An Unconditionally Stable Subcell Model for Thin Wires in the ADI-FDTD Method

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Abstract - In this paper, a method for the implementation of thin wire with the alternating-direction implicit finite-difference time-domain (ADI-FDTD) method is discussed. The magnetic field around the wire is modified according to the "standard" subcell model, which results in the modification to the tridiagonal matrix system in the ADI-FDTD. The perfect-electric-conductor (PEC) condition is directly incorporated within the tridiagonal matrix equation. This method is efficient, stable, and suitable to ADI-FDTD scheme with large time step size. The validity of this method is confirmed through numerical examples.

Index Terms - ADI-FDTD method, PEC condition, subcell model, thin wire.

I. INTRODUCTION

The alternating-direction-implicit finite-difference time-domain (ADI–FDTD) method is an unconditionally stable alternative to the standard fully explicit FDTD method [1-5]. The main advantage of the ADI-FDTD method is that the Courant–Friedrich–Levy (CFL) condition that is always required by the conventional FDTD method can be totally removed. Hence, the ADI-FDTD method is extremely useful for problems where a very fine mesh is needed over a large geometric area. Nevertheless, from the implementation point of view, the ADI-FDTD method is more complicated than the conventional FDTD method. This is because the field components of the conventional FDTD method can be directly updated, but in the ADI-FDTD method they have to be implicitly updated by solving a tridiagonal matrix system.

In many electromagnetic problems analyzed numerically with the FDTD method, thin wires need to be modeled. A wire is considered thin when its diameter is less than the selected mesh size. It is certainly possible to select a sufficiently small mesh, so that the wire diameter occupies one or more computational cells, but this approach often results in a very fine discretization and excessive computational resources. As an alternative, a "standard" subcel1 model which is based on the near-field physics can be used [6]. This model assumes that the circumferential component of the magnetic field and the radial component of the electric field vary as 1/r near the wire, where r is the radial distance from the wire axis. In such a case, a modification to the standard FDTD algorithm is easily available. However, the modification to the ADI-FDTD algorithm is complicated, due to its implicit calculation by solving a tridiagonal matrix system.

$$H_x^{n+\frac{1}{2}}\left(i_0, j_0 \pm \frac{1}{2}, k+\frac{1}{2}\right) = H_x^n\left(i_0, j_0 \pm \frac{1}{2}, k+\frac{1}{2}\right) + \frac{\Delta t}{2\mu\Delta z}\left[E_y^n\left(i_0, j_0 \pm \frac{1}{2}, k+1\right) - E_y^n\left(i_0, j_0 \pm \frac{1}{2}, k\right)\right]$$
$$-\frac{\Delta t}{2\mu\Delta y}\left(\frac{2}{\ln\left(\Delta y/r_0\right)}\right)\left[E_z^{n+\frac{1}{2}}\left(i_0, j_0 + \frac{1}{2} \pm \frac{1}{2}, k+\frac{1}{2}\right) - E_z^{n+\frac{1}{2}}\left(i_0, j_0 \pm \frac{1}{2} - \frac{1}{2}, k+\frac{1}{2}\right)\right], \ \left(k_1 \le k \le k_2\right) \quad (1)$$

$$E_z^{n+\frac{1}{2}}\left(i_0, j_0, k+\frac{1}{2}\right) = E_z^n\left(i_0, j_0, k+\frac{1}{2}\right) + \frac{\Delta t}{2\varepsilon\Delta x}\left[H_y^n\left(i_0+\frac{1}{2}, j_0, k+\frac{1}{2}\right) - H_y^n\left(i_0-\frac{1}{2}, j_0, k+\frac{1}{2}\right)\right]$$
$$-\frac{\Delta t}{2\varepsilon\Delta y}\left[H_x^{n+\frac{1}{2}}\left(i_0, j_0+\frac{1}{2}, k+\frac{1}{2}\right) - H_x^{n+\frac{1}{2}}\left(i_0, j_0-\frac{1}{2}, k+\frac{1}{2}\right)\right] \quad (2)$$

$$\left[1+\frac{s_1\Delta t^2}{2\varepsilon\mu\Delta y^2}\right]E_z^{n+\frac{1}{2}}\left(i_0, j_0, k+\frac{1}{2}\right) - \frac{s_1\Delta t^2}{4\varepsilon\mu\Delta y^2}\left[E_z^{n+\frac{1}{2}}\left(i_0, j_0+1, k+\frac{1}{2}\right) + E_z^{n+\frac{1}{2}}\left(i_0, j_0-1, k+\frac{1}{2}\right)\right]$$
$$= E_z^n\left(i_0, j_0, k+\frac{1}{2}\right) + \frac{\Delta t}{2\varepsilon\Delta x}\left[H_y^n\left(i_0+\frac{1}{2}, j_0, k+\frac{1}{2}\right) - H_y^n\left(i_0-\frac{1}{2}, j_0, k+\frac{1}{2}\right)\right]$$
$$-\frac{\Delta t^2}{4\mu\varepsilon\Delta y\Delta z}\left[E_y^n\left(i_0, j_0+\frac{1}{2}, k+1\right) - E_y^n\left(i_0, j_0-\frac{1}{2}, k+1\right) - E_y^n\left(i_0, j_0+\frac{1}{2}, k\right) + E_y^n\left(i_0, j_0-\frac{1}{2}, k\right)\right]$$
$$-\frac{\Delta t}{2\varepsilon\Delta y}\left[H_x^n\left(i_0, j_0+\frac{1}{2}, k+\frac{1}{2}\right) - H_x^n\left(i_0, j_0-\frac{1}{2}, k+\frac{1}{2}\right)\right] \quad (3)$$

This paper presents the implementation of the thin wires for the ADI-FDTD method. The magnetic field around the wire is modified according to the "standard" subcell model, which results in the modification to the tridiagonal matrix system. For the perfect conductor boundary condition, the tangential electronic field component along the wire needs to be set to zero, which is directly incorporated within the tridiagonal matrix system. This method is unconditionally stable and has high accuracy. The theory proposed in this article is validated through numerical examples.

II. FORMULATIONS

The field components around thin wire are shown in Fig. 1. r_0 is the radius of the wire. i_0 and j_0 denote the indices of spatial increments of thin wire. Indices $k_1 \le k \le k_2$ denote the height of the wire.

In the ADI-FDTD method, the calculation for one discrete time step is performed using two procedures. According to the "standard" subcell model [3], the numerical formulations of the $E_z^{n+1/2}$, $H_x^{n+1/2}$ components near the thin wire in the first procedure for the ADI-FDTD method are given in eqs. (1) and (2) above. Obviously, updating of $E_z^{n+1/2}$ component, as shown in eq. (2), needs the unknown $H_x^{n+1/2}$ component at the same time; thus the $E_z^{n+1/2}$ component has to be updated implicitly. By substituting eq. (1) into eq. (2), the equation for $E_z^{n+1/2}$ field is given in eq. (3), where $s_1 = 2/\ln\left(\Delta y/r_0\right)$. Thus, $E_z^{n+1/2}$ fields along a particular y-directed line ($i = i_0$) are updated simultaneously by solving the tridiagonal matrix equation through eq. (4) and repeated for each k ($k_1 \le k \le k_2$), where $\chi = \Delta t^2/4\varepsilon\mu\Delta y^2$, $E_{z,j_0}^{n+1/2}$ is the unknown z-direction electronic field, r_{j_0} is the right side of eq. (3). For other k value ($k < k_1$ or $k > k_2$), and other i value ($i \ne i_0$), standard ADI-FDTD formulation is applied.

$$
j_0 \rightarrow
\begin{bmatrix}
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & 1+2\chi & -\chi & 0 & 0 & 0 & \cdots \\
\cdots & -\chi & 1+\chi+s_1\chi & -s_1\chi & 0 & 0 & \cdots \\
\cdots & 0 & -s_1\chi & 1+2s_1\chi & -s_1\chi & 0 & \cdots \\
\cdots & 0 & 0 & -s_1\chi & 1+s_1\chi+\chi & -\chi & \cdots \\
\cdots & 0 & 0 & 0 & -\chi & 1+2\chi & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{bmatrix}
\begin{bmatrix}
\cdots \\ E^{n+1/2}_{z,j_0-2} \\ E^{n+1/2}_{z,j_0-1} \\ E^{n+1/2}_{z,j_0} \\ E^{n+1/2}_{z,j_0+1} \\ E^{n+1/2}_{z,j_0+2} \\ \cdots
\end{bmatrix}
=
\begin{bmatrix}
\cdots \\ r_{j_0-2} \\ r_{j_0-1} \\ r_{j_0} \\ r_{j_0+1} \\ r_{j_0+2} \\ \cdots
\end{bmatrix}
\quad (4)
$$

$$\uparrow j_0-1 \quad \uparrow j_0 \qquad \uparrow j_0+1$$

$$
\begin{bmatrix}
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & 1+2\chi & -\chi & 0 & 0 & \cdots \\
\cdots & -\chi & 1+\chi+s_1\chi & -s_1\chi & 0 & \cdots \\
\cdots & 0 & -s_1\chi & 1+s_1\chi+\chi & -\chi & \cdots \\
\cdots & 0 & 0 & -\chi & 1+2\chi & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{bmatrix}
\begin{bmatrix}
\cdots \\ E^{n+1/2}_{z,j_0-2} \\ E^{n+1/2}_{z,j_0-1} \\ E^{n+1/2}_{z,j_0+1} \\ E^{n+1/2}_{z,j_0+2} \\ \cdots
\end{bmatrix}
=
\begin{bmatrix}
\cdots \\ r_{j_0-2} \\ r_{j_0-1} \\ r_{j_0+1} \\ r_{j_0+2} \\ \cdots
\end{bmatrix}
\quad (5)
$$

$$\uparrow j_0-1 \qquad \uparrow j_0+1$$

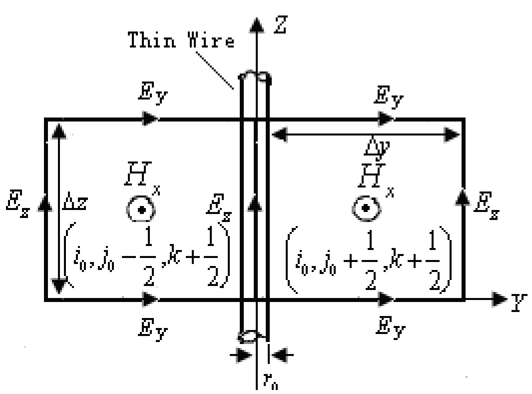

Fig. 1. Field components around a thin wire ($k_1 \le k \le k_2$).

The thin wire is often seen as perfect electric conductor. The tangential electronic field value E_z along the wire must be set to zero. So in the matrix systems (4), the values of $E^{n+1/2}_{z,j_0}$ are not calculated. Then the matrix system (4) becomes (5) above.

Thus, components $E_z^{n+1/2}$ are updated by solving the modified tridiagonal matrix system through Eqs. (5). This ADI-FDTD method incorporates the PEC condition into the matrix system. It seems to be complicated, but it is unconditionally stable and has higher accuracy. If we set the tangential electric field values $E_z^{n+1/2}$ to be zeros directly after solving the tridiagonal matrix system (4), the ADI-FDTD method is unstable, even for very small time step size, which is validated through numerical examples in the next section.

In the second procedure, components E_z^{n+1}, H_y^{n+1} can similarly be treated as $E_z^{n+1/2}$ and $H_x^{n+1/2}$ in the first procedure, which is not shown here due to space limitation.

III. NUMERICAL VALIDATION

To validate the theory presented in this paper, a simple numerical simulation is studied. A thin wire of length 10 cm and radius of 1 mm is embedded in a shielding enclosure, as shown in Fig. 2. The physical dimension of the enclosure is $15cm \times 15cm \times 15cm$. In the middle of the enclosure, a small current source is applied along the z direction. The time dependence of the excitation function is:

$$g(t) = \exp[-\alpha(t-t_0)^2] \quad (6)$$

where α and t_0 are constants, and equal to $1.26\times10^{10}\,s^{-2}$ and 1.0×10^{-9} s, respectively. In such a case, the highest frequency of interest is 2 GHz.

The space increments used in the FDTD simulation are $\Delta x=\Delta y=\Delta z=5mm$. The total mesh size is $30\times30\times30$. Observation point A is 5 cells diagonally far from the source. We apply the ADI-FDTD technique to compute the time domain electric field at the observation point. The time step is $\Delta t = 1\big/c\sqrt{(1/\Delta x)^2+(1/\Delta y)^2+(1/\Delta z)^2}$ = 9.62ps, which is the maximum time step to satisfy the limitation of the 3D CFL condition in the conventional FDTD method. For simplicity, both the thin wire and the enclosure are perfect electric conductors.

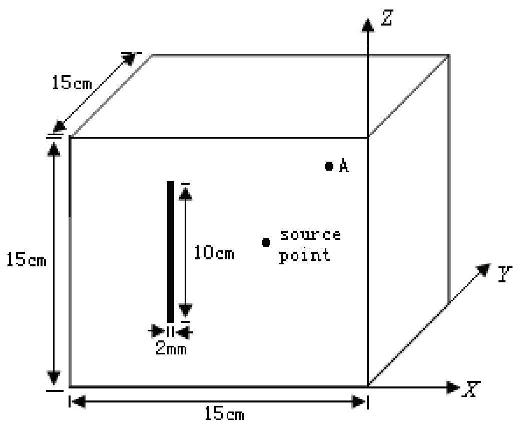

Fig. 2. Geometric configuration of the numerical simulation.

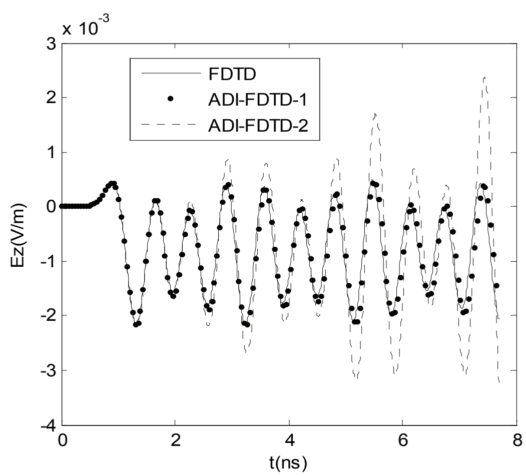

Fig. 3. The electric field component E_z at observation point A calculated by different methods.

To ensure the ADI-FDTD method be symmetric up to the numerical noise level, the excitation field should be directly incorporated within the tridiagonal matrix and the time discretization of the source is done appropriately within each full time step [7].

Figure 3 gives the results of the electric field component E_z at observation point A calculated by ADI-FDTD- 2 methods for large number of time steps.

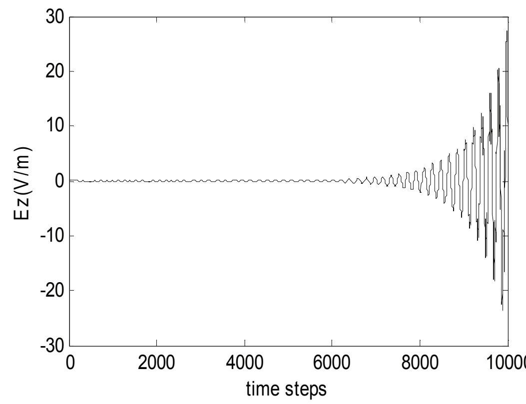

Fig. 4. The electric field component E_z at observation point A calculated by ADI-FDTD-2 methods for large number of time steps.

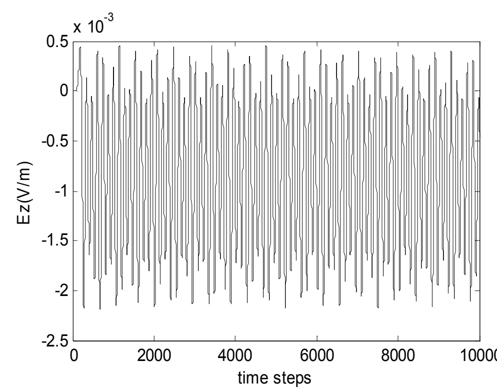

Fig. 5. The electric field component E_z at observation point calculated by ADI-FDTD -1 methods for large number of time steps.

The ADI-FDTD-1 denotes the method that incorporates the PEC condition into the matrix system. The ADI-FDTD-2 denotes the method that set the tangential electric field values E_z to be zeros directly after solving the tridiagonal matrix system. For comparison, the result obtained by the conventional FDTD method is also plotted in this figure. In all the methods, the time step sizes are 9.62ps. It can be seen from this figure that the results calculated by ADI-FDTD-1 agree well with the results obtained by the conventional FDTD, which shows that the ADI-FDTD-1 method has higher accuracy. The value calculated by ADI-FDTD-2 deviate from that of conventional FDTD significantly. It is apparent that ADI-FDTD-2 can't obtain correct results.

To confirm the stability of these two methods, the ADI-FDTD program is tested for a long time history. Figure 4 shows the result of the electric field component E_z at observation point A calculated by ADI-FDTD-2 method for large number of time steps, the time step size is 9.62ps. It can be seen from this figure that the result of method 2 starts to be unstable after 6,000 time steps. To confirm the stability of method 1, the ADI-FDTD program is tested for 10,000 time steps with time step size 384.80ps which is 40 times as that of conventional FDTD method. No instability problem is observed. This illustrates that the ADI-FDTD scheme based on the method that incorporates the PEC condition into the matrix system is unconditionally stable, and the programs based on the method that set the tangential electric field values E_z to be zeros directly are not stable, even for small number of time steps.

IV. CONCLUSION

Based on the assumption of a 1/r dependence of the magnitude of local fields with radial distance r from the wire center, the magnetic field around the thin wire is modified, which results in the modification to the tridiagonal matrix system in the ADI-FDTD method. Meanwhile, due to the implicit calculation of the ADI-FDTD method, the perfect conductor boundary condition must be incorporated within the tridiagonal matrix system. The matrix needs to cross out the rows and columns for which the value doesn't need to be calculated. This method is unconditionally stable and has higher accuracy, which is validated through numerical examples. It needs to be noted that the stability of the ADI-FDTD-1 method is numerically verified rather than theoretically proven. The theoretical analysis of the stability is under research and will be reported in a future publication.

This unconditionally stable subcell model can be used to calculate the induced currents on the coupled wires in cavity, simulate the inner conductor of the coaxial feed, and analyze the radiation from the shielding cable, thus it has important effects on the electromagnetic numerical simulations.

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