Stochastic Optimization of a Patch Antenna

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Abstract **−** The paper describes an efficient technique for optimizing the shape of the patch of a multi-band antenna by means of Genetic Algorithms and the hybrid FEM–RBCI method, for the analysis of open-boundary scattering and radiation electromagnetic field problems. The admissible rectangular patch area is logically and regularly subdivided into rectangular sub-areas, coinciding with the trace of the tetrahedral edge element mesh on the patch surface. In this way the relevant matrices of the finite element algebraic system, computed at the beginning of the optimization, remain unchanged, even if the patch is changed by inserting some metallic sub-areas. Moreover, in order to reduce the computing time of the iterative solver, the solution of a similar patch configuration is used as the initial guess for the solver.

Keywords: finite element method, genetic algorithms, micro-strip antennas, and optimization methods.

I. INTRODUCTION

Antenna design is a topic of great importance in electromagnetics. It involves the selection of physical parameters to achieve optimal gain, pattern performance, bandwidth, and so on, subject to some specified constraints. If a trial and error process is used for antenna design, the designer is required to have great experience and intuition; so innovative design methods are required [1,2]. In addition to producing results with excellent performance, this also gives unconventional and non-intuitive physical realizations.

For personal communications, multi-band antennas are of particular interest. In this paper, we consider a dual-band antenna design using a single patch, and an efficient technique is described for optimizing the shape of the patch by means of Genetic Algorithms (GA) [3,4] and the hybrid FEM/RBCI (Finite Element Method – Robin Boundary Condition Iteration) method, for the analysis of open-boundary electromagnetic scattering [5] and radiation [6] problems.

In Section 2 the FEM/RBCI method for the micro-strip antenna is briefly recalled. In Section 3 the optimization procedure is outlined. Results are shown in Section 4 and our conclusions follow in Section 5.

II. THE FEM-RBCI METHOD FOR A MICROSTRIP ANTENNA

Consider a patch antenna recessed in a perfectly conducting (PEC) plane; the cavity is filled with a homogeneous lossless material with relative dielectric permittivity ε_r and relative magnetic permeability μ_r. The original microstrip antenna is a rectangle of size $w_a \times l_a$, residing on top of a parallelepipedal cavity the dimensions of which are $w_c \times l_c$ and depth h (see Fig.1).

A relevant application is to design a patch that operates at the two frequencies of the Global Positioning System (GPS): 1227 and 1572 MHz.

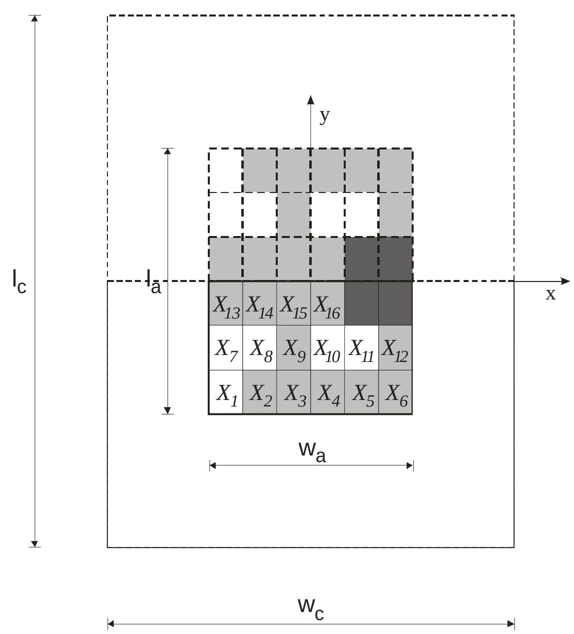

Fig. 1. Top view of cavity and patch.

In order to make the antenna work in this frequency range, the following values are selected: w_a=5.36 cm,

l_a=7.02 cm, $w_c=2w_a$, $l_c=2l_a$, h=0.48 cm, $\varepsilon_r=4.4$, $\mu_r=1$. The same device was optimized in [2]. The antenna is powered by a coaxial cable and irradiates in an unbounded vacuum medium.

To apply FEM-RBCI, the vacuum region is truncated, at a distance of d=5 cm from the cavity, to a bounded one by means of a fictitious boundary B_F, which encloses the antenna aperture and the patch (see Fig.2). In the bounded domain thus obtained, the vector Helmholtz equation holds for the electric field,

$$\nabla\times\left(\mu_r^{-1}\nabla\times\bar{E}\right)-k_0^2\varepsilon_r\bar{E}=0 \qquad (1)$$

where μ_r and ε_r are the relative magnetic permeability and electrical permittivity, respectively, and k_0 is the free-space wave number, given by $k_0^2=\omega^2\varepsilon_0\mu_0$, with ω being the angular frequency and μ_0 and ε_0 the free-space permeability and permittivity, respectively. Homogeneous Dirichlet $\left(\hat{n}\times\bar{E}=0\right)$ conditions hold on the PEC surfaces of the cavity, the PEC plane, and the patch surface.

A Robin (mixed) boundary condition is assumed on B_F,

$$\hat{n}\times\nabla\times\bar{E}+jk_0\hat{n}\times(\hat{n}\times\bar{E})=\bar{U} \qquad (2)$$

where n is the outward normal to B_F and $\bar{U}$ is an unknown vector tangent to B_F.

The internal conductor of the coaxial cable is assumed to carry an impressed density current J_{int} which represents the known source of the antenna. Since the source is electrically short and small, it can be modeled as a current filament [7]. The source can be expressed as,

$$\bar{J}_{int}=I_{int}\,\delta\left(x-x_f\right)\delta\left(y-y_f\right)\hat{z} \qquad (3)$$

where $x_f=w_a/3$ and $y_f=0$ specifies the feed position, I_{int} denotes the electric current magnitude, and $\delta(x)$ is the Dirac delta function.

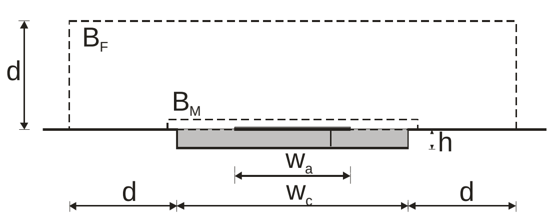

Fig. 2. Cross section of the FEM domain at plane y=0.

Discretizing the domain by tetrahedral edge elements, the FEM leads to the matrix equation,

$$\mathbf{AE}=\mathbf{B}_0+\mathbf{BU} \qquad (4)$$

where **A** is a complex and symmetric matrix, $\mathbf{B}_0$ is due to the source, **B** is a rectangular matrix, whose entries are the Kronecker delta, and links the vector $\bar{U}$ with the right hand side of the FEM system, **E** is the array of the expansion coefficients for the electric field and **U** is the array whose generic entry is given by,

$$U_j=\int_{B_F}\bar{U}\cdot\bar{w}_j\,dS \qquad (5)$$

in which $\bar{w}_j$ is the generic edge form function. Since **U** is unknown, system of equation (4) cannot be solved.

Let us now consider another surface, B_M, lying between the antenna and the fictitious boundary (see Fig. 2). At minimum, B_M can be selected as coinciding with the antenna aperture itself. The total field outside B_M can be expressed as [8],

$$\begin{aligned}\bar{E}(\bar{r})=\int_{B_M}\Big[&\bar{\bar{G}}(\bar{r},\bar{r}')\cdot\left(\hat{n}'\times\nabla'\times\bar{E}(\bar{r}')\right)+\\&+\nabla\times\bar{\bar{G}}(\bar{r},\bar{r}')\cdot\left(\hat{n}'\times\bar{E}(\bar{r}')\right)\Big]ds'\end{aligned} \qquad (6)$$

where the dyadic Green's function, which takes account of the presence of the ground plane, is given by [8],

$$\begin{aligned}\bar{\bar{G}}(\bar{r},\bar{r}')=&\,\bar{\bar{G}}_0(\bar{r},\bar{r}')-\bar{\bar{G}}_0(\bar{r},\bar{r}'')+\\&+2\hat{z}\hat{z}g_0(\bar{r},\bar{r}'')\end{aligned} \qquad (7)$$

where r'' is the symmetrical of r' with respect to the ground plane and,

$$\bar{\bar{G}}_0(\bar{r},\bar{r}')=\left(\bar{\bar{I}}+k_0^{-2}\,\nabla\nabla\right)g_0(\bar{r},\bar{r}') \qquad (8)$$

$$g_0(\bar{r},\bar{r}')=\frac{1}{4\pi\left|\bar{r}-\bar{r}'\right|}e^{-jk_0\left|\bar{r}-\bar{r}'\right|}\ . \qquad (9)$$

A similar expression for $\nabla\times\bar{E}$ is easily obtained from equation (6), so that an integral equation is derived which links $\bar{U}$ to $\bar{E}$ [5]. Note that, since B_M and B_F do not intersect with each other, singularities are avoided in this integral equation. The discrete form of the equation reads [5],

$$\mathbf{U}=\mathbf{ME} \qquad (10)$$

where **M** is a rectangular matrix in which null columns appear for the internal edges not involved in the computation.

Equations (4) and (10) together form the global algebraic system of the FEM-RBCI method, which can be conveniently solved by an iterative scheme as follows:

1) Select an arbitrary first guess for **U**; 2) solve equation (4) for **E**, by means of a standard conjugate gradient solver (COCG); 3) obtain an improved guess for **U** by means of equation (10); 4) if the procedure has converged, i.e. a user-selected end iteration tolerance τ is satisfied, stop; otherwise go to 2). This scheme can be seen as a two-block Gauss-Seidel iterative method,

$$\begin{aligned} \mathbf{E}^{(n)} &= \mathbf{A}^{-1}\mathbf{B_0} + \mathbf{A}^{-1}\mathbf{B}\mathbf{U}^{(n)} \\ \mathbf{U}^{(n+1)} &= \mathbf{M}\mathbf{E}^{(n)} \end{aligned} . \qquad (11)$$

In this way the symmetry of matrix $\mathbf{A}$ is fully exploited. Moreover, since this procedure converges in a few iterations, it also minimizes the number of multiplications of the dense matrix $\mathbf{M}$ by a vector.

III. OPTIMIZATION OF THE ANTENNA

The admissible rectangular patch surface is regularly subdivided into rectangular sub-areas, which coincide with the trace of the tetrahedral finite element mesh on the patch itself (see Fig. 1). The optimal antenna is designed by making these sub-areas metallic or not, although the four sub-areas at the middle-right part of the patch are always filled with metal in order to fix the feed point.

Only the lower half of the domain is considered, in order to exploit the symmetry of the problem. We can therefore operate on 16 sub areas, the design variables x_1, x_2, …, x_{16}, each of which can assume the value 1 (filled) or 0 (empty): they form the GA chromosome, giving a total of 2^{16} different configurations.

The objective function to minimize is chosen as [2],

$$f = |S_{11}|_1 + 0.1|S_{11}|_2 + |S_{11}|_3 \qquad (12)$$

where $|S_{11}|_k$, for k=1, 2, 3, refers to the return loss at the three frequency points: 1227, 1400 and 1572 MHz, respectively. The return loss is defined as,

$$|S_{11}| = \left|\frac{Z_{in} - Z_0}{Z_{in} + Z_0}\right| \qquad (13)$$

where Z_{in} is the input impedance at the feed and Z_0=50Ω. After the electric field E along the source edges has been obtained, by means of the FEM/RBCI, the voltage drop along the current filament can be calculated. Thus, the input impedance Z_{in} can be obtained.

We point out that the objective function f as given in equation (12) will not guarantee pattern integrity for the shaped patch: the objective function can be combined with a penalization criterion to drive the GA search towards topologies for which pattern connectivity is maintained [9]. However, at this point our focus will only be on optimization of the return loss.

During the optimization procedure, the mesh remains unchanged, hence the domain discretization is only performed once at the beginning of the optimization. Moreover, before starting the evaluation of $|S_{11}|_k$ for a GA population, a structure in which all the sub-areas are non-metallic, except for those near the source current filament (dark gray areas in Fig.1), is selected. For this first patch configuration, corresponding to the null chromosome, the FEM/RBCI matrices **A**, **B** and **M** are computed and stored. Adding a sub-area of metal to the patch is equivalent to forcing a homogeneous Dirichlet boundary condition for the electric field on the edges lying on that sub-area. This, in turn, is equivalent to dropping the corresponding rows and columns in matrix **A**. Matrices **B** and **M** remain unchanged for a fixed k. In this way, the whole preprocessing phase consists of modifying some Dirichlet boundary conditions, and the FEM matrices are recomputed only three times, once for each frequency in equation (12), during the fitness evaluation of a GA population, thus saving a great amount of computing time.

Moreover, before evaluating the objective functions, the GA population is ordered, taking into account the Hamming distance between the chromosomes, starting from the chromosome with more bits equal to 0, in such a way that configurations having similar patch shapes will be contiguously ordered. Then, by using solution **E** for the electric field of the previous configuration as the initial guess for the iterative conjugate gradient solver (COCG) in the FEM/RBCI analysis of the next configuration, the number of iterations, of both the solver and the FEM/RBCI, is reduced and a further saving in the overall computing time is obtained.

Fig.3 is a flowchart of the whole optimization procedure as described above.

IV. RESULTS

The formulation described in Section 2 was implemented in ELFIN [10], a finite element code developed by the authors for electromagnetic CAD research, which employs zero-order tetrahedral edge elements to solve three-dimensional electromagnetic scattering and radiation problems.

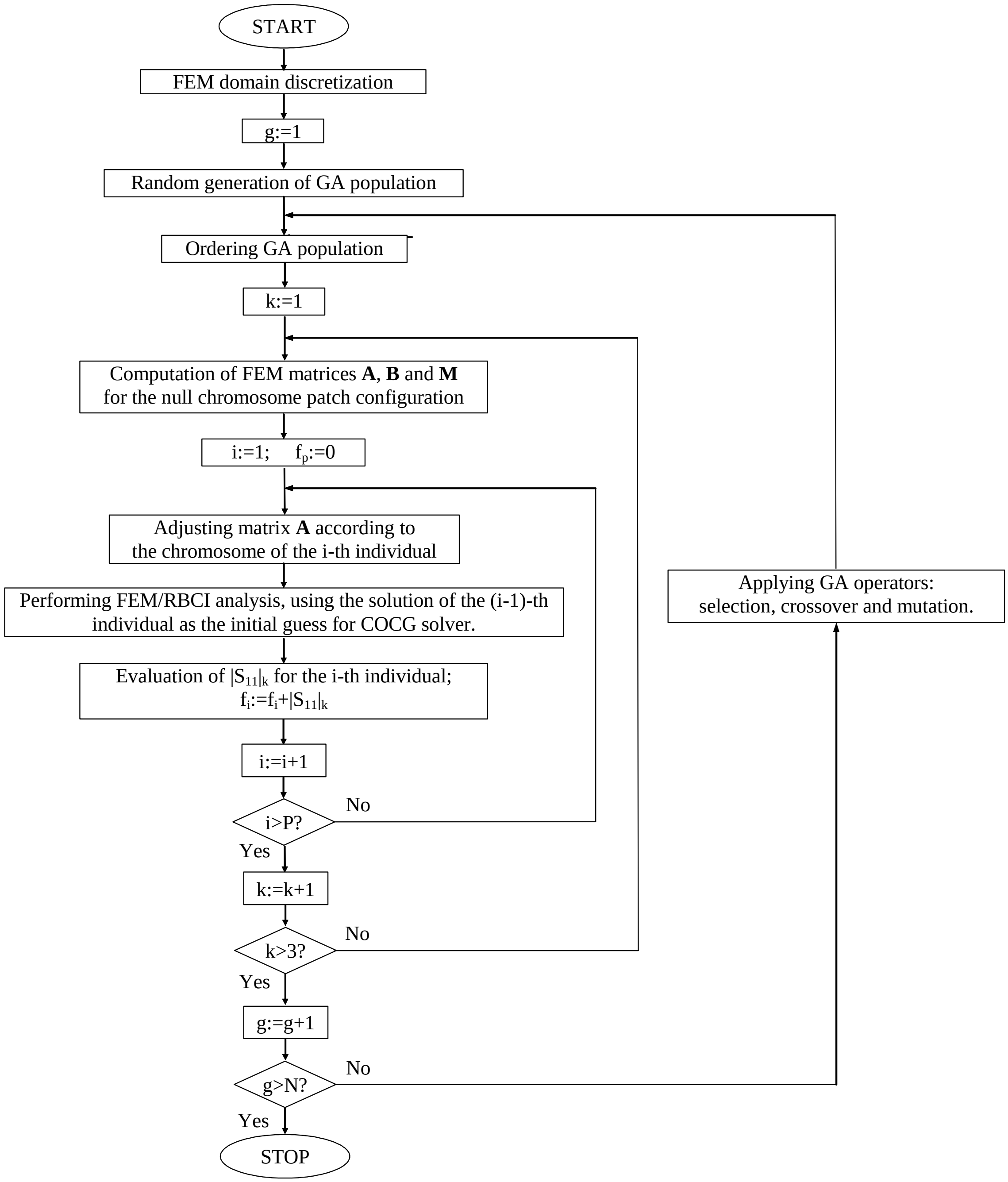

Fig. 3. A flowchart of the whole optimization procedure.

The finite element mesh used was made up of 10133 elements with 12661 edges. The feed source was discretized with four edges.

The COCG solver [11] with diagonal preconditioning was used for the solution of the various FEM complex symmetric systems of equation (4); the stopping criterion used for COCG was that described as criterion 2 in [12], with an end-iteration tolerance of δ=0.05%. The RBCI end-iteration tolerance was set to τ = 1% and convergence was reached, on average, in about five iterations.

In order to calculate the return loss, for a fixed configuration and a single frequency, the following computing times are required: a time T_p for the preprocessing phase, a time T_c for the construction of the FEM/RBCI system, i.e. for the computation of matrices

A, **B** and **M**, and a time T_s for the solution of the global algebraic system, equations (4) and (10). The whole computing time T for a single evaluation of |S11| is thus,

$$T = T_p + T_c + T_s \tag{14}$$

with the above data, T_p is about 35% of T, T_c is about 10% of T and T_s is about 55% of T. Solving the problem on a 3.2 GHz Pentium IV workstation with 4Gb RAM, T is about 50 s.

In optimization by GAs, the population size was set to P=30 individuals: each individual is a 16-bit chromosome relating to a patch configuration. The reproduction process, which randomly creates a new generation from the old one, was chosen by tournament selection with a shuffling technique, to choose random pairs for mating, and elitism was also used. The crossover process, by means of which individuals exchange portions of chromosomes from one generation to the other, was 2-point crossover with a probability p_c varying from 0.3 to 0.7 as the optimization proceeds. The mutation process, by means of which some random flips in the chromosomes of an individual are made, was employed with a probability p_m varying from 0.05 to 0.01 as the optimization proceeds. This choice of GA parameters is the same as discussed in [4]. The evolution was halted after N=30 generations.

The whole computing time T_{CPU} to find the optimum for a standard optimization procedure is therefore,

$$T_{CPU} = N \times P \times 3 \times T \tag{15}$$

Using the strategy described in Section 3, the preprocessor is called only once at the start of the optimization so T_p is added just once. Moreover, the computation of matrices **A**, **B** and **M**, occurs only three times for each GA generation. Finally, using the solution of the previous configuration as the initial guess for the iterative conjugate gradient solver (COCG) in the FEM/RBCI analysis, the time T_s is, on average, reduced by about 7% (about 15% when the new configuration is very similar to the previous one). Hence the whole computing time T_{cpu} is reduced to,

$$T_{CPU} = T_p + N \times 3 \times T_c + N \times P \times 3 \times 0.93 \times T_s \tag{16}$$

Further computing time is saved when an individual has a null Hamming distance from the previous one: in this case, in fact, the fitness is not recalculated but is simply allotted the same value as its twin.

Implementing all these tricks, the time required to carry out the optimization, T_{CPU} , is reduced from 1.35 10^5 s to 0.6 10^5 s.

The optimum configuration for the patch antenna was that with sub areas x_1, x_7, x_8, x_{10}, x_{11} empty and all the others filled with metal (light gray areas in Fig. 1). The objective function value calculated for the optimal configuration is f_{min} = 0.94. The return loss of the optimized patch is shown in Fig. 4, as per design; the resonant frequencies occur at 1.23 and 1.57 GHz. The best (minimum) and average values of the objective function f, through the various generations, are plotted in Fig. 5. The history of GA optimization shows a good convergence by the algorithm.

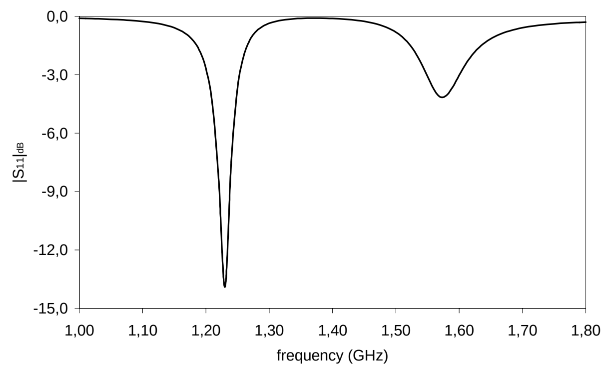

Fig. 4. Return loss of the optimal patch.

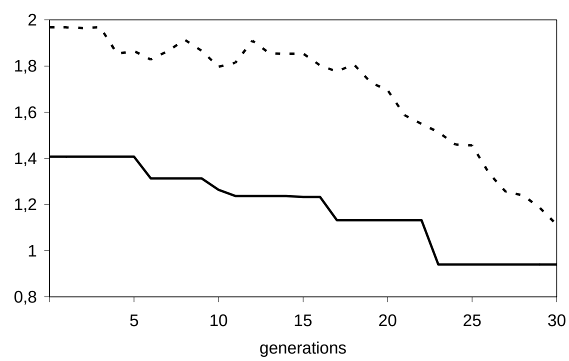

Fig. 5. Best (solid) and average (dotted) objective function f over GA generations.

V. CONCLUSIONS

In this paper optimization of a microstrip antenna has been performed by means of Genetic Algorithms and a hybrid Finite Element – Robin Boundary Condition Iteration method. The goal was to design a patch antenna for personal communications that operates at the two GPS frequencies.

A strategy to make the optimization procedure more efficient has been outlined. The optimum was reached in about half the time required by the standard procedure.

The optimized patch performs well in the design frequency bandwidth and has an unconventional and non-intuitive shape.

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