A NUMERICAL TECHNIQUE TO DETERMINE ANTENNA PHASE CENTER LOCATION FROM COMPUTED OR MEASURED PHASE DATA

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ABSTRACT

In many antenna applications, it is a requirement to have knowledge of the antenna phase center properties. Many interpretations of antenna phase center and its apparent location exist. This paper will present a formulation for determination of antenna phase center location derived using the antenna phase response. A fortran program, developed from the formulations, is used to calculate phase center location for several examples which include both computed and simulated measured phase data. A simple numerical technique is presented that processes measured phase data allowing accurate phase center determination.

1. INTRODUCTION

The purpose of this paper is to present a numerical technique that will allow accurate determination of antenna phase center location for any far field observation angle. This numerical technique allows for phase center determination from both computed and measured phase response data.

Knowledge of antenna phase center is most useful in applications such as reflector antenna feed positioning and RF position determination systems such as GPS (Global Positioning System). In these applications, it is sometimes necessary to assign a reference point to the system antenna from which radiation may be said to emanate and to which positioning information can be referenced.

For these systems, this reference point is defined as the phase center of the antenna. When referenced to the phase center, the fields radiated by an antenna are spherical waves with identical spherical wavefronts or equiphase surfaces [1].

The following sections of this paper will present the two most common approaches for determination of antenna phase center. It will be shown that these approaches agree only if the antenna has the radiating phase properties of an ideal point source.

Additionally, later sections of this paper will expand on the phase center definition to provide a numerical technique for determination of antenna phase center location for any far field observation angle.

2. THE CENTER OF MINIMUM PHASE VARIATION

In previous works, it is generally agreed that the antenna phase center is the location from which radiation may be said to emanate and that this location can be found from the center of curvature of the radiated phase contours of the antenna [1-14]. Since the radiated phase contours of an antenna are angular dependent, it follows that the antenna phase center location is also angular dependent.

One of the most common approaches used in previous works to determine the so called "antenna phase center" is to locate a single point on the antenna boresight axis that, when used as the far field reference coordinate origin, results in minimum variation in the antenna's far field phase response as a function of observation angle. This approach produces a single point weighted average location for all observation angles, restricted to lie along the antenna boresight axis [4-6, 9-12]. Since antenna phase center is angular dependent, this single point location cannot be defined as the true antenna phase center. More appropriately, this single point location can be defined as the center of minimum phase variation.

Given that the antenna phase center is the center of phase curvature, this point will only correspond to the true antenna phase center (for all observation angles), if the antenna radiates a true equi-spherical wavefront. For practical antennas, the phase front curvature will generally project an antenna phase center that is not located on the boresight axis.

Since the center of minimum phase variation cannot be measured or calculated implicitly, it must be determined once the phase response of the antenna is known. One method used to determine this point is to rotate the antenna about various points along the boresight axis until a rotation point is found which produces minimum variation in the observed phase response. A similar numerical technique is to perform a least mean square error curve fit of the phase response to that of a point source located along the boresight axis. The point source location that results in a "best fit" is then taken as the solution. In either method, this point, the center of minimum phase variation, is then taken as the phase center with the understanding that it is only an approximation to the phase center location for a narrow beamwidth of observation in the main pattern lobe [14]. In some applications, it may be necessary to have a more accurate knowledge of the antenna phase center location for all observation angles.

The next section of this paper will present a more complete derivation of antenna phase center that is accurate for all observation angles and that does not restrict the antenna phase center to lie along the boresight axis.

3. ANTENNA PHASE CENTER

This section of the paper presents an approach for determination of antenna phase center [1] that is valid for all observation angles in any given far field sweep plane.

Each of the far field components radiated by an antenna can be written (assuming the usual $e^{j(\omega t-\beta r)}/r$ dependence) as:

$$E = E(\theta,\phi)e^{jF(\theta,\phi)}\,a \tag{1}$$

where a is a unit vector, $E(\theta,\phi)$ and $F(\theta,\phi)$ represent, respectively, the (θ,ϕ) variations of the amplitude and phase, where θ and ϕ are the usual angular co-ordinates in spherical geometry as shown in Figure 1.

The amplitude pattern of the far field is independent of the co-ordinate system in which the antenna is located. The phase function, $F(\theta,\phi)$, however, is sensitive to the location of the co-ordinate origin (since it is implicitly referenced to the origin) and if there exists an origin which reduces $F(\theta,\phi)$ to a constant, then this origin is said to be the phase center of the antenna [1,13]. Since this definition of phase center depends upon the polarization of the field and the planes which contain the angular variables θ and ϕ, these quantities must be specified whenever the concept of phase center is used.

Considering a sweep in only the ϕ plane, $\theta = 90$ degrees, the phase is function of ϕ whatever the origin chosen but over a small range of ϕ there may exist a point PC such that $F(\phi)$ is practically constant. If PC is chosen as the phase center for a given aspect angle ϕ_p, then the range of ϕ for which the fixed point PC can be used as the phase center will depend on the allowable tolerance on $F(\phi)$. To find the point PC, use is made of the evolute of a plane equiphase contour. The evolute is the locus of the center of curvature of the contour, and the center of curvature corresponds to the location of an origin which leads to no change in the phase function over an increment $\Delta\phi$. Knowledge of F as a function of ϕ for any origin near the antenna is sufficient to determine the evolute of a far field equiphase contour.

In the co-ordinate system of Figure 2, OP = r is the distance from the origin to a point on an equiphase contour S. The ray DP is normal to the tangent line of S at P, therefore DP, or an extension of DP, must pass through the center of curvature.

In the following development [1], point D at x = -d is found, then r is made very large so that γ is approximated by ϕ. Knowing d and γ for each point on the curve, a pencil of lines such as DP can be determined.

The locus of the phase center, or equivalently, the evolute of S, is traced by the envelope curve of the rays.

Let an equiphase contour in the x-y plane be given by

$$F(\phi) - \beta r = C \tag{2}$$

where C is an arbitrary constant. Then,

$$r = f(\phi) = (F(\phi) - C)/\beta = (F - C)/\beta \tag{3}$$

and

$$dr/d\phi = F'/\beta \tag{4}$$

where the prime denotes differentiation with respect to ϕ.

From Figure 2 we also have,

$$x = r\cos(\phi) \tag{5}$$

$$y = r\sin(\phi) \tag{6}$$

$$\tan(\gamma)\tan(\delta) = -1 \tag{7}$$

$$\tan(\gamma) = y/(x + d) \tag{8}$$

$$\phi = \tan^{-1}(y/x) \tag{9}$$

then

$$d = (y/\tan(\gamma)) - x \tag{10}$$

Substituting for $\tan(\gamma)$ gives

$$d = -y\tan(\delta) - x \tag{11}$$

If we let r be represented as a function of ϕ as in (3), $r = f(\phi)$, we may substitute (9) to give

$$r = f(\tan^{-1}(y/x)) \tag{12}$$

then

$$x^2 + y^2 = r^2 = \{f(\tan^{-1}(y/x))\}^2 \tag{13}$$

so

$$x^2 + y^2 - \{f(\tan^{-1}(y/x))\}^2 = 0 \tag{14}$$

which is a function containing both the spatial terms and the phase front properties.

We seek an expression for $\tan(\delta)$ in (11) which may be obtained by noting that

$$\tan(\delta) = \tan(\gamma + \pi/2) \tag{15}$$

and

$$\tan(\gamma) = -dx/dy \tag{16}$$

thus

$$\tan(\delta) = dy/dx \tag{17}$$

Carrying out the implicit differentiation on (14) yields

$$\tan(\delta) = \frac{-x + f(\phi)f'(\phi)\left(-\frac{y}{x^2+y^2}\right)}{y - f(\phi)f'(\phi)\left(\frac{x}{x^2+y^2}\right)} \tag{18}$$

$$= \frac{\frac{dr}{d\phi}\sin(\phi) + r\cos(\phi)}{\frac{dr}{d\phi}\cos(\phi) - r\sin(\phi)}$$

Substituting (3) and (4) gives

$$\tan(\delta) = \frac{\frac{F'}{\beta}\sin(\phi) + \frac{F-C}{\beta}\cos(\phi)}{\frac{F'}{\beta}\cos(\phi) - \frac{F-C}{\beta}\sin(\phi)} \tag{19}$$

which relates the angular variable δ to the observation variable ϕ and the field phase properties through F. Substituting into (11) to include the phase center geometry yields

$$d = \frac{\lambda}{2\pi}\left(\frac{F'(F-C)}{(F-C)\sin(\phi) - F'\cos(\phi)}\right) \tag{20}$$

where, as noted earlier, the prime denotes differentiation with respect to ϕ. Changing to a new variable $u = 2\pi \cos(\phi)$ and writing d in terms of λ, we obtain

$$\frac{d}{\lambda} = \frac{-dF/du}{1 + \frac{u\,dF/du}{\beta r}} \tag{21}$$

where the d on the right hand side of (21) denotes differentiation, not to be confused with the distance parameter d. Since $\beta r >> 1$ we may approximate (21) as

$$d/\lambda = -dF/du \tag{22}$$

We may approximate (22) by computing the change in F for an incremental change in u, ie.

$$\frac{d_i}{\lambda} = -\left(\frac{F(\phi_i + \Delta\phi) - F(\phi_i)}{2\pi(\cos(\phi_i + \Delta\phi) - \cos(\phi_i))}\right) \tag{23}$$

where ϕ_i is a given value of ϕ and $\Delta\phi$ is a small angular increment. As $\Delta\phi$ becomes small, equation (23) approximates the derivative of $F(\phi)$ with respect to the variable $2\pi \cos(\phi)$. Once d is found as a function of ϕ, rays such as DP can be drawn by setting $\gamma = \phi$. The evolute of the equiphase contour is then the envelope curve of the rays. Once the rays such as DP are constructed for all observation angles, the phase center location on the evolute can be determined. From (23) it can be seen that, for an antenna to exhibit minimal phase center movement, it must have a near constant phase response. It should also be noted that, regardless of the location of the co-ordinate origin chosen, the formulation will always calculate the same phase center location in relation to the antenna position.

The phase center location on the evolute is calculated by finding the point of intersection of successive rays as shown in Figure 3 [15]. The rays are shown for two successive observation angles, ϕ_1 and ϕ_2. The values of d_1 and d_2 are calculated using (23). The phase center location for an observation angle ϕ_1 is located at a point $PC(x_a,y_a)$ which is defined by a length L from the co-ordinate origin and an angle Ψ, defined from the $\phi = 0$ degrees axis.

From the geometry defined in Figure 3,

$$L = (x_a + y_a)^{1/2} \tag{24}$$

$$x_a = k + d_1 \tag{25}$$

$$y_a = k \tan(\Omega_1) \quad 26)$$

and

$$k = \frac{\frac{\tan(\Omega_2)}{\tan(\Omega_1)}(d_1 - d_2)}{1 - \frac{\tan(\Omega_2)}{\tan(\Omega_1)}} \tag{27}$$

where

$$\Omega_1 = \pi - \phi_1 \tag{28}$$

and

$$\Omega_2 = \pi - \phi_2 \tag{29}$$

The value of Ψ can be found from $\tan^{-1}(y_a/x_a)$ and by locating $PC(x_a,y_a)$ in the proper quadrant. It should also be noted that the positive x direction is to the left of the origin, which is consistent with a positive value of d_i.

In order to calculate the antenna phase center location L, and the corresponding angle Ψ, a fortran program was written which performs the calculations of equations (23) through (29) for any given phase response as a function of observation angle. This fortran program allows calculation of antenna phase center location for all observation angles in a given angular sweep plane. Successful use of this formulation and fortran program has been previously demonstrated in the literature [2, 15-19].

The following sections of this paper will provide example phase center calculations and some details regarding special considerations for use of the above formulations with measured phase data.

4. EXAMPLE CALCULATIONS USING COMPUTED PHASE DATA

This section of the paper presents two examples of antenna phase center calculation using the formulations developed in section 3 and the resulting fortran program. These example calculations are based upon the use of computed antenna phase response data. The next section of the paper will demonstrate the use of the formulations with simulated measured phase response data where some special considerations must be taken into account. For the following phase center determinations, a 1 degree angular increment was used in the generation of the phase response and all resulting formula calculations.

The first case to be considered is that of an ideal point source where the phase center is located at the point source for all observation angles. To demonstrate that the formulations are independent of the co-ordinate origin selected, the point source is arbitrarily located 1.65λ away from the co-ordinate origin along the +y-axis. The co-ordinate system reference is taken from Figure 1.

The far field phase response for the point source in a ϕ sweep (θ = 90 degrees) is shown in Figure 4. The phase center location, calculated using the formulations of section 3, is presented by L, the radial distance from the coordinate origin, and Ψ, the angular location, where Ψ and ϕ have the same 0 degree reference axis. L and Ψ are presented in Figures 5 and 6, respectively. As expected, the phase center location is calculated to be at the point source for all observations angles.

The next example is that of a 10 element log-periodic dipole antenna with the following parameters:

τ = 0.92
σ = 0.172
Element radius = 1.4 mm
Longest dipole length = 0.395 m
Longest element half-wavelength frequency = 380 MHz
Shortest element half-wavelength frequency = 805 MHz

The antenna boresight axis was oriented along the y-axis with the dipole elements parallel with the z-axis. The co-ordinate origin was located at the front antenna element. All antenna characteristics were calculated at a frequency of 432 MHz. The radiation characteristics of the log-periodic antenna were computed using a fortran program previously developed by the authors.

The amplitude and phase characteristics of this antenna are presented in Figures 7 and 8, respectively. The phase center location parameters, L and Ψ, are presented in Figures 9 and 10, respectively. It can be seen that the phase center location about boresight (ϕ = 90 degrees) is relatively stable at approximately 0.66λ away from the co-ordinate origin at an angle of 270 degrees. This is consistent with the active region of the log-periodic antenna at a frequency of 432 MHz. This phase center location is essentially fixed for some ±45 degrees about the boresight axis which is consistent with the 3 dB beamwidth of the antenna.

As a far field observer moves outside of the main lobe of the antenna, the phase center properties change significantly. This is most noticeable in regions of amplitude nulls which exhibit rapid phase changes. In the null regions behind the antenna, the phase center exhibits a large displacement away from the antenna (approximately 8 wavelengths) and it reverses location about the antenna as the observer moves from one side of the null to the other. This behavior is typical for all antennas exhibiting similar null characteristics.

In general, the phase center of an antenna exhibits small displacements in regions within the main lobe of the antenna. The broader the antenna pattern lobe, the more constant or fixed the phase center location becomes. In regions of rapid amplitude changes or nulls, the phase center tends to exhibit rapid and large displacement about the antenna.

5. EXAMPLE CALCULATIONS USING MEASURED PHASE DATA

The formulations of section 3 can be applied to measured phase response data as easily as they are to computed phase response data. However, in many cases, the measured phase response can be contaminated with slight variations in the phase contour due the presence of noise or inaccuracies within the measurement equipment or test setup. This section will present a discussion of how these measurement variations affect phase center location determination. A simple numerical correction technique will be presented that, when applied to measured phase response, will reduce the affects of phase measurement noise and variations.

In previous sections, it was shown that the antenna phase center location is a function of the antenna's radiated phase contour. From equations (23) and (27), it is also evident that the phase center location is a function of the first and second derivatives of the phase contour. This indicates that the phase center location is very sensitive to small variations in the phase contour. Phase center calculations made using measured phase response data may have large errors since the data may be contaminated with measurement noise. This measurement noise would appear as rapid variations in the phase contour over small angular segments which would cause discontinuities in the derivatives of the phase contour. These phase derivative discontinuities in the measured phase response data must be removed through a filtering process. The technique used here to numerically filter the measured phase response data is a cubic spline interpolation [20].

As an example of this numerical filtering process, consider the computed phase response of the log-periodic antenna as presented in the previous section. For this example calculation, the log-periodic's computed phase response was corrupted with a ±1 degree random noise to simulate a noisy measured phase response. Using the noise corrupted phase response, the phase center location of this antenna was calculated and compared to the results obtained in the previous section. This comparison is graphically presented in Figure 11. For this comparison, the phase center parameter L is only considered since the same discussion applies for the parameter Ψ. Also, information is only presented for observation angles of $\phi = 0$ to 180 degrees.

From Figure 11, it can be seen that no correlation exists between the two calculated phase center locations. Using the noisy phase response data results in a calculated phase center location that has no meaning. The noisy phase response has a contour that exhibits rapid changes over small angular segments. This drastically affects the phase center calculations since they are a function of the phase contour derivatives. In order to improve the phase center calculations for the noisy phase response data, a filtering process must be applied to smooth the phase response contour. This smoothing will improve the first and second derivatives of the phase contour which will allow more accurate calculation of antenna phase center location.

In this example, the filtering process used is a cubic spline interpolation which is applied directly to the noisy phase response. The noisy phase response is interpolated over the 0 to 180 degree angular sweep using 10 degree sample points. To ensure more accuracy at the ends of the sweep, the interpolation uses additional sample points at -5 and 185 degrees. The solution to the cubic spline interpolation is taken at 1 degree intervals.

Using the interpolated phase response, the phase center location was again calculated and compared to the results obtained using the computed phase. This comparison, again considering only the L parameter, is presented in Figure 12. The comparison demonstrates a significant improvement in the calculation of antenna phase center location, however, the results are still unsatisfactory in that they do not accurately predict the phase center location as well as expected.

To examine the effects of the noise on the phase center calculations, it is necessary to examine the derivatives of the phase contour. The first, second and third derivatives of the interpolated phase response were calculated and compared to those of the computed phase response. These derivatives are compared in Figures 13, 14 and 15. From these derivative comparisons, it is apparent that the phase center calculations are inaccurate because of the disagreement between the phase derivatives. This is most noticeable in the second and third derivative calculations. The third derivative of the interpolated phase response has obvious discontinuities every 10 degrees corresponding to the interpolation sample points. This is expected since a cubic spline interpolation guarantees only continuous first and second derivatives at the sample points. Also, the cubic spline interpolation does not guarantee agreement between the interpolated and original data's second derivatives.

In order to improve the prediction of antenna phase center location with the noisy phase response data, the filtering technique must make a better prediction of the second and third derivatives of the interpolated phase response. This can be accomplished by interpolating the first derivative of the interpolated phase response using the 10 degree sample points. Figure 16 presents a comparison of the interpolated phase response's second derivative where the first derivative of the interpolated phase is itself interpolated at the sample points. A significant improvement in the comparison is seen.

As an alternative to interpolating the first derivative of the interpolated phase response, the values of d_i in equation (23) can be interpolated. To demonstrate this technique, the phase center location of the noisy phase response is calculated by first interpolating the phase response and then interpolating the values of d_i. In the region of the main beam, the sample points for interpolation of d_i were taken at 30 degree intervals to increase the filtering process. Figure 17 shows the recalculated phase center location compared to that calculated from the computed phase response. A significant improvement in the accuracy of the phase center location is obtained.

In order to predict the antenna phase center location accurately from measured phase response data, which is susceptible to noise or irregular phase variations, it is necessary to filter the phase response. This filtering technique requires that the measured phase data be interpolated using a cubic spline interpolation routine at appropriate sample points. Also, the filtering technique requires that the values of d_i in the phase center calculations be interpolated for a higher level of accuracy. The sample points should be chosen such that an accurate interpolation of phase can be made with as much angular separation between the sample points as possible.

6. A DISCUSSION OF THE USE OF AND ACCURACY ASSOCIATED WITH THE PHASE CENTER CALCULATIONS

The technique used in this work to calculate antenna phase center location is intended for computed phase data or measured phase data recorded under laboratory conditions. Computed phase data will generally not exhibit rapid fluctuation over small $\Delta\phi$ and will not require numerical filtering. Measured data may exhibit some noise or fluctuations due to set up or equipment limitations, however, the numerical filtering technique should allow accurate determination of the antenna phase center properties.

The calculations of the phase center location are a function of the antenna's phase response and the $\Delta\phi$ chosen for the angular sweep. In general, a greater accuracy can be obtained by reducing $\Delta\phi$ to as small a value as possible. For most antennas that exhibit a broad pattern beamwidth, a $\Delta\phi$ of 1 degree is sufficient. If the antenna has a narrow pattern beamwidth or exhibits rapid phase change over small angular regions, then it is necessary to reduce the value of $\Delta\phi$.

The value of $\Delta\phi$ chosen for a specific antenna must be left to the judgement of the individual performing the phase center calculations.

In the interpolation filtering process, a larger increment in the angular sample points increases the level of filtering. In general, a larger angular increment in the sample points will provide the greatest accuracy. The sample point increment should be selected such that the data interpolation remains valid.

One limitation with the use of this technique and measured data applies to measurements obtained outside laboratory conditions. Measured antenna phase data obtained under less than ideal conditions may include the affects of scattering, multipath or other interfering signals which the user may want to consider in the phase center calculations. Application of the numerical filter may remove some of these affects over small angular regions.

7. CONCLUSIONS

A formulation for calculation of antenna phase center location, accurate for all observation angles in a given angular sweep, was presented. A fortran program based upon these formulations was developed and used to calculate antenna phase center location for several examples. It was shown that antenna phase center location is relatively fixed over the main pattern lobe, however, large phase center variations can occur in regions of amplitude nulls or rapid phase variations.

A simple numerical filtering technique was presented that improves phase center calculations for noisy or measured phase data.

8. PROGRAM AVAILABILITY

Copies of the fortran program for calculation of antenna phase center location can be obtained by contacting Dr. Steven R. Best at Parisi Antenna Systems.

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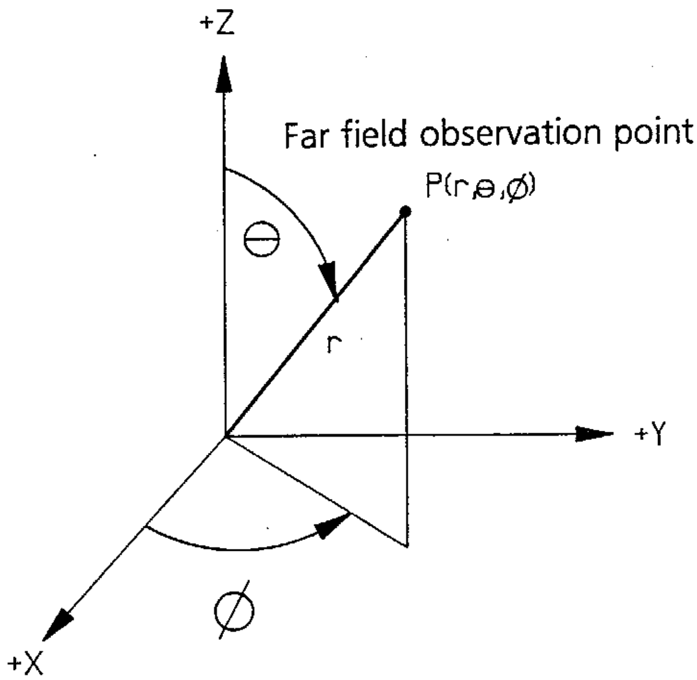

Figure 1. Geometry of the Reference Coordinate System.

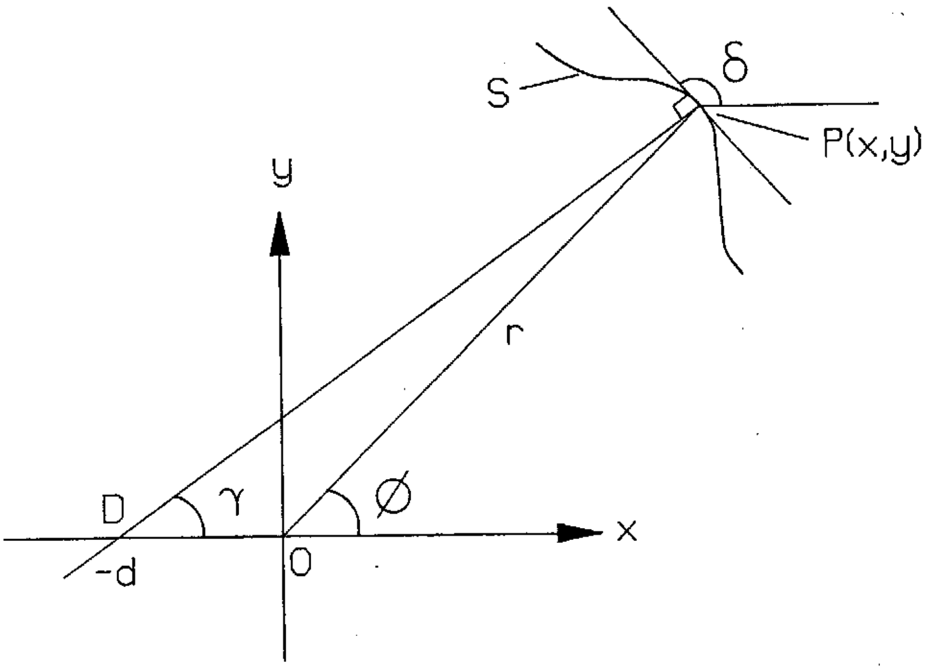

Figure 2. Geometry of the Coordinate System Used for Phase Center Formulations.

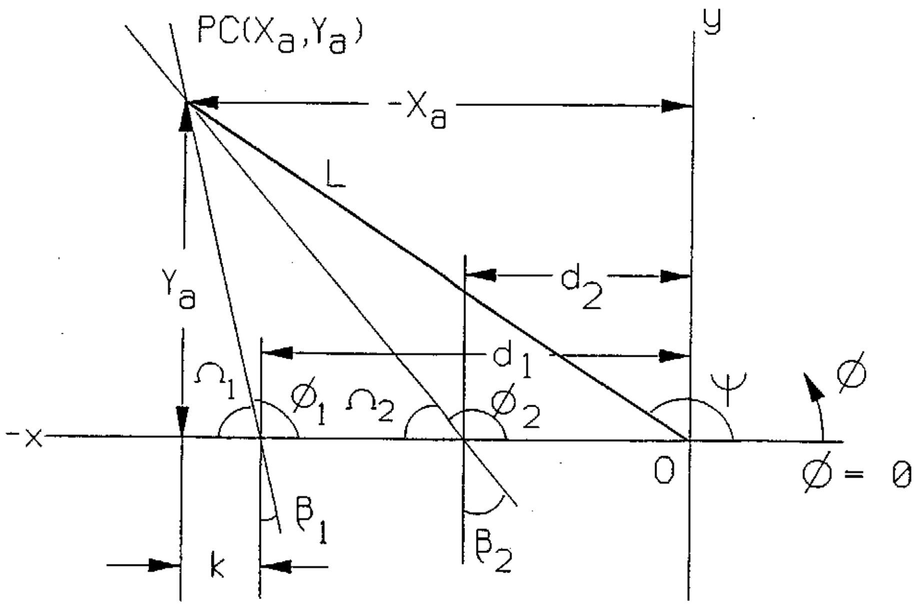

Figure 3. Geometry of the Coordinate System Used to Calculate the Phase Center Location.

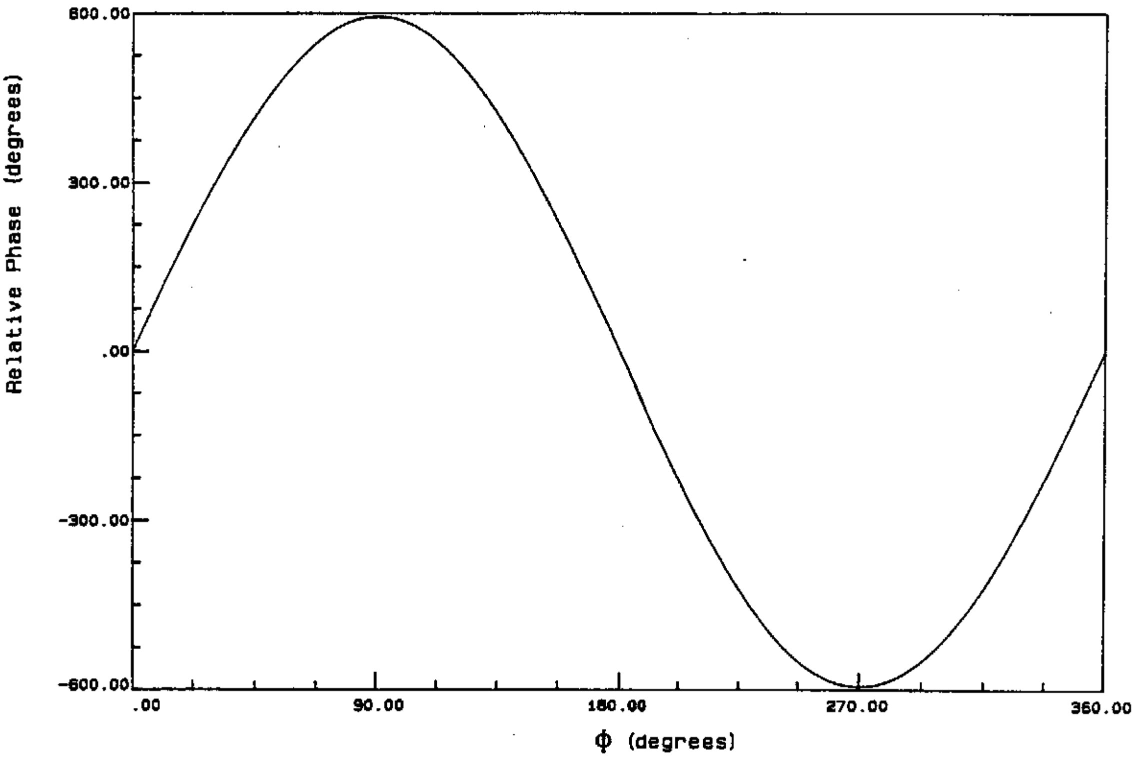

Figure 4. Computed Phase Response of a Point Source Located 1.65λ from the Coordinate Origin.

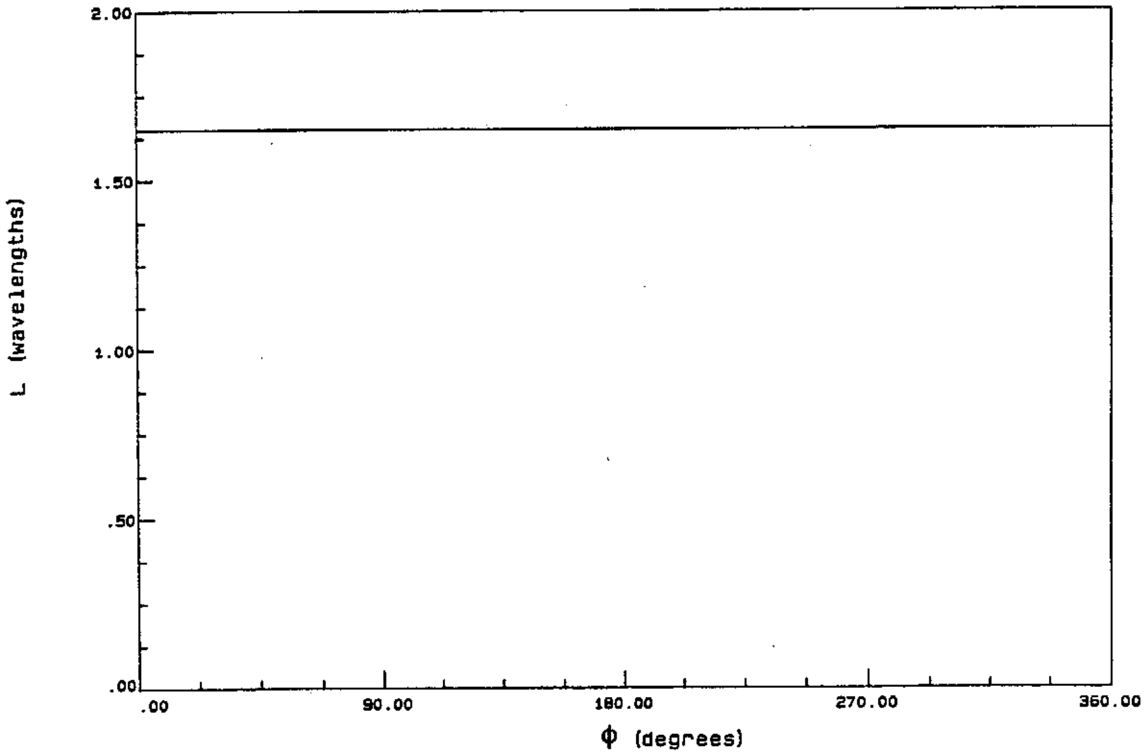

Figure 5. Phase Center Location, L, for the Point Source.

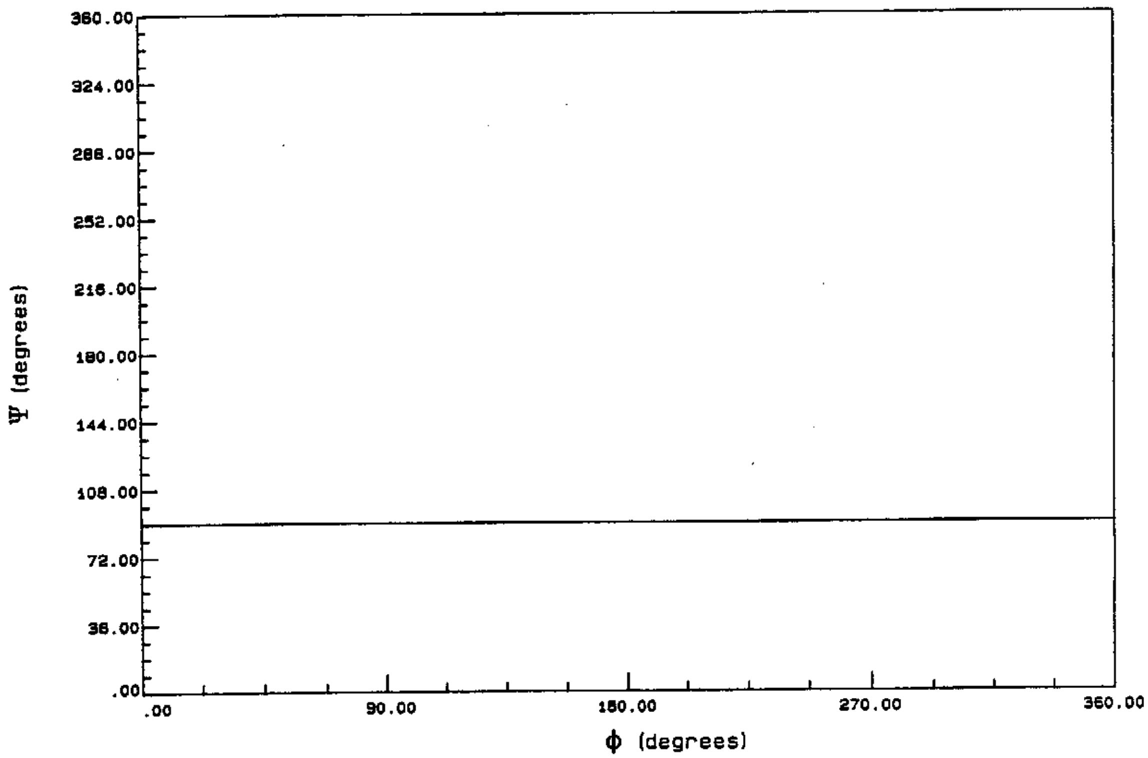

Figure 6. Phase Center Location, Ψ, for the Point Source.

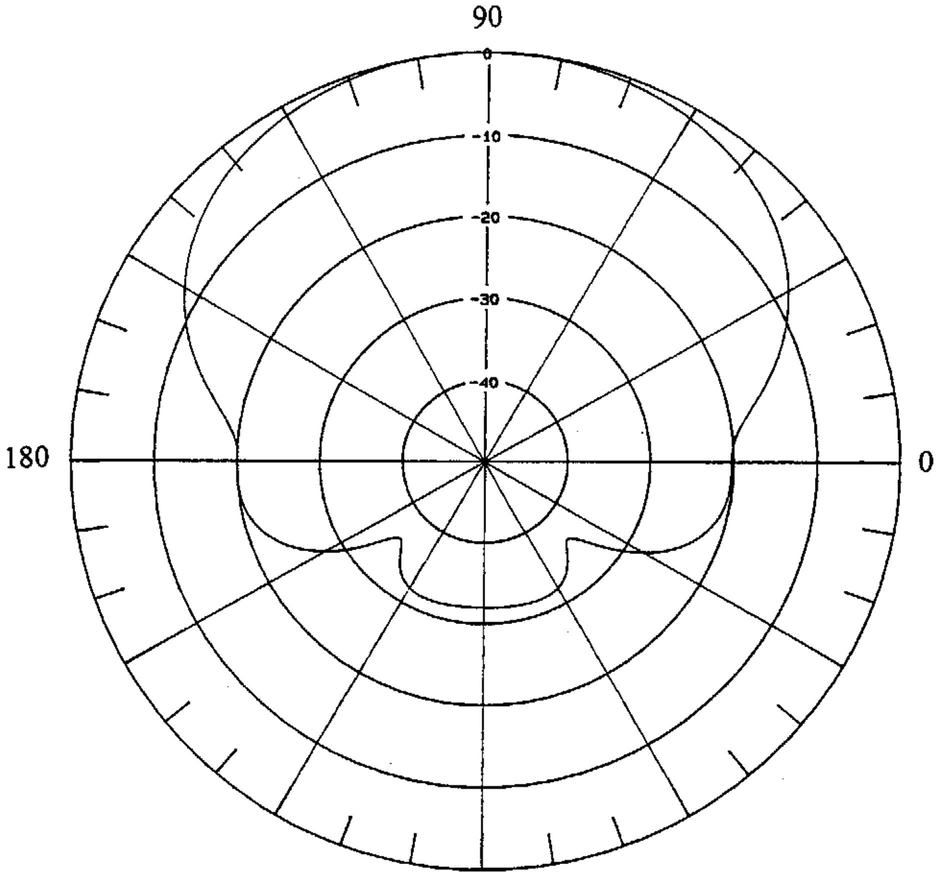

Figure 7. Amplitude Pattern Response of the Log-Periodic Antenna.

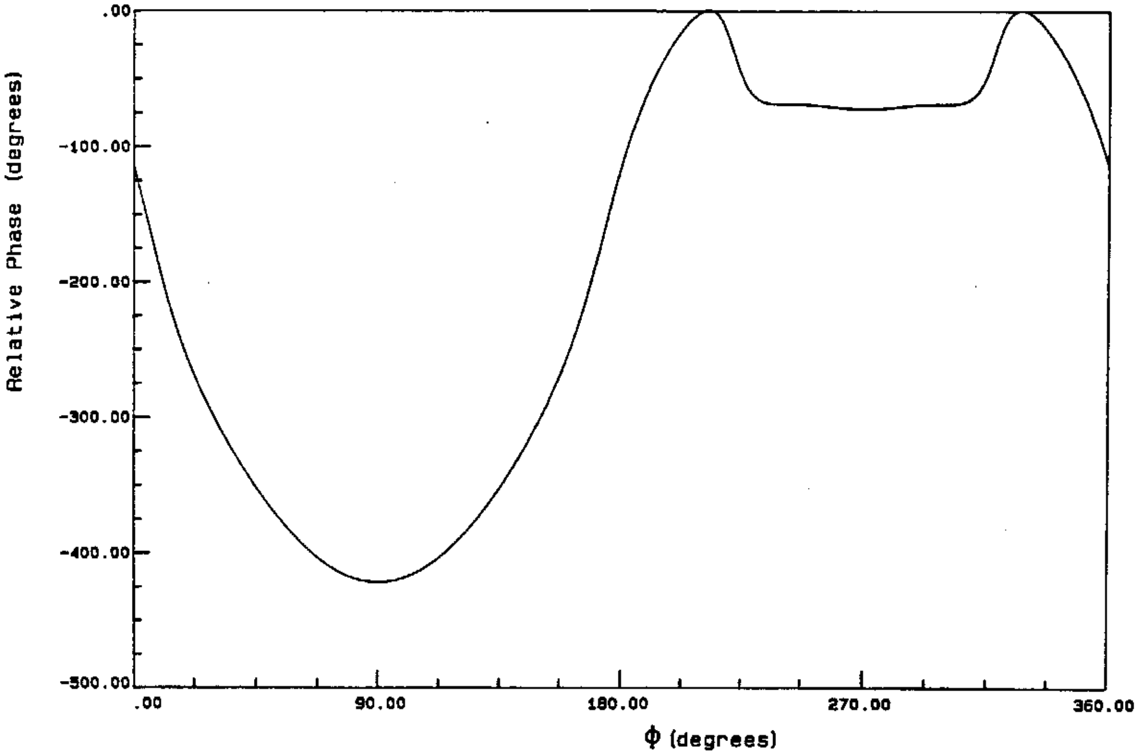

Figure 8. Phase Pattern Response of the Log-Periodic Antenna.

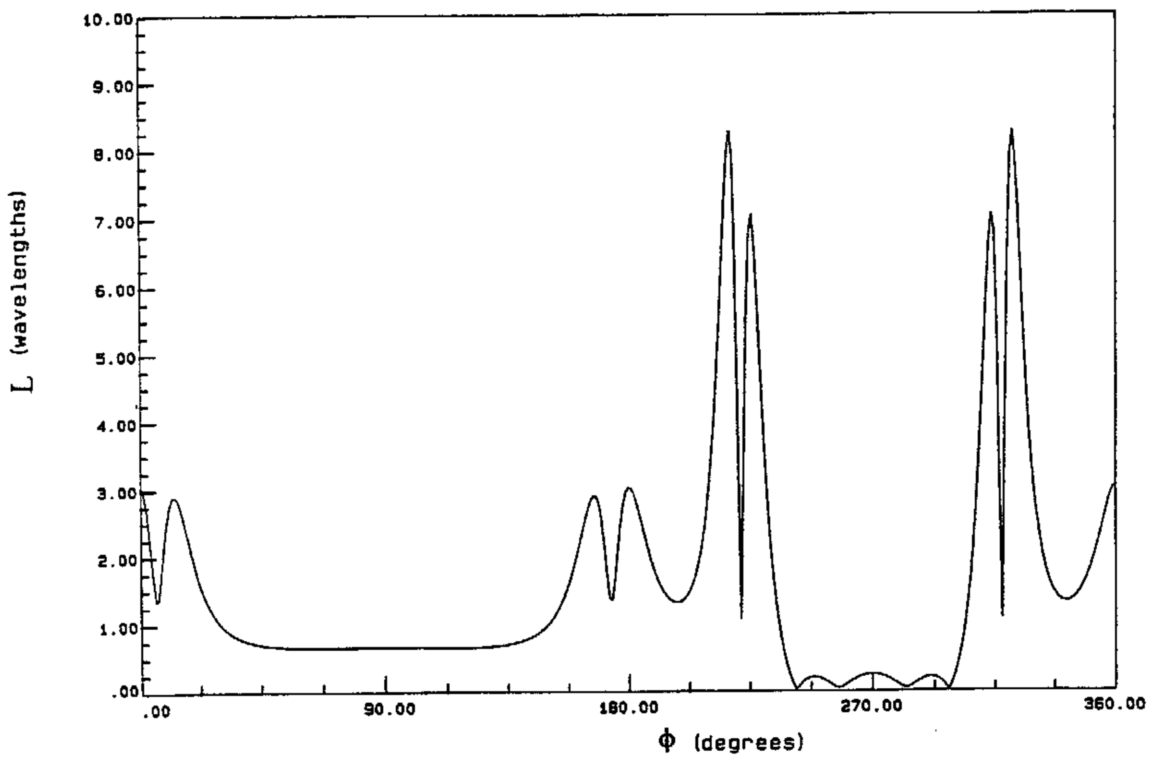

Figure 9. Phase Center Location, L, of the Log-Periodic Antenna.

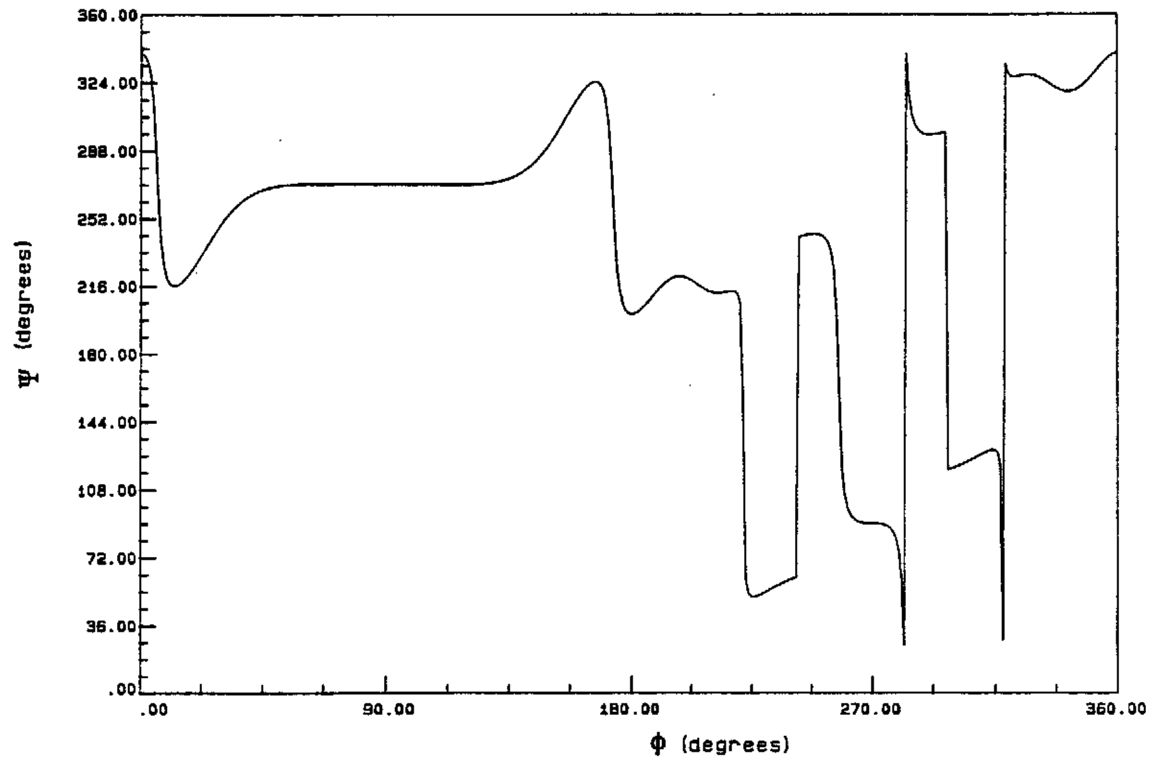

Figure 10. Phase Center Location, Ψ, of the Log-Periodic Antenna.

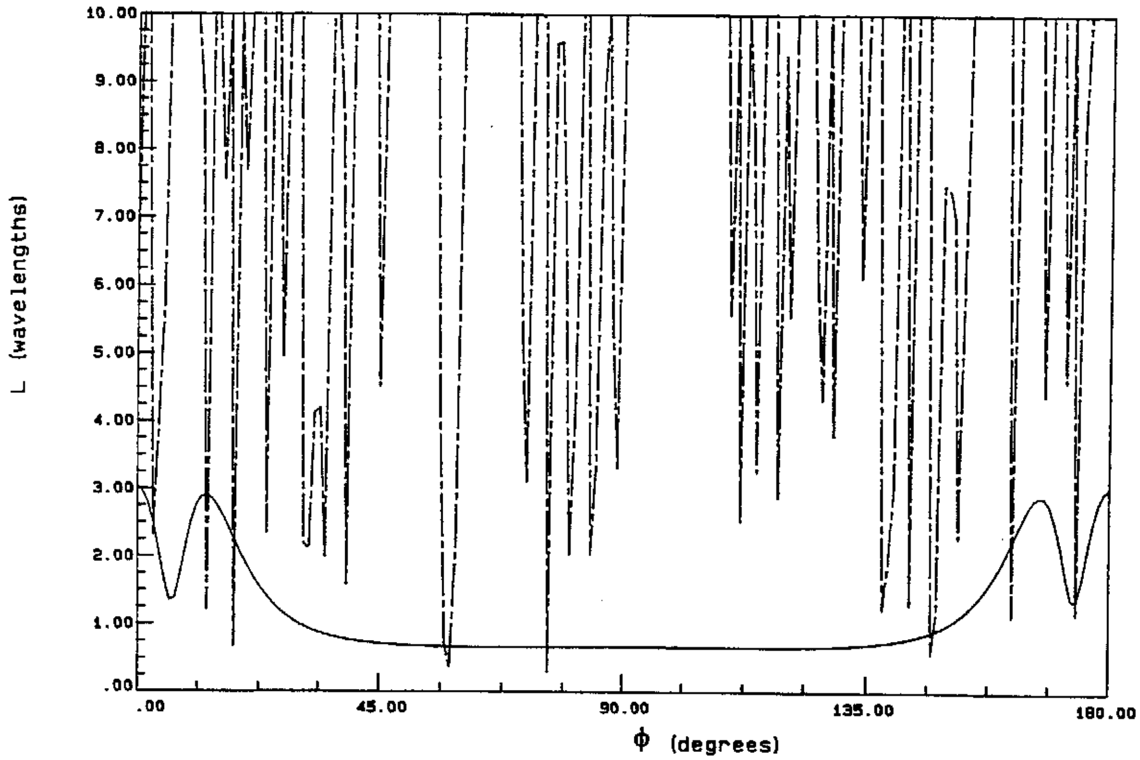

Figure 11. Comparison of the Log-Periodic Antenna's Phase Center Location, L, Calculated Using the Computed Phase Response (———) and the Noisy Phase Response (— - —).

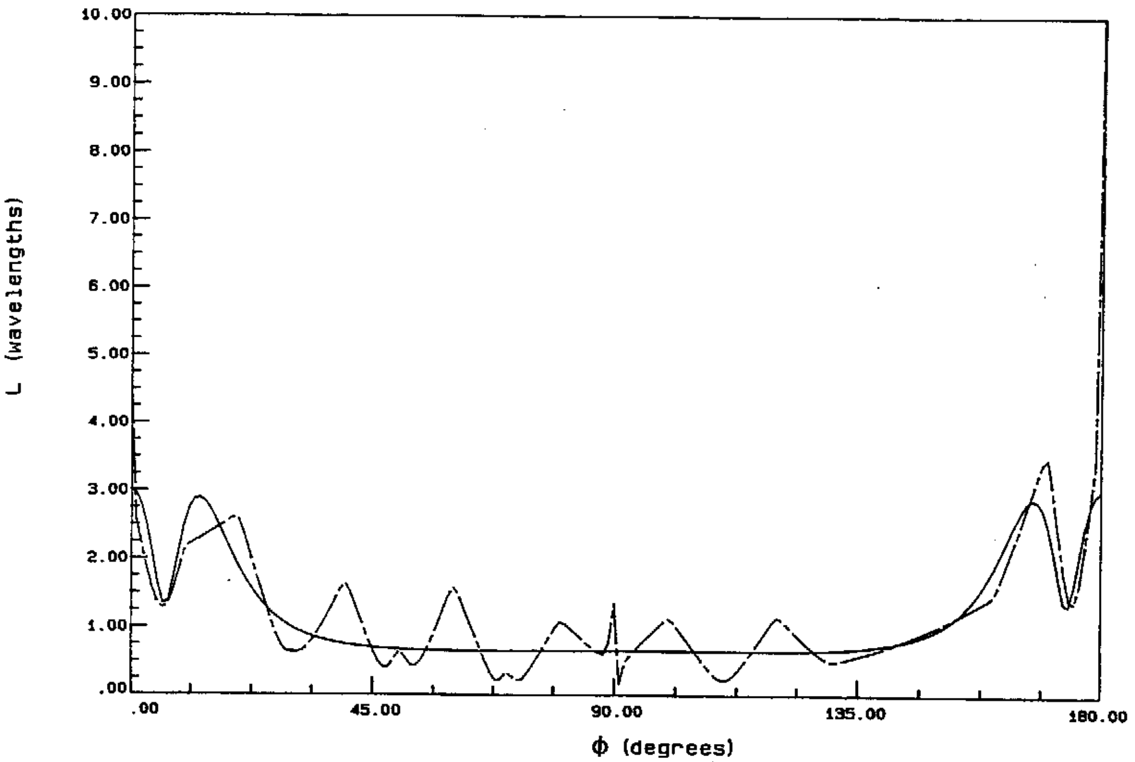

Figure 12. Comparison of the Log-Periodic Antenna's Phase Center Location, L, Calculated Using the Computed Phase Response (———) and the Interpolated Noisy Phase Response (— - —).

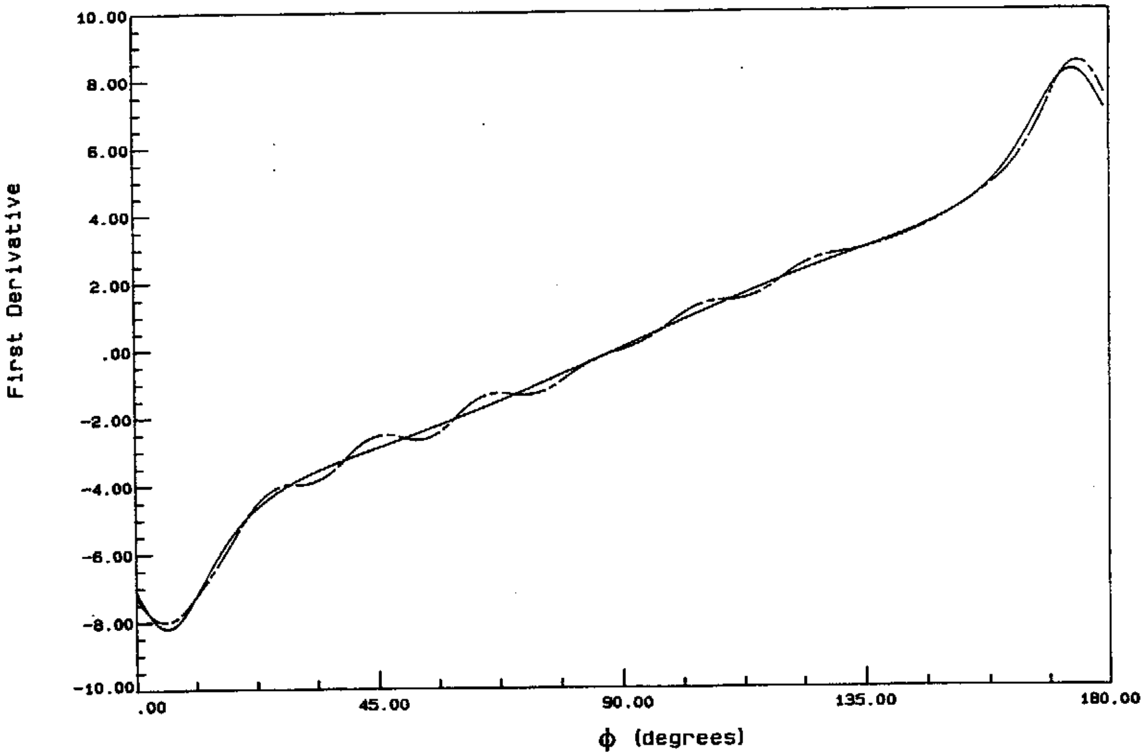

Figure 13. Comparison of the Calculated First Derivative of the Log-Periodic Antenna's Computed Phase Response (——) and Interpolated Noisy Phase Response (— - - —).

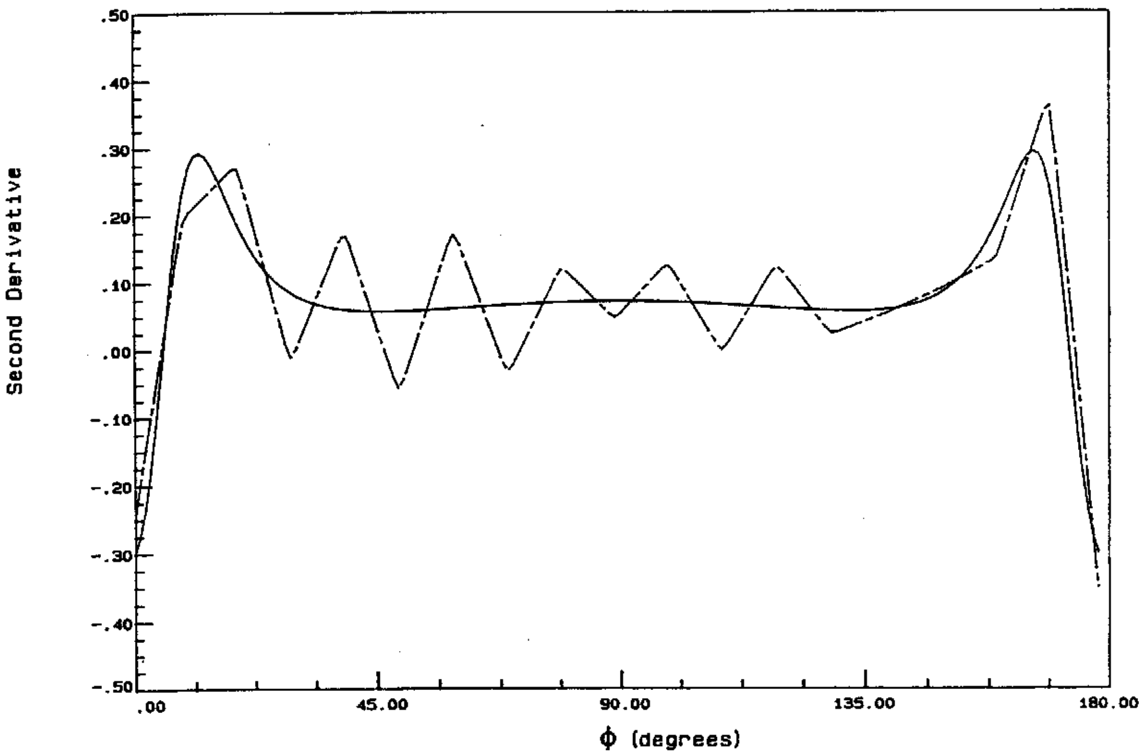

Figure 14. Comparison of the Calculated Second Derivative of the Log-Periodic Antenna's Computed Phase Response (——) and Interpolated Noisy Phase Response (— - - —).

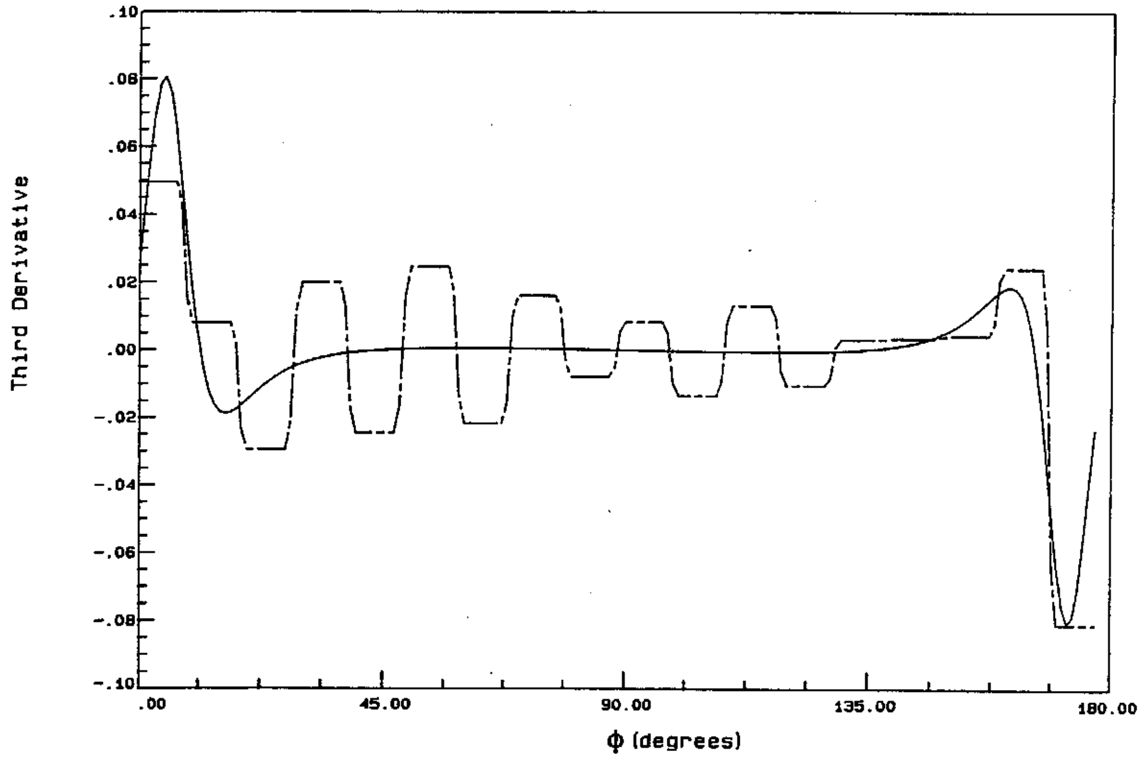

Figure 15. Comparison of the Calculated Third Derivative of the Log-Periodic Antenna's Computed Phase Response (——) and Interpolated Noisy Phase Response (—-—).

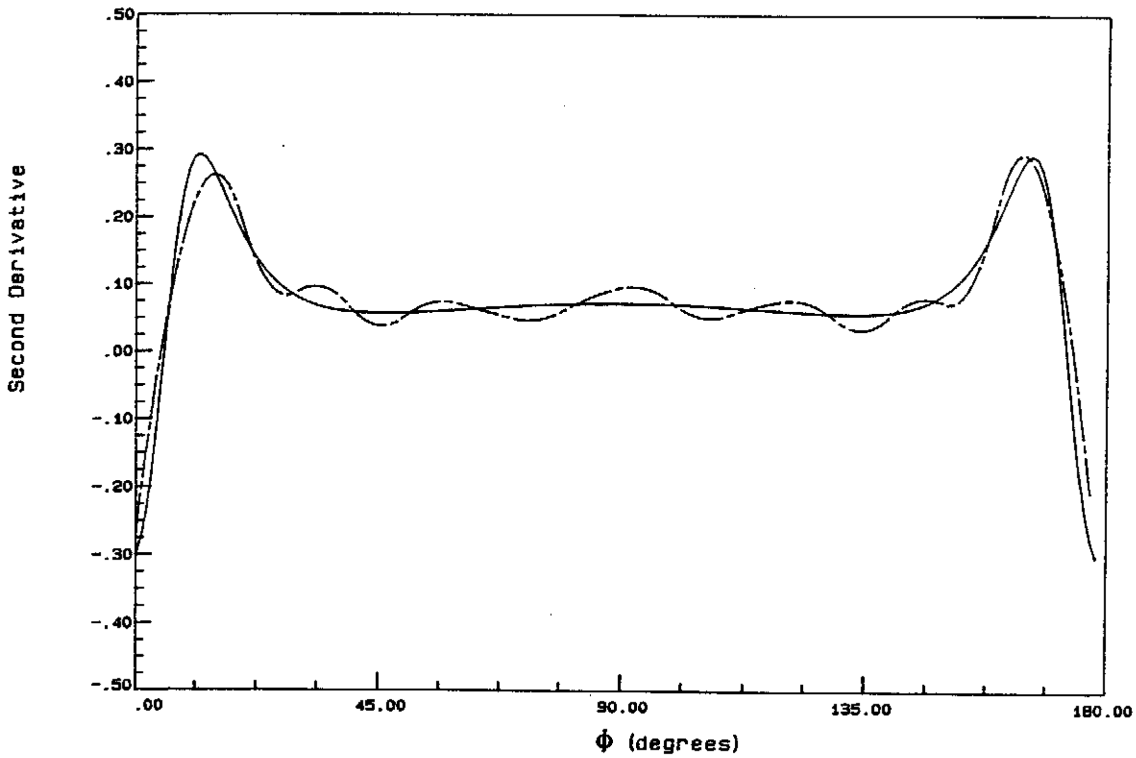

Figure 16. Comparison of the Calculated Second Derivative of the Log-Periodic Antenna's Computed Phase Response (——) and Interpolated Noisy Phase Response (—-—) where the First Derivative of the Noisy Phase Response was Interpolated.

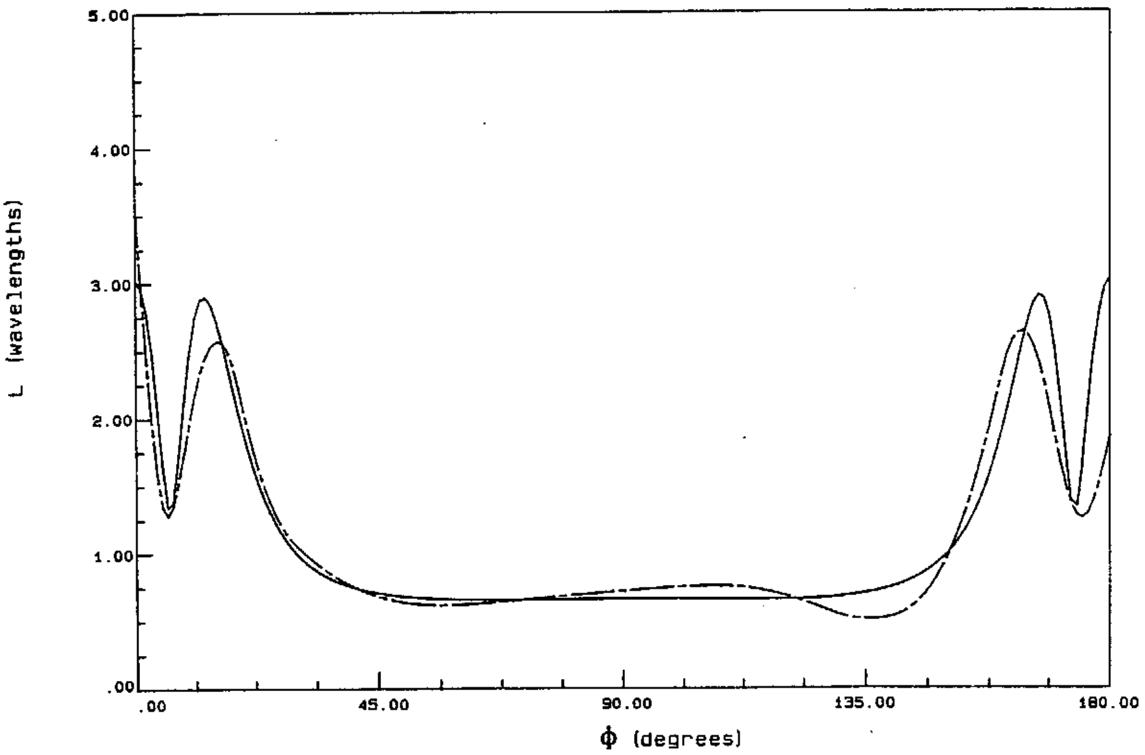

Figure 17. Comparison of the Log-Periodic Antenna's Phase Center Location, L, Calculated Using the Computed Phase Response (——) and Interpolated Noisy Phase Response (— - - —) where the values of d_i have been Interpolated.