# THE APPLICATION OF THE FINITE ELEMENT METHOD IN DESIGN OF ELECTRIC MOTORS

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*ABSTRACT. The classical approach to design of electric motors is based on the concept of simplified magnetic circuit analysis. This approach fails in today's design situation due to utilization of new materials and new designs. Application of sophisticated numerical methods becomes inevitable. Some problems arising in the application of the finite element method in design of electric motors are discussed in this paper.*

*Electric motors are always part of a system and their behaviour within the system, which is characterized by integral parameters (torques and reactances), should be known at the design stage. Computation of these parameters from a finite element magnetic field solution is described here.*

*The computation of torque/angle characteristics from finite element field solution by application of two basic approaches:*
- *global virtual work method and cubic spline interpolation technique,*
- *Maxwell stress tensor integration,*

*is described. The applicability of both approaches is illustrated by computation of the torque in a switched reluctance motor and the advantages of virtual work approach are emphasised.*

*The main problem in application of the finite element method to computation of magnetic fields in electric motors is that the field sources (currents) and load angle are unknown. External environment (terminal voltage and mechanical load) are known, and the magnetic field solution is iterated until the external constraints are satisfied. To avoid finite element mesh rotation an iterative process was implemented in which only the fundamental harmonic of the lumped stator winding distribution is taken into consideration instead of the three-phase winding excitation. The direct and quadrature reactances are computed from the finite element magnetic field solution utilizing flux linkage and stored energy approaches. The procedure is illustrated by computation of the reactances of a permanent magnet synchronous electric motor.*

## 1. INTRODUCTION

Design of electric motors is a process of synthesis based on geometrical modelling and analysis. The traditional approach to analysis in the design of electric motors was based on the concept of magnetic circuit analysis resulting with simple analytical models. These models incorporate equivalent circuits postulated on the basis of experience and intuition. Results of such an approach are qualitatively correct but quantitatively inaccurate and have to be corrected with empirically introduced correction parameters based on previous experience and measurements on prototypes. These parameters are usually valid for very narrow classes of motors significantly similar to those which were produced and measured before the new design. From the designers point of view the main advantage of this approach is its simplicity and ease of application.

This traditional approach to design of electric motors fails in new situations. Utilization of new magnetic materials enables developing of new designs. Precise prediction of motor characteristics before a motor is manufactured is one of the essential prerequisites for good, concurrent design and cost-effective production. Accordingly, more sophisticated methods of analysis based on electromagnetic field theory which enable more accurate modelling of electric motors are required to be implemented in the design procedures. Numerical field analysis is the only available tool capable of dealing with problems which arise in the design of the new generation of electric motors. The most widely spread numerical method for electromagnetic field analysis in electric motors is the finite element method (FEM). Its application to the calculation of two main integral characteristics of electric motors: torques and reactances is described here.

## 2. CALCULATION OF TORQUES

One of the most important characteristics of electric motors is torque as a function of rotor position. It can be derived from the FEM solution of the magnetic field in the electric motor. Two distinct approaches for directly calculating the torque from the FEM solution are Maxwell stress tensor integration and the virtual work approach.

## 2.1 Maxwell Stresses

The torque on a rigid body enclosed by a surface S via Maxwell's magnetic stress tensor integration can be obtained by calculating the surface integral [1]:

$$\vec{T}=\oint_S (\vec{r}\times\bar{\bar{t}})\,dS \tag{1}$$

where n is an unit vector normal to the integration surface S, r is a distance vector, and t is Maxwell stress tensor defined by:

$$\bar{\bar{t}}=\vec{H}\cdot(\vec{B}\cdot\vec{n})-\frac{1}{2}(\vec{H}\cdot\vec{B})\cdot\vec{n} \tag{2}$$

In the case of the exact solution the integral in (1) is independent of the chosen integration surface. Approximate and discrete nature of the FEM solution introduces dependency of the accuracy of torque calculation upon the accuracy of the calculation of the local flux densities and upon the choice of the integration surface (3D models) or integration contour (2D models). High accuracy of the calculation of the local flux densities, which are obtained from potential solutions (scalar or vector) by differentiation, can be assured by increasing the FEM mesh density. The most convenient choice for the integration surface (contour) in the FEM models of electrical motors is in the airgap between the stator and the rotor. In a practical applications of this approach it is advisable to evaluate the torque using several surfaces (contours) and to average the results, or to apply relatively complicated iterative methods for selection of the best surface (contour) [2].The application of this approach in normal design practice is very complicated.

The main advantage of the Maxwell stress tensor integration approach to torque calculations is that it requires only one field solution to obtain the torque for one position. If it is necessary to obtain a complete torque-angle characteristics this advantage becomes obsolete.

## 2.2 Virtual Work

Since the extremization solution procedure used in the finite element method optimizes the calculation of stored energy, the virtual work method seems to be more stable and accurate approach to torque calculation. The torque can be calculated from the coenergy of a system by [3,4]:

$$T=\frac{\partial W_c}{\partial \alpha} \tag{3}$$

where $W_c$ is the total coenergy of a system and $\alpha$ is the angular displacement. Total coenergy of a system can be calculated from the FEM solution by:

$$W_c=\sum_e \left\{\int_{V^e} H^e B^e dV^e-\int_{V^e}\left(\int_0^{B^e} H^e dB^e\right)dV^e\right\} \tag{4}$$

The main disadvantage of this approach is the calculation of the coenergy derivatives with respect to a small perturbation of the rotor position. The numerical realisation of this method can give rise to a significant round-off error in computing a finite difference approximation of the required derivative because of near identical coenergies for small displacements. On the other hand using increased displacements of the rotor in order to achieve greater differences in the system coenergy decreases the accuracy of the finite difference approximation.
In order to overcome those difficulties, a cubic spline interpolation of the coenergy function through points calculated in several rotor positions is introduced [5]. The interpolation of the coenergy on the i-th segment is defined by [6]:

$$\begin{aligned}W_c(\alpha)=\;&m_{i-1}\frac{(\alpha_i-\alpha)^2(\alpha-\alpha_{i-1})}{h_{i-1}^2}-\\&-m_i\frac{(\alpha-\alpha_{i-1})^2(\alpha_i-\alpha)}{h_{i-1}^2}+\\&+y_{i-1}\frac{(\alpha_i-\alpha)^2[2(\alpha-\alpha_{i-1})+h_{i-1}]}{h_{i-1}^3}+\\&+y_i\frac{(\alpha-\alpha_{i-1})^2\;[2(\alpha_i-\alpha)+h_{i-1}]}{h_{i-1}^3}\end{aligned} \tag{5}$$

where:

$$h_i=\alpha_{i+1}-\alpha_i \;\;;\;\; m_i=\frac{\partial W_c(\alpha)}{\partial\alpha}\Big|_{\alpha=\alpha_i} \;\;;\;\; i=0,1...n$$
$$y_i=W_c(\alpha_i) \;\;;\;\; \alpha\in[\alpha_{i-1},\alpha_i] \;\;;\;\; i=1,...n$$

The unknown coefficients of the interpolation $m_i$ are derived from the condition of smooth joints (equal first derivatives) at points $\alpha_i$ (i=1,..n-1). The first derivatives at the first point (i=0) and the last point (i=n) of interpolation are set to the zero, which is natural for this problem. The torque is calculated by derivation of the cubic spline interpolation of the coenergy function with respect to displacement α.

### 2.3 Example: Torque in a Switched Reluctance Motor

The usage of switched reluctance motors (SRM) has increased in recent years due to their simplicity and controllability. As a variable speed driver they are much more efficient compared to variable speed induction motors. The motor has salient poles on both the stator and the rotor, the windings on the stator are of simple form and there are no windings on the rotor. This results in a relatively simple construction of such motors. The currents in the stator circuits are switched on and off in accordance with the rotor position and with simple control the motor develops the torque-speed characteristics typical for series-connected d.c. motors. The main characteristics of the magnetic field analysis in SRM are:

- complex geometry and very small airgap between stator and rotor,
- deep saturation of magnetic material in normal operation,
- eddy currents can be neglected.
- the influence of edge effects must be taken into account.

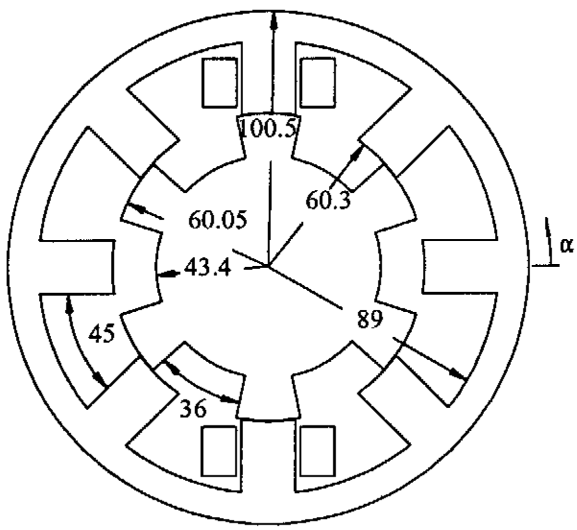


**Figure 2.1** Cross-section of SRM motor

Taking into consideration the aforesaid one can conclude that a 3D nonlinear static model of SRM is sufficient for design analysis. The importance of 3D calculation is emphasized in the case of deep saturation of magnetic material and/or in the case of rotor-stator position with maximum reluctance. In those cases the magnetic field is leaked out of iron parts of the motor.

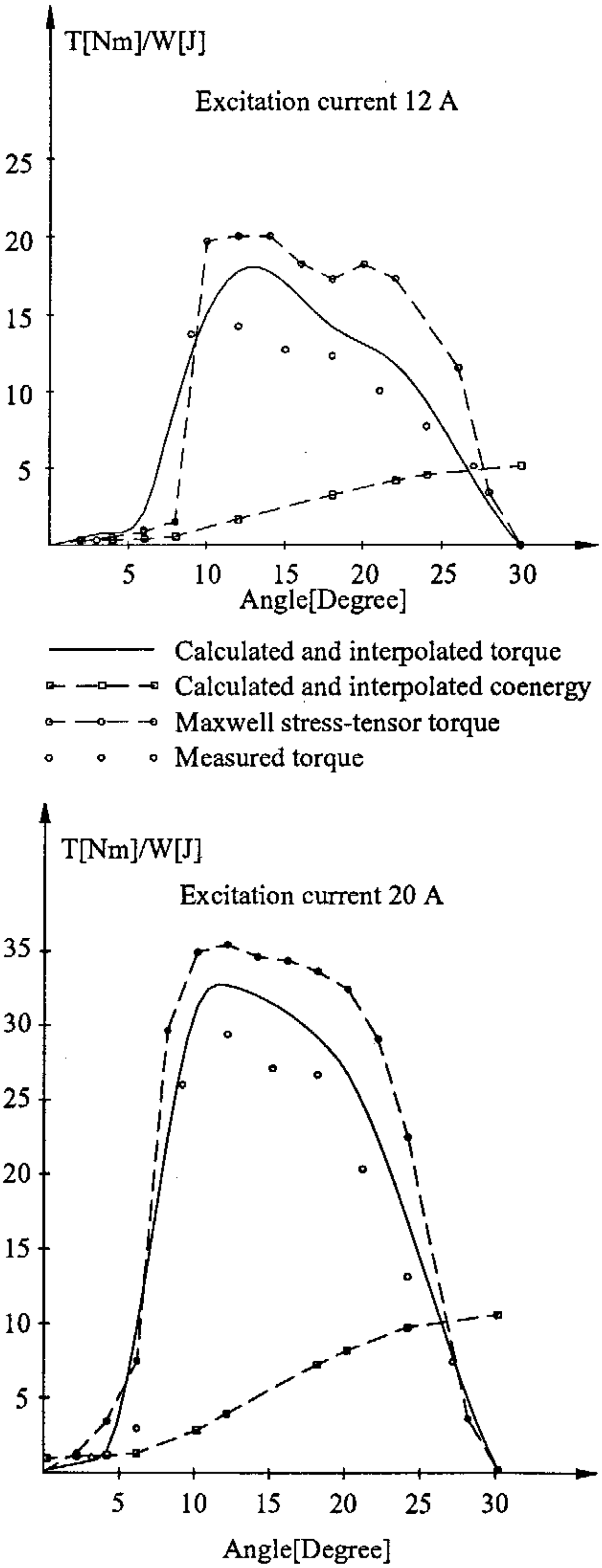


**Figure 2.2** Calculated and measured torques and coenergies

Because of the static model, a total scalar potential approach to the numerical field calculation is chosen. The main advantages of this approach are [7]:

- one unknown per node,
- current regions are not included in the discretization,
- the influence of current sources is taken into account by applying the Biot-Savart law.

The finite element discretization of the SRM model is based on the application of twenty-node isoparametric brick elements. This ensures good approximation of the motor geometry as well as giving a good approximation to the field values (gradients of the potentials). The space around the active part of the SRM must be included in the finite element mesh.

The above mentioned approach was applied to the computation of the torque/angle characteristics of the SRM shown in Figure 2.1. The surface for Maxwell stress tensor integration was chosen to be a cylinder through the middle of the airgap. The airgap was discretized by three layers of finite elements. Total coenergy of the system was calculated from the same FEM solution. The resulting curves showing the comparison of the virtual work approach and the Maxwell stress tensor integration with the measurements are shown in Figure 2.2.

## 3. INDUCTANCE CALCULATIONS

The main objective of inductance calculations in electric motor design is the computation of the reactances in the direct and quadrature axis system ($X_d$ and $X_q$). The two-axis system was introduced into classical theory of electric machines as a mean of facilitating analysis of salient pole machines [8]. Reactances $X_d$ and $X_q$ are the basis for further representation of the motor in system studies. Two fundamental approaches to inductance calculations are:

- calculation of inductance from flux linkage,
- calculation of inductance from stored magnetic energy.

Both approaches can be applied in the case of the FEM magnetic field solution.

### 3.1 FEM Modelling of Electric Motors

A specific problem in applying the FEM to the design of electric motors is that in most cases the source (currents in stator slots) and load angle (relative position rotor-stator) are not specified. Under normal operating conditions for electric motors the state of the system is defined by the external quantities (terminal voltage and mechanical load) while source currents and load angle which are necessary for the FEM problem formulation are unknown. Solving a problem under these new constraints requires an iterative application of the FEM solution. The iterative scheme starts with the initial estimate of the load angle and field source currents which can be obtained by the classical design approach. The FEM solution is then applied and a first estimate for problem external sources can be determined. The derived terminal voltage is then compared with the specified conditions and the inputs to the FEM solution are iterated until a solution which agrees with the load point specification is obtained.

One way of forming the FEM models of electric motors is to specify source currents densities in stator slots which correspond to phase currents determined from the three phase excitation current system. Utilization using that approach requires rotating the FEM mesh as load angle varies, which complicates the iterative scheme of the FEM application. In most electric motors the fundamental component of excitation is dominant, while higher harmonics can be neglected. It enables decomposition of the lumped stator winding distribution into Fourier components and only the fundamental component of the current sheet is retained [9]. Stator currents in the FEM model of the motor are then assigned to the stator slots proportional to the area under the current sheet density distribution associated with each slot. The three phase sum of the fundamental components rotates in synchronism with the rotor, which means that changing load angle can be simulated by changes in the fundamental current sheet distribution while the FEM mesh remains the same.

### 3.2 Definition of Reactances

The quantities obtainable from the FEM magnetic field solution are direct and quadrature axis flux linkages and energies. In order to find them, taking into account saturation of magnetic material, a three step procedure for FEM solution is given in [10]:

Step 1. Nonlinear solution of the FEM problem where the sources are excitation current together with total armature current (both d and q components)

Step 2. The permeability for each element achieved in nonlinear iteration procedure (step 1.) is fixed.

Step 3. Two linear solutions of the FEM problem with separately applied d and q components of armature current while excitation current is set to zero. Permeabilities in the FEM mesh for this step are the ones fixed in step 2.

Direct axis flux linkage $\psi_d$ and stored magnetic energy $W_d$ are calculated from the FEM solution in step 3 when the direct axis component of armature current is applied, while quadrature axis flux linkage $\psi_q$ and stored magnetic energy $W_q$ are calculated from the FEM solution in step 3 when the quadrature axis component of armature current is applied. The reactances are then defined as:

$$X_d = \frac{\omega\,\Psi_d}{I_d} \quad ; \quad X_d = \frac{2\,\omega\,W_d}{3\,I_d^2} \tag{7}$$

$$X_q = \frac{\omega\,\Psi_q}{I_q} \quad ; \quad X_q = \frac{2\,\omega\,W_q}{3\,I_q^2} \tag{8}$$

where $I_d$ and $I_q$ are the direct and quadrature axis components of armature current respectively.

### 3.3 Example: Synchronous Permanent Magnet Motor (SPMM)

The high energy density of rare earth permanent magnets and relatively low costs of their utilization has permitted them to replace classical DC excitation systems of electric motors. The elimination of the excitation winding (copper losses, brushes...) results in more reliable and mechanically simpler motor. The rare earth permanent magnets have near linear characteristics over normal operating conditions. This fact greatly simplifies their modelling in the FEM. They can be replaced with simple current sheets surrounding a material having a permeability equal to the recoil permeability of the permanent magnet material which is, in the case of rare earth materials, slightly greater than the one of free space. The geometry of such motors enables utilization of a two-dimensional model. FEM approach based on magnetic vector potential is applied in the solution procedure. The cross-section of the analyzed motor can be seen in Figures 3.2 to 3.4. The fundamental component of decomposition of lumped stator winding into Fourier component is given as [9]:

$$ni(\xi) = \frac{9}{\pi^2} N_s \sqrt{2}\, I_a \sin(\xi - \alpha) \tag{9}$$

where:

- $I_a$ is the root mean square value of the armature current,
- $N_s$ is the number of turns in the stator winding per pole and per phase,
- $\xi$ is the electrical angle with respect to the d axis,
- $\alpha$ is the electrical angle between the phasor of the armature current and the d axis.

Current $I_i$ associated with i-th slot of the stator winding is:

$$I_i = \frac{9}{\pi^2} N_s \sqrt{2}\, I_a \sin(\xi - \alpha)\, \Delta\xi \tag{10}$$

A set of equations which define the external environment in steady state operation of a permanent magnet electric motors can be deduced from the d-q theory by adopting the Park transformation in rotor reference frame. The equations are [11]:

$$\begin{aligned} V_q &= RI_q + \omega\,\phi_d \\ V_d &= RI_d - \omega\,\phi_q \end{aligned} \tag{11}$$

$V_q$ and $V_d$ are q and d components of the terminal voltage, $I_q$ and $I_d$ are the corresponding current components, R is ohmic resistance of the winding, $\omega$ is steady state frequency and $\Phi_d$ and $\Phi_q$ are the corresponding flux linkages. These equations can be expressed in phasor form and combined to obtain the phase stator voltage $V_a$ in phasor form:

$$\bar{V}_a = (R + j\,X_q)\,\bar{I}_a + j(X_d - X_q)\,\bar{I}_d + \bar{E}_0 \tag{12}$$

$E_0$ is the open circuit voltage ($I_d$=$I_q$=0) resulting only from permanent magnets (excitation) and can be considered as a constant. This voltage is computed from the FEM solution of the model excited only by permanent magnets. A phasor diagram derived from (12) is presented in Figure 3.1.

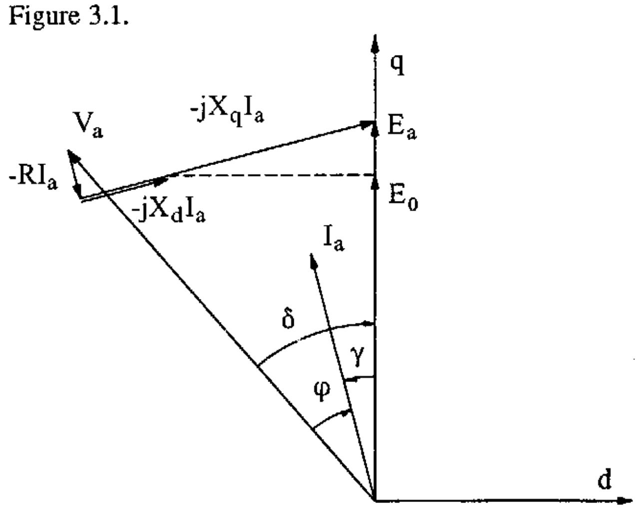


**Figure 3.1** Phasor diagram of SPMM

Unknown reactances $X_d$ and $X_q$ are obtained from the FEM field solution using the idea of distributed turns in the flux linkage calculations as described in [10]:

$$\Psi_d = \int_0^{\pi} 2A(\xi)\,\frac{6N_s}{\pi^2}\sin\xi\,d\xi \tag{13}$$

$$\Psi_q = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2A(\xi)\,\frac{6N_s}{\pi^2}\cos\xi\,d\xi \tag{14}$$

Stored magnetic energy is calculated from the FEM solution by:

$$W_m = \sum_e \int_{S^e} \Big( \int_0^{B^e} H^e\,dB^e \Big)\,dS^e \tag{15}$$

The iterative procedure described in section 3.1 was performed and after three iterations an agreeable solution ( successive solutions for $X_d$ and $X_q$ differed by less than 3.5%) was obtained. FEM field solutions for step 1 and step 3 in the third iteration are illustrated in Figures 3.2 to 3.4.

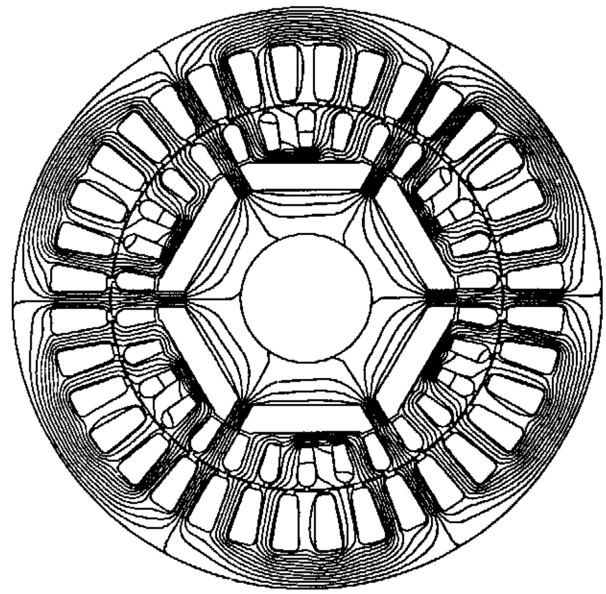

**Figure 3.3** Linear FEM solution in q axis (third iteration, step 3)

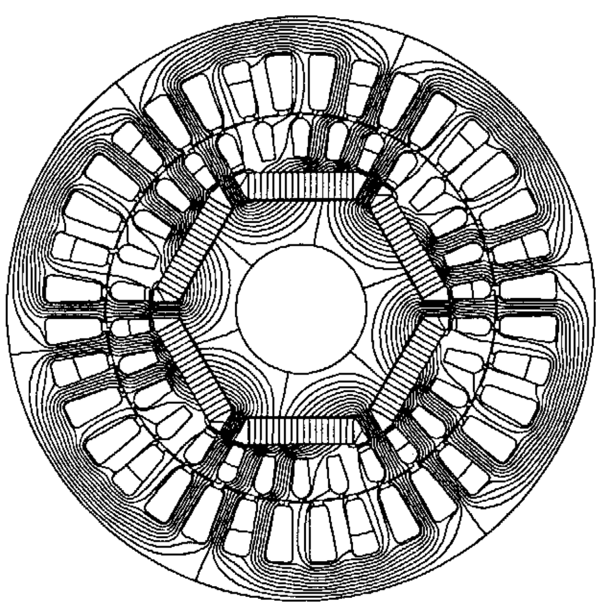

**Figure 3.2** Nonlinear FEM magnetic field solution (third iteration, step 1)

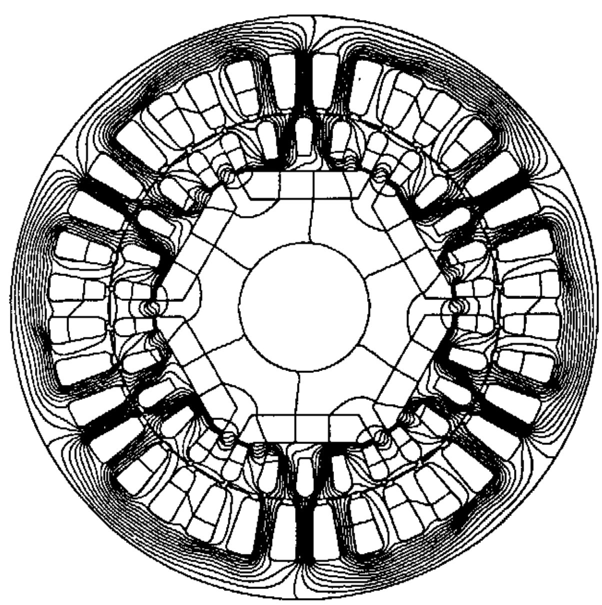

**Figure 3.4** Linear FEM solution in d axis (third iteration, step 3)

The results of the computation of the reactances calculated from flux linkages as well as the reactances calculated from stored energy are given in Table 1.

| | Energy | | Flux Linkage | |
|---|---|---|---|---|
| | $X_d$ | $X_q$ | $X_d$ | $X_q$ |
| Iteration 1 | 55.44 | 87.83 | 53.42 | 78.88 |
| Iteration 2 | 43.97 | 93.80 | 42.80 | 89.82 |
| Iteration 3 | 42.76 | 96.50 | 41.60 | 92.15 |

**Table 1.** Reactances through iterations 1 to 3

## 4. CONCLUSIONS

The application of the finite element method in computation of integral parameters of electric motors cannot be avoided in today's design practice. The problem in applying of the method is that a certain level of specialist's knowledge is required from a designer. Some aspects of that knowledge are clarified in this paper. The most important parameters of electric motors are torques and reactances. In order to calculate them additional computations must be performed after the FEM solution.

The torques can be computed by the application of Maxwell stress tensor integration or by the virtual work approach. The virtual work approach is improved by the introduction of cubic spline interpolation of coenergy/angle dependence. This results in the smooth approximation of the coenergy/angle curve and well defined differentiation with respect to rotor/stator position. The method is based on computation of the total coenergy of a system and thus less sensitive to meshing of the model. The described approach was applied on a test example and compared with the Maxwell stress method and measurements.

Computation of reactances of electric motors requires iterative application of FEM magnetic field solution because the field sources and load angle are unknown. The iterative procedure is simplified because only the fundamental component of the lumped stator winding distribution is taken into consideration instead of three-phase winding excitation. Three phase sum of the fundamental components rotates in synchronism with the rotor and rotating of rotor the FEM mesh is unnecessary. Direct and quadrature axis reactances are computed from the FEM solution by flux linkage and stored energy approaches. The method was tested on a synchronous permanent magnet motor.

The final conclusion is that the stored energy approach to torque and inductance calculations is better for use in traditional design procedures because of the nature the FEM (minimization of energy functional) and weak dependence on local errors due to bad meshing of the model.

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