Ultrawide band Negative Refraction Based on Moving Media Concept

Amir Jafargholi[1] and Ali Jafargholi[2]

[1]Institute of Space Science and Technology
Amirkabir University of Technology, 424 Hafez Ave., P.O. Box: 15875-4413, Tehran, Iran
jafargholi@ieee.org

[2]Department of Electrical Engineering
Sharif University of Technology, Azadi Ave., P.O. Box: 11365-11155, Tehran, Iran
jafargholi@sharif.edu

Abstract **─** The anomalous wave propagation in a homogeneous, isotropic and lossless medium that moves with a constant velocity along the axis of a rectangular waveguide; and its interesting properties lead us to the fact that the moving media may be regarded as double negative metamaterial (DNG). In this paper, the correct sign of the permittivity and permeability of an equivalent stationary media, for each TE and TM modes and for both low and high dielectric velocities are investigated and it is shown that this medium acts as a DNG material over specific frequency ranges.

Index Terms **─** Moving Media, Metamaterial.

I. INTRODUCTION

Theory of negative index was first introduced by Veselago [1]. He showed in his theoretical investigations that the DNG material has negative refraction property. After works of Rotman [2], Pendry [3], and Smith [4], a standard procedure was established for the design of bulk artificial media with negative parameters. Negative refraction has been realized in artificial structures which is made of wires and SRRs [5], photonic crystals [6] and discrete elements [7], hole arrays in thin metal films [8], thin layers of dielectrics [9], vertical loops on a substrate [10], waveguide arrangements [11], and even a natural material based on colossal magnetoresistance [12]. The narrow bandwidth of aforementioned proposed artificial materials is due to resonant nature of these structures and this fact is main limiting factor for negative refraction applications.

In this paper it will be shown that some of the drawbacks might be overcome, by using other types of artificial structures which would have a larger bandwidth, such as moving media. Though moving media, [13-14] have been previously categorized in the bianisotropic groups, the proposed media can also be considered as artificial negative refractive structures. Moreover, similar to SRR the proposed media show DNG properties just in certain directions [3]. In contrast to SRR structures showing resonance and narrow-band DNG properties, the proposed moving media have wide bandwidth which meets the law of entropy.

Recently, Grzegorczyk et al studied the wave refraction phenomenon as a function of frequency and medium velocity in an isotropic non-dispersive moving medium which its velocity is parallel to the interface where refraction occurs, using spectral domain approach [15]. They investigated the effects of motion of the medium on a rotation of refraction. They showed that it can either enhance or attenuate the natural negative refraction of the medium. However, it should be emphasized that our approach is fundamentally different with [15] in three aspects: it involves permittivity and permeability definition of an equivalent stationary media based on moving medium parameters, negative refraction is obtained for both TE and TM modes for each frequency range, and we consider that the

Submitted On: May 9, 2012
Accepted On: August 22, 2012

direction of medium movement to be in the same direction of wave propagation.

In this paper, we consider stationary medium with equivalent permittivity and permeability for both TE and TM waveguide modes. The range of validity for each quantity is clearly stated. It has been demonstrated that this media acts as an ultrawide band negative refractive structure. It is to be noted that treatment of this issue at present is purely theoretical, however developing waveguide which filling medium with specified physical constituent moving with constant velocity may not be impossible in future, however, the applications of metamaterial in waveguide have been already addressed in literatures, [16]-[18-19].

II. QUANTITIES DEFINITIONS

The moving media are not a new concept in electromagnetic and it has been discussed in [13-14]. In [13], it has been showed that for a general moving media, the cutoff frequency of ordinary waveguide is lowered by a factor which depends upon the velocity of the medium. It has been also shown that change in the cutoff frequency, propagation constant and transverse-wave impedance are modified independent of the guide geometry.

In [14], Du and Compton showed that for a slowly moving medium, $n\bar{v}<1$, where n is the index of refraction and v is the velocity of the medium, $\bar{v}$, divided by the velocity of light in vacuum, there are two critical frequencies, separating three frequency ranges in each of which there is a different type of propagation. Furthermore, for a high-speed medium i.e. $n\bar{v}>1$, it is found that there is no cutoff phenomenon at all, although there is one critical frequency separating two frequency ranges in which the propagation is different.

In Fig. 1, the anomalous properties of this medium are shown, [14]. We assume that a homogeneous, isotropic and lossless media that moves with a constant velocity along the axis of a waveguide acts as an equivalent medium to a rectangular waveguide which is filled with stationary media and modified permittivity and permeability. First, we define the main quantities of such equivalent rectangular waveguide compared with same quantities of moving media. Then, the restrictions on the range of validity of derived quantities are specified, and finally the correct sign of equivalent permittivity, ε_e, and permeability, μ_e, of moving media will be determined. In a dielectric loaded rectangular waveguide the cutoff frequency, f_c, is,

$$f_c = \frac{k_c}{2\pi\sqrt{\mu\varepsilon}} = \frac{k_c}{2\pi n} \qquad (1)$$

where k_c is, in general, a different propagation constant for each mode (i.e. for a rectangular waveguide $k_c^2 = \left(\frac{p\pi}{a}\right)^2 + \left(\frac{q\pi}{b}\right)^2$ .where a and b are the dimensions of the waveguide, and p, q specify the mode) and,

$$n = \sqrt{\mu\varepsilon} = \sqrt{\mu_r\varepsilon_r}\cdot\sqrt{\mu_0\varepsilon_0} \qquad (2)$$

which in this equation, ε_r and μ_r are the dielectric permittivity and permeability respectively. And the wave impedance of TE and TM modes are equal to

$$Z^{TE} = \frac{\omega\mu_r}{\beta_r} \qquad (3)$$

$$Z^{TM} = \frac{\beta_r}{\omega\varepsilon_r} \qquad (4)$$

where $\beta_r = \omega\sqrt{\mu_r\varepsilon_r}$ is the propagation constant.

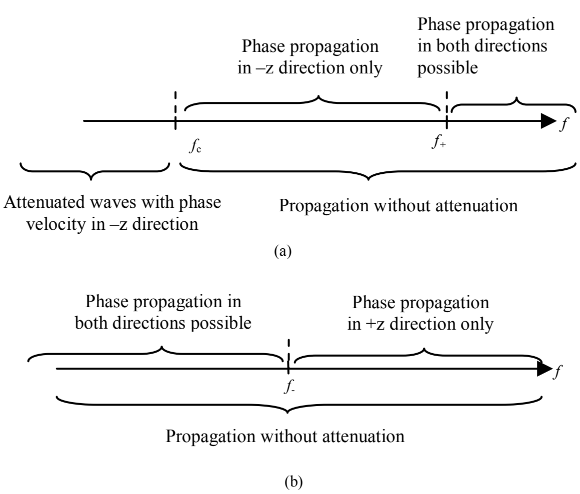


Fig. 1. Frequency ranges for wave propagation in the waveguide, with the medium moving in the +Z direction, (a) the low velocity case: $n\bar{v}<1$. (b) the high velocity case: $n\bar{v}>1$, [14].

Now, for an equivalent medium which is filled with stationary media and modified permittivity, ε_e, permeability, μ_e ; propagation constant, β_e, and comparing (3) and (4) with TE and TM impedance of moving media, based on [14], one can show that [17]

$$Z^{TE} \Rightarrow \frac{\omega\mu_e}{\beta_e} \equiv \frac{\omega\mu a}{h+\omega\Omega} \quad (5)$$

$$Z^{TM} \Rightarrow \frac{\beta_e}{\omega\varepsilon_e} \equiv \frac{h+\omega\Omega}{\omega\varepsilon a} \quad (6)$$

Where ε and μ are the permittivity and permeability of the moving material in rectangular waveguide, and a and Ω are defined as

$$\Omega = \frac{(n^2-1)\overline{v}}{(1-n^2\overline{v}^2)c} \quad (7)$$

$$a = \frac{1-\overline{v}^2}{1-n^2\overline{v}^2} \quad (8)$$

Here, $h = -\omega\Omega \pm a\sqrt{k^2-k_c^2}$ and c is the velocity of light.

$$\overline{v} = \frac{v}{c} \quad (9)$$

From equations (5) and (6), we have

$$\frac{\mu_e}{\varepsilon_e} = \frac{\mu_r}{\varepsilon_r} \rightarrow \mu_e = \frac{\mu_r}{\varepsilon_r}\varepsilon_e \quad (10)$$

Using (10) and (2), we get

$$n_e = \sqrt{\mu_e\varepsilon_e} = |\varepsilon_e|\sqrt{\frac{\mu_r}{\varepsilon_r}} = |\mu_e|\sqrt{\frac{\varepsilon_r}{\mu_r}} \quad (11)$$

The modal impedances of an ordinary rectangular waveguide depend on the modal indexes. As it mentioned before, in [13-14] it has been shown that for a general moving media, the transverse-wave impedance are modified independent on the guide geometry. Hence in the equivalent medium, it is expected that the equivalent transverse-wave impedance is independent from the modal indexes and it related to the velocity of the moving medium.

III. CRITICAL FREQUENCIES

As stated earlier we assume that a rectangular waveguide which is filled with stationary media and modified permittivity and permeability acts as an equivalent medium to a homogeneous, isotropic and lossless media that moves with a constant velocity along the axis of a waveguide.

When the velocity of the moving medium is small, i.e. $n\overline{v} < 1$, the cutoff frequency is [14],

$$f = f_c = \frac{k_c}{2\pi\sqrt{\mu_0\varepsilon_0}} \cdot \sqrt{\frac{1-n^2\overline{v}^2}{n^2(1-\overline{v}^2)}} \quad (12)$$

Comparing (1) and (12) it is evident that for frequencies below the cutoff frequency

$$f < f_c \Rightarrow \frac{1}{\sqrt{\mu_e\varepsilon_e}} < \sqrt{\frac{1-n^2\overline{v}^2}{n^2(1-\overline{v}^2)}} \quad (13)$$

or

$$n_e > \sqrt{\frac{n^2(1-\overline{v}^2)}{1-n^2\overline{v}^2}} \quad (14)$$

Using (11), (14) can be written as

$$n_e = |\varepsilon_e|\sqrt{\frac{\mu_r}{\varepsilon_r}} > \sqrt{\frac{n^2(1-\overline{v}^2)}{1-n^2\overline{v}^2}} \quad (15)$$

Combination of (2) and (15) leads to the following inequalities for small values of velocity

$$\varepsilon_e > \varepsilon_r\sqrt{\frac{1-\overline{v}^2}{1-n^2\overline{v}^2}} \quad (16)$$

or

$$\varepsilon_e < -\varepsilon_r\sqrt{\frac{1-\overline{v}^2}{1-n^2\overline{v}^2}} \quad (17)$$

Waveguide impedances for TE and TM modes are, [14]

$$Z^{TE} = \mp i\frac{\omega}{\omega_C}\eta \cdot \left[1-\left(\frac{f}{f_C}\right)^2\right]^{-1/2} \quad (18)$$

$$Z^{TM} = \pm i\frac{\omega_C}{\omega}\eta \cdot \left[1-\left(\frac{f}{f_C}\right)^2\right]^{1/2} \quad (19)$$

Where, η is the intrinsic impedance of medium.

By comparing (18) and (19) with (3) and (4) respectively, the correct sign for the permittivity can be determined. For TE mode, the equivalent permittivity and permeability in +z direction are negative, whereas for -z direction, they are positive. Similarly, for TM mode, the equivalent permittivity and permeability in +z and -z directions are positive and negative, respectively. When the frequency is less than f_c, the fields are attenuated along the guide axis, but unlike an ordinary waveguide below cutoff, there is a phase velocity $v_P = 1/\Omega$ in the negative z-direction for both solutions, [14]. Based on what stated, one may conclude that the moving media acts as DNG and double positive (DPS) for TM and TE respectively.

When the frequency is slightly above f_c, there is no attenuation, but the two waves both have different phase velocities in the negative z-direction. For large enough frequency we have from [14]

$$f_+ = \frac{k_c}{2\pi\sqrt{\mu_0\varepsilon_0}} \cdot \sqrt{\frac{1-\bar{v}^2}{n^2-\bar{v}^2}} \tag{20}$$

This means that the waves can propagate in either direction without attenuation, but again with different phase velocities. For frequency above f_+, we have

$$\frac{1}{n_e} > \frac{1}{\sqrt{\mu_r\varepsilon_r}} = \frac{1}{n} \to n_e < n \tag{21}$$

Using (11), (21) can be written as

$$|\varepsilon_e| < \varepsilon_r \Rightarrow -\varepsilon_r < \varepsilon_e < \varepsilon_r \tag{22}$$

Equ. (22) shows that it is possible to have negative and positive values for permitivity. With the following wave impedances defined in [14]

$$Z^{TE} = \pm\eta \cdot \left[1-\left(\frac{f_C}{f}\right)^2\right]^{-1/2} \tag{23}$$

$$Z^{TM} = \pm\eta \cdot \left[1-\left(\frac{f_C}{f}\right)^2\right]^{1/2} \tag{24}$$

We note that both TE and TM waves in +z and -z directions can exist and equivalent permittivity and permeability are positive for +z direction and negative for -z direction.

For frequency $f_c < f < f_+$, one can show that

$$f_c < f < f_+ \Rightarrow \begin{cases} n_e > n \\ n_e < \sqrt{\dfrac{n^2(1-\bar{v}^2)}{1-n^2\bar{v}^2}} \end{cases} \tag{25}$$

This condition leads to

$$\begin{cases} n_e > n \\ n_e < \sqrt{\dfrac{n^2(1-\bar{v}^2)}{1-n^2\bar{v}^2}} \end{cases} \Rightarrow \begin{cases} |\varepsilon_e| > \varepsilon_r \\ |\varepsilon_e| < \varepsilon_r\sqrt{\dfrac{1-\bar{v}^2}{1-n^2\bar{v}^2}} \end{cases} \tag{26}$$

From which we have

$$\begin{cases} \varepsilon_e < -\varepsilon_r \ , \ \varepsilon_e > \varepsilon_r \\ -\varepsilon_r\sqrt{\dfrac{1-\bar{v}^2}{1-n^2\bar{v}^2}} < \varepsilon_e < \varepsilon_r\sqrt{\dfrac{1-\bar{v}^2}{1-n^2\bar{v}^2}} \end{cases} \tag{27}$$

Again defining of ψ as

$$\psi = \sqrt{\frac{1-\bar{v}^2}{1-n^2\bar{v}^2}} \tag{28}$$

The second inequality in (27) can be expressed

$$\begin{cases} if \quad n^2 < 1 \to \psi < 1 \\ if \quad n^2 > 1 \to \psi > 1 \end{cases} \Rightarrow \begin{cases} \varepsilon_r\psi < 1 \\ \varepsilon_r\psi > 1 \end{cases} \tag{29}$$

Hence, the limit for the range of validity of (27) may be shown by doted region in Fig. 2. When $n\bar{v} < 1$ then $n^2\bar{v}^2 < 1$ and $\bar{v}^2 < 1/n^2 < 1$, thus

$$\frac{v}{c} < \frac{1}{\sqrt{\mu_r\varepsilon_r}} \tag{30}$$

This is the necessary condition for (27) to have a solution. Now we should reject one of these two solutions of (27). Figure 2, shows that if $n^2 < 1$, (27) has no solution for permittivity and no wave could propagate. However, the doted region is the valid region for (27).

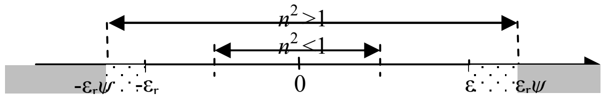


Fig. 2. Region of solution for permittivity, $n\bar{v} < 1$.

Since the only propagating mode is in -z direction, therefore the moving media for both TE and TM waves in -z direction act as DNG medium. For the case where the velocity of the moving medium is high, i.e. $n\bar{v} > 1$, the cutoff frequency is [14],

$$f = f_{-} = \frac{k_c}{2\pi\sqrt{\mu_0\varepsilon_0}} \cdot \sqrt{\frac{1-\bar{v}^2}{n^2-\bar{v}^2}} \tag{31}$$

For frequencies below the cutoff

$$f < f_{-} \Rightarrow \frac{1}{n_e} < \sqrt{\frac{1-\bar{v}^2}{n^2-\bar{v}^2}} \tag{32}$$

$$n_e > \sqrt{\frac{n^2-\bar{v}^2}{1-\bar{v}^2}} \tag{33}$$

Further, using (11), we can show

$$\varepsilon_e > \varepsilon_r \sqrt{\frac{\varepsilon_r}{\mu_r} \cdot \frac{n^2-\bar{v}^2}{1-\bar{v}^2}} \tag{34}$$

or

$$\varepsilon_e < -\varepsilon_r \sqrt{\frac{\varepsilon_r}{\mu_r} \cdot \frac{n^2-\bar{v}^2}{1-\bar{v}^2}} \tag{35}$$

Similar to previous method of solution, defining

$$\psi = \sqrt{\frac{n^2-\bar{v}^2}{1-\bar{v}^2}} \tag{36}$$

Leads to

$$\begin{cases} if \quad n^2 < 1 \rightarrow \psi < 1 \\ if \quad n^2 > 1 \rightarrow \psi > 1 \end{cases} \Rightarrow \begin{cases} \varepsilon_r\psi < 1 \\ \varepsilon_r\psi > 1 \end{cases} \tag{37}$$

The range of validity of (33) is shown in Fig. 3. It seems in Fig. 3, we note that both negative and positive signs for permittivity are acceptable.

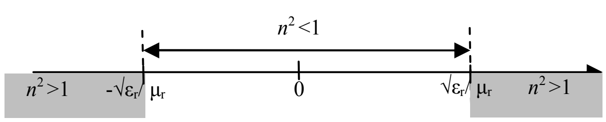


Fig. 3. Region of solution for permittivity, $n\bar{v} > 1$.

Again, TE and TM wave impedances can be expressed as

$$Z^{TE} = \frac{\omega\mu_e}{\beta_e} \equiv \pm \frac{\omega\mu a}{\sqrt{k^2a^2 - k_c^2 a}} \tag{38}$$

$$Z^{TM} = \frac{\beta_e}{\omega\varepsilon_e} \equiv \pm \frac{\sqrt{k^2a^2 - k_c^2 a}}{\omega\varepsilon a} \tag{39}$$

And both TE and TM waves may propagate in +z and -z directions simultanously, and the equivalent permittivity and permeability in +z and -z directions are positive and negative, respectively. Finally, for frequency above the cutoff frequency

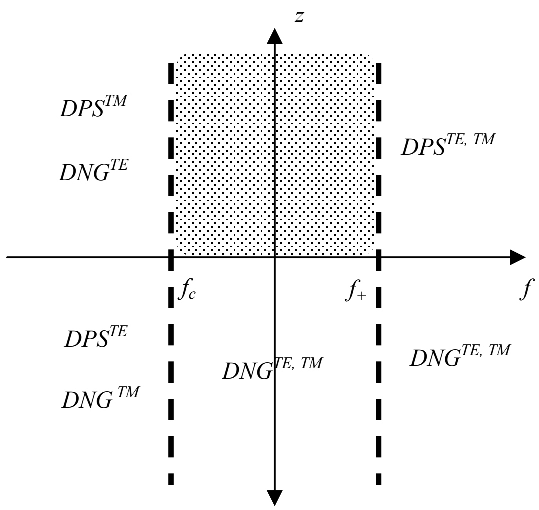
$$n_e < \sqrt{\frac{n^2-\bar{v}^2}{1-\bar{v}^2}} \tag{40}$$

One can show that

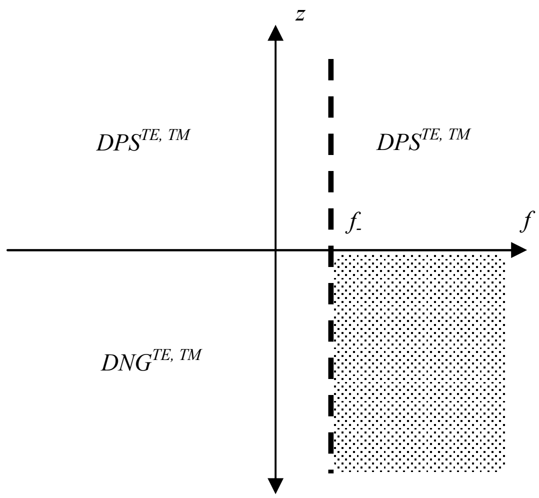
$$-\sqrt{\frac{\varepsilon_r}{\mu_r} \cdot \frac{n^2-\bar{v}^2}{1-\bar{v}^2}} < \varepsilon_e < \varepsilon_r \sqrt{\frac{\varepsilon_r}{\mu_r} \cdot \frac{n^2-\bar{v}^2}{1-\bar{v}^2}} \tag{41}$$

Based on TE and TM impedances (38), and (39) and the propagation mode [14], the only propagating mode is in +z direction and so the moving media for both TE and TM waves in +z direction act as DPS medium.

As a summary, for a slowly moving medium there are two critical frequencies, separating three frequency ranges with different type of propagation. For frequencies below the cutoff, in TE mode, the equivalent permittivity and permeability in +z direction are negative, whereas in -z direction, they are positive. Similarly, for TM mode, the equivalent permittivity and permeability in +z and -z directions are positive and negative, respectively. For frequency $f_c < f < f_+$, the only propagating mode is -z and so the moving media for both TE and TM waves in -z direction act as DNG medium. For frequency above f_+, for both TE and TM modes, the equivalent permittivity and permeability in +z and -z directions are positive and negative, respectively.



(a)



(b)

Fig. 4. Moving media as a DNG or DPS equivalent medium for each frequency band and propagation mode, (a) $n\bar{v}<1$, (b) $n\bar{v}>1$.

IV. CONCLUSION

The anomalous wave propagation in a homogeneous, isotropic and lossless medium that moves with a constant velocity along the axis of a rectangular waveguide is investigated. The correct sign of the permittivity and permeability of an equivalent stationary medium, for each TE and TM modes and for both low and high dielectric velocities are extracted and it demonstrated that this media acts as an metamaterial structure over specific frequency ranges.

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Amir Jafargholi received the PhD degree in electrical engineering from K.N. Toosi University of Technology, Tehran, Iran, in 2011. He is the coauthor of about 50 scientific contributions published in international books, journals and peer-reviewed conference proceedings. His research interest includes the applications of metamaterials in the analysis and synthesis of antennas. Dr. Jafargholi was a recipient of a Student’s Best Thesis National Festival award for his BS thesis, on May 2006. He was a recipient of the 22th Khawarizmi International and 13th Khawarizmi Youth Award on Jan. 2009 and Oct. 2011, respectively. He was also the recipient of Research Grant Awarded in Metamaterial 2010.

Ali Jafargholi was born in Tehran, Iran, on November 3, 1989. He received the B.S degree in electrical engineering from Sharif University of Technology, Tehran, Iran, in 2012, and is currently working toward the M.S degree in communication engineering. His research interests include metamaterial applications to antenna designs.