qquad (14b)$$

where equation (14a) is a Green's Function. Combining equations (10) and (14), the electric field is

$$E_z = -\frac{1}{j\omega\varepsilon} \int_{-l/2}^{l/2} I(z') \left[\left(\frac{d^2}{dz^2} + \beta^2 \right) G(z,z') \right] dz' \qquad (15)$$

Equation (15) is referred to as Pocklington's integrodifferential equation [4], and it is used to determine the current along the surface of the wire, given an incident electric field.

As mentioned earlier, the selected weighting function varies depending upon the requirements for the modeling. If the segment length is kept electrically small (1/10th wavelength or shorter), then the point-matching method is commonly used for a weighting function. That is, the current is assumed to only exist on the center of the wire segment as a delta function. Since

$$\int_{-\infty}^{\infty} I_z \cdot \delta(z = z_0) dz = I_z(z = z_0) = I_{z=z0} \qquad (16)$$

equation (15) can be reduced to

$$E_z = -\frac{1}{j\omega\varepsilon} I(z'_{z0}) \int_{-l/2}^{l/2} \left[\left(\frac{d^2}{dz^2} + \beta^2 \right) G(z,z') \right] dz' \qquad (17)$$

Also, since we assume the wire segment is very thin (a<<λ), then equation (14a) reduces to

$$G(z,z') = G(R) = \frac{e^{-j\beta R}}{4\pi R} \qquad (18)$$

Recall in Equation (6) the electric field was a function of the unknown current and the impedance. Equation (17) fills this function, and the impedance matrix can be created, allowing the currents to be found.

Finding the Electric Fields from the RF Currents

Once the currents are known, the electric and magnetic fields can be found at any point in space (due to those currents), by using the Herzian dipole equations. The wire segments must be electrically short (λ>>L), but this requirement is necessary for the MoM technique and is already implemented. The general electric fields can be found using Equations (19) and (20).

$$E_r = \frac{IL\cos\theta}{2\pi\varepsilon_0} \left(\frac{1}{cr^2} + \frac{1}{j\omega r^3} \right) \qquad (19)$$

$$E_\theta = \frac{IL\sin\theta}{4\pi\varepsilon_0} \left(\frac{j\omega}{c^2 r} + \frac{1}{cr^2} + \frac{1}{j\omega r^3} \right) \qquad (20)$$

Summary

The MoM technique is a fairly straightforward and intuitive technique. Creating a wire structure, finding the current distribution over the entire structure, and then finding the fields due to those currents is intuitive to most engineers. The MoM technique requires the creation of a system of N linear equations with N unknowns, where each unknown is the current on a single segment. The unknown currents are solved by using matrix techniques. Once the current is known, the fields can be found using the Herzian dipole equations.

References

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[3] T.H. Hubing, "Modeling the Electromagnetic Radiation from Electrically Small sources with Attached Wires," Ph.D. Dissertation, North Carolina State University, 1988

[4] C.A. Balanis, *Advanced Engineering Electromagnetics*, Wiley, 1989, Sect. 12.4.1.