Convex Optimization Based Sidelobe Suppression for Adaptive Beamforming with Subarray Partition

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Abstract **─** This paper proposes a convex optimization based method to suppress sidelobe in adaptive beamforming at subarray level. Usually phase shift rather than amplitude tapering is implemented at element level to maximize signal to noise ratio. Nevertheless, sidelobe control can be realized at subarray level. The proposed approach is realized by adding a constraint minimizing the difference between weights at subarray level and element-level Chebyshev synthesis into the optimal conditions. Compared with penalty function method, simulations show that for a uniform linear array, the proposed method can suppress sidelobe level considerably, especially the lobes close to mainlobe. Furthermore, it is able to produce better shaped main lobe, which is extremely close to the referenced pattern.

Index Terms - Adaptive beamforming, convex optimization, sidelobe control, and subarray level.

I. INTRODUCTION

Adaptive beamforming is a significant realm in phase array radar. To reduce the complexity of hardware and alleviate computational load, subarray partition is usually considered [1, 2]. In addition, low sidelobe is often required to guarantee the performance against non-stationary cluster or reverberation [3, 4]. Many pattern synthesizing methods were developed to generate low sidelobe [5-7]. However, element tapering reduces the global gain of antennas, which results in signal-to-noise (SNR) loss. Thus, forming adaptive pattern with low sidelobes digitally at subarray level is a feasible solution.

One of the most used algorithms is minimized variance distort response (MVDR). Its apparent disadvantage is high sidelobes when sample matrix is used instead of the true covariance matrix [4]. Much relevant work has been done in recent years to deal with sidelobe controlling. Carlson proposed the diagonal sample matrix inversion (LSMI) [8] to solve the problem by adding an appropriate number to the covariance matrix. It was proved to be simple and effective, but there is no close solution for the optimal number. Penalty function method has to select a loading value k^2 to determine the adaptive weights. As the value of k^2 increases, the property of sidelobe suppression improves but interference cancelling degrades [3]. Second order cone (SOC) [9] is the method using optimization to control sidelobe. Although the method in [9] can get desired low sidelobe exactly, sidelobe region and sampling density should be defined beforehand. A small number of samplings may result in unexpected high sidelobe jittering, while a large number of samplings will lead to intensive computation. Furthermore, inappropriate sidelobe region definition may cause high sidelobe close to the main lobe in [9]. Sparse constraint is utilized to suppress the sidelobe level in [10]. However, it may fail to null mainlobe interference when its mainlobe region is set relative broad to enhance robustness of the beamformer against steering vector error. More recently, iterative algorithms based on evolutionary has been exploited in pattern synthesis [11, 12]. Despite their advantages, they have the drawback of complicated feeding network and high computational burden [13].

This paper proposes a new alternative convex optimization based approach to suppress sidelobe in adaptive beamforming at subarray level. It saves

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the trouble of definition of threshold, sidelobe region, and sampling density. We construct the formula by imposing interferences cancelling on optimization conditions while setting object to minimize the difference between element level equivalent weights and Chebyshev tapering. Simulation results indicate that the convex optimization based approach can suppress sidelobe effectively, which simultaneously tune out inferences from sidelobe or mainlobe. The validity and feasibility of the proposed algorithm are verified with Matlab simulation.

II. SIGNAL MODEL

A uniform linear array (ULA) with N isotropic elements is considered in this paper. Interval between elements is half of the carrier wavelength, i.e., $d = \lambda/2$. The array is divided into L non-overlapped subarrays. When K far field narrow band signals from θ_1, θ_2,...., θ_K are imposed on the array, the received data at element level $\vec{x}_{ele}$ is [14],

$$\vec{x}_{ele}(m) = A\vec{s}(m) + N(m) \tag{1}$$

where $\vec{s}(m) = [s_1(m),...,s_K(m)]^T$, represents the K incident signals at the m^{th} snapshot. $(.)^T$ denotes the transposition operator. $N(m) = [n_1(m), n_2(m),..., n_N(m)]^T$ is the Gaussian sensor noise with zero mean and variance $\sigma_n^{\ 2}$. A is an array manifold vector and $A = [\vec{a}(\theta_1), \vec{a}(\theta_2),..., \vec{a}(\theta_K)]$. $\vec{a}(\theta_i)$ is the steering vector and $\vec{a}(\theta_i) = [e^{j\varphi_1}\ e^{j\varphi_2}\ ...\ e^{j\varphi_N}]^T$. φ_n is the phase difference of the n^{th} sensor relative to the origin from θ_i. Thus φ_n is defined as,

$$\varphi_n = \frac{2\pi \sin\theta_i}{\lambda} \cdot x_n \tag{2}$$

where x_n is the coordinate of n^{th} sensor. It is assumed that the array is symmetric to the origin and N is oven. Thus,

$$x_n = \left(n - \frac{N+1}{2}\right)d\ , n = 1, 2, ..., N\ .$$

The element to subarray transformation matrix is given by [2],

$$T_d = D_{\vec{w}} \cdot D_{\theta_d} \cdot T\ , \tag{3}$$

where θ_d is the direction of the desired signal. $\vec{w}$ is an $N\times1$ vector, denoting element amplitude tapering. T is an $N\times L$ matrix, containing zeros and ones, which are determined by the division of subarrays. Both $D_{\theta d}$ and $D_{\vec{w}}$ are diagonal matrices, i.e., $D_{\theta_d} = diag\{a_1(\theta_d), a_2(\theta_d),..., a_N(\theta_d)\}$, $D_{\vec{w}} = diag\{w_1, w_2,..., w_N\}$. When no amplitude tapering is imposed at element level, i.e., $w_i = 1$, $D_{\vec{w}} = I$, where I represents identity matrix.

We denote the weights vector at subarray level by $\vec{w}_{sub}$. Then the pattern with subarray partition is given by,

$$f(\theta) = \left(T_d \cdot \vec{w}_{sub}\right)^H \cdot \vec{a}(\theta) \tag{4}$$

where $(\cdot)^H$ is the Hermitian transposition operator.

The received data at subarray level, $\vec{x}_{sub}$, as shown in Fig. 1, can be obtained by [2],

$$\vec{x}_{sub} = T_d^H \cdot \vec{x}_{ele}\ . \tag{5}$$

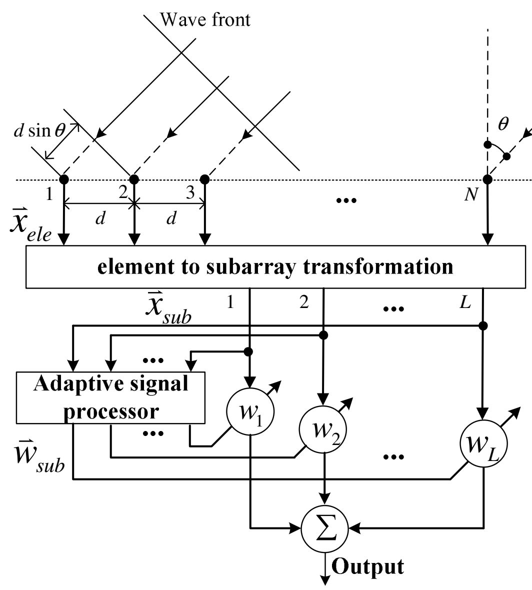

Fig. 1. A functional block diagram of an adaptive ULA with subarray partition.

The covariance matrix is defined as,

$$\begin{aligned} R_{sub} &= E\left\{\vec{x}_{sub} \cdot \vec{x}_{sub}{}^H\right\} \\ &= E\left\{T_d{}^H \cdot (\vec{x}_{ele} \cdot \vec{x}_{ele}{}^H) T_d\right\} \\ &= T_d{}^H E\left\{\vec{x}_{ele} \cdot \vec{x}_{ele}{}^H\right\} T_d \\ &= T_d{}^H R_{ele} T_d \end{aligned}\ . \tag{6}$$

In practice, it is difficult to get the theoretical value of R_{ele}, generally its approximate estimation is used instead [15],

$$\hat{R}_{ele} = \frac{1}{N_{sap}} \sum_{m=1}^{N_{sap}} \vec{x}_{ele}(m) \cdot \vec{x}_{ele}^{\ H}(m), \tag{7}$$

where N_{sap} is the sampling rate, and $N_{sap} = 2N$ in this paper [16].

III. CONVEX OPTIMIZATION BASED ADAPTIVE BEAMFORMING WITH SIDELOBE CONTROL

A subarray level adaptive beamformer is achieved by grouping the array elements into subarrays on which conventional beamforming is performed and applying an adaptive beamforming algorithm to the subarray outputs as shown in Fig. 1. Signals in each subarray channel are multiplied by a complex weight. The number of adaptive coefficients is therefore the number of subarrays L instead of the total number of elements N, which reduces the computational load manifestly in $\vec{w}_{sub}$ calculation.

In many practical applications, to get higher SNR, antenna is preferred to be used without attenuation. This may invalidate sidelobe suppression contributed by element amplitude tapering. Nevertheless, we may try to minimize the difference between $\vec{w}_{ref}$ and $T \cdot \vec{w}_{sub}$ with constraint of interference nulling. Here, $\vec{w}_{ref}$ is the optimal element tapering for quiescent low sidelobe pattern synthesis. $T \cdot \vec{w}_{sub}$ can be regarded as the equivalent element amplitude tapering with subarray partition.

Chebyshev tapering is often desirable in linear array since it gives smallest possible beamwidth for a given low, uniform sidelobe level [13, 17]. Therefore, we set $\vec{w}_{cheby}$ as reference element tapering. To form adaptive beam with sidelobe restriction at subarray level, the following optimization problem is proposed,

$$\begin{aligned} &\min \ \left\| T \cdot \vec{w}_{sub} - \vec{w}_{cheby} \right\|^2 \\ &s.t.: \ \left(T \cdot \vec{w}_{sub} \right)^H \vec{a}\left(\theta_k\right) = 0 \\ &\qquad\qquad k = 1, 2, ..., K < L \end{aligned} \tag{8}$$

where $\|\bullet\|$ denotesthe Euclid norm. θ_k represents the direction of the k^{th} interference. $\vec{w}_{sub}$ is the optimization variables. Interferences are assumed incoherent with each other. However, in general, we have no prior information of interferences, i.e., θ_k is unknown. Nevertheless, in the situation of strong interferences and small signal, the optimal weight vector tends to be orthogonal to the interference subspace [15]. Thus we can modify equation (8) as,

$$\begin{aligned} &\min \ \left\| T \cdot \vec{w}_{sub} - \vec{w}_{ref} \right\|^2 \\ &s.t. \quad \vec{w}_{sub}^{H} \cdot J = 0 \end{aligned} \tag{9}$$

where J denotes the interference subspace (ISS) at subarray level, which can be obtained from $\hat{R}_{sub}$. Rank the eigenvalues of $\hat{R}_{sub}$ in descending order as $\lambda_1 \geq \cdots \geq \lambda_K > \lambda_{K+1} \geq \cdots \geq \lambda_L$. Eigenvectors, $\vec{u}_1, \vec{u}_2, \cdots, \vec{u}_K$, corresponding to the first K large eigenvalues, span the ISS. To determine the dimension of ISS, we use the Akaike information criterion (AIC) [18],

$$\begin{aligned} &AIC(k) = -2\ln\left[\frac{\prod_{i=k+1}^{L} \lambda_i^{\frac{1}{L-k}}}{\frac{1}{L-k}\sum_{i=k+1}^{L}\lambda_i}\right]^{(L-k)N_{sap}} + 2k(2L-k) \\ &\hat{K} = \min\left\{AIC(k), k = 0,1,...,L-1\right\} \end{aligned} \tag{10}$$

where L is the number of eigenvalues. $\hat{K}$ is the estimation of the dimension of ISS. The standard form of convex optimization problems is given in [19],

$$\begin{aligned} &\min \quad f_0(x) \\ &s.t. \quad f_i(x) \leq 0, \ i = 1, ..., m \\ &\qquad\quad a_i^T x = b_i, \ \ i = 1, ..., p \end{aligned} \tag{11}$$

where $f_0, ..., f_m$ are convex functions. When the objective functions as well as the inequality constraint functions are convex, and the equality constraint functions $h_i(x) = a_i^T x - b_i$ are affine, equation (11) is a convex problem [19]. Extending the objective function in equation (9) yields,

$$\begin{aligned} &\left\| T \cdot \vec{w}_{sub} - \vec{w}_{ref} \right\|^2 \\ &= \vec{w}_{sub}^H T^H T \vec{w}_{sub} - 2\,\mathrm{Re}\left[\vec{w}_{ref}^H T \vec{w}_{sub}\right] + \vec{w}_{ref}^H \vec{w}_{ref} \end{aligned} \tag{12}$$

It is obvious the objective function is quadratic. In addition, equality constraint in equation (9) is apparently affine. Thus equation (9) belongs to convex optimization. Several public solvers are available to solve convex optimization such as CVX, SeDuMi, YALMIP, etc. YALMIP is used in our simulation, since the toolbox makes the development of optimization problems in general and avoid simmediate use of other solvers error-prone, even when parameterized matrices and variables are complex [20].

To facilitate the use of YALMIP toolbox, we modify the problem expression slightly. A new scalar non-negative variable t is introduced. It is obviously that $\vec{w}_{sub} = \arg\min_{\vec{w}_{sub}} \left\| T \cdot \vec{w}_{sub} - \vec{w}_{cheby} \right\|^2$ is equivalent to the following expression,

$$\min_{\vec{w}_{sub}} t \qquad s.t. \ \left\| T \cdot \vec{w}_{sub} - \vec{w}_{ref} \right\|^2 \le t \ . \qquad (13)$$

Therefore equation (9) can be converted into the following form [9],

$$\begin{aligned} &\min t \\ &s.t.: \vec{w}_{sub}^H J = 0 \qquad\qquad (14) \\ &\begin{bmatrix} I_{N\times N} & T \cdot \vec{w}_{sub} - \vec{w}_{ref} \\ \left(T \cdot \vec{w}_{sub} - \vec{w}_{ref}\right)^H & t \end{bmatrix} \ge 0 \end{aligned}$$

when t reaches its minimum, we get the optimal $\vec{w}_{sub}$. When the YALMIP toolbox is installed in Matlab properly, optimization can be realized with following commands. It is assumed that the reader is familiar with Matlab.

```
  t = sdpvar(1);
  % real part of wsub
wsub_r=sdpvar(subN,1,'full');
  % imaginary part of wsub
wsub_i=sdpvar(subN,1,'full');
wsub = wsub_r+j.*wsub_i;
  % equality constraint
cond=set((wsub)'*J==0);
  M = [eye(eleN),(T*wsub-w_ref);…
     (T*wsub-w_ref)',t];
 %combine   equality   constraint   and
  inequality constraint
  cond = cond+set(M>= 0);
obj = t;
solvesdp(cond,obj);
```

The number of iterations grows with problem size as $O(\sqrt{N})$ [21]. In our simulations, the algorithm converged after 8~10 iterations.

IV. SIMULATION RESULTS

For all simulations in this paper, a uniform linear array (ULA) consisting of 96 elements with $\lambda/2$ spacing is used. ULA is utilized because of its merit of simplest geometry and excellent directivity [22]. In our simulations, the array is divided into 16 symmetrical subarrays as [8, 7, 6, 7, 5, 6, 5, 4, 4, 5, 6, 5, 7, 6, 7, 8], so that grating lobes are suppressed. An element-level Chebyshev tapering with -40 dB sidelobe magnitude is set as referenced weights for the proposed method. We compare our method with penalty function [17], where subarray-level Chebyshev tapering with the same sidelobe level is chosen. Its scalar weighting factor k = 20. We assume that there is no look direction gain constraint. Thus the formula for penalty function we chose is given as equation (14) in [17],

$$\vec{w}_{pen} = k^2 \left(R_{sub} + k^2 I\right)^{-1} \vec{w}_{cheb} \qquad (15)$$

where I is a 16 order identity matrix. $\vec{w}_{cheb}$ is a 16×1 vector. Signal of interest is neglected since it is usually possible to form the interference covariance matrix with signal absent in radar applications [23]. Interference to noise ratio (INR) is 30 dB.

A. Performance of beam pattern control

In this section, θ_d = 5°. We compare penalty function method with the proposed algorithm in two scenarios: Two sidelobe interferences from [11°, −9°] and one mainlobe interference from 6.5°. In the first example, two sidelobe interferences in direction of 11° and −9° are imposed. Patterns are plotted in Fig. 2. Both adaptive patterns are able to achieve high interference inhibition gain, lower than -70 dB. However, it is observed that the proposed method performs outweighs penalty function in maintaining low sidelobe, especially lobes close to mainlobe. The first sidelobe level can be as low as -41dB. Additionally, the proposed method produces a better shaped mainlobe, of which the width is the same as reference pattern.

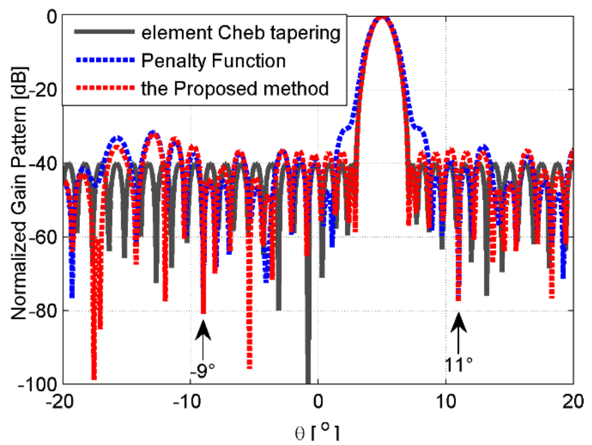

Fig. 2. Normalized pattern with two sidelobe interferences from 11° and –9°.

Figure 3 illustrates adaptive patterns in presence of a mainlobe interference from 6.5°, 1.5° away from the desired signal. Both the two algorithms can prohibit interference effectively. However, the proposed method dose better in suppressing sidelobes near the mainlobe.

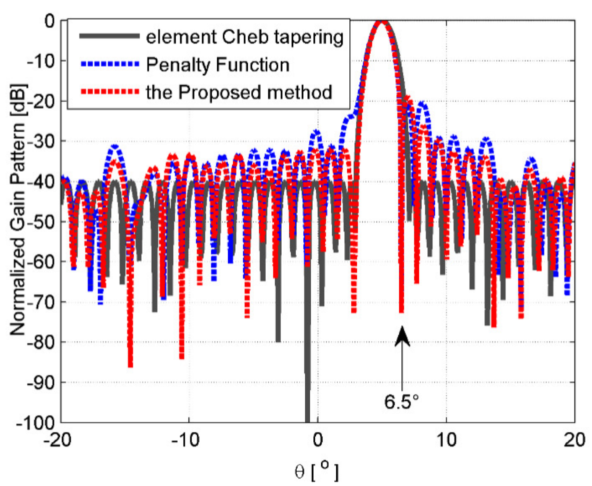

Fig. 3. Normalized pattern with one mainlobe interference from 6.5°.

B. SINR comparison

Figure 4 depicts the output SINR versus scanning angle θ. Interference comes from −3°. The SINR is calculated via [10],

$$SINR\left(\theta\right)_{ouput} = \frac{\sigma_s^2 \vec{w}^H \vec{a}\left(\theta\right) \vec{a}^H\left(\theta\right) \vec{w}}{\vec{w}^H R_{sub} \vec{w}} \tag{16}$$

where $\vec{w}$ denotes adaptive weights at subarray level. $\vec{a}\left(\theta\right)$ is subarray-level steering vector. σ_s^2 is the power of desired signal and $\sigma_s^2 = 1$ in this simulation. According to linearly constrained minimum variance (LCMV) criterion, the optimum adaptive weights at subarray level we use to calculate optimal SINR are given as follows [24, 25],

$$\vec{w}_{opt} = \mu R_{sub}^{-1} \vec{a}(\theta_d)_{sub} \tag{17}$$

where μ is a positive constant and does not affect the calculation of SINR.

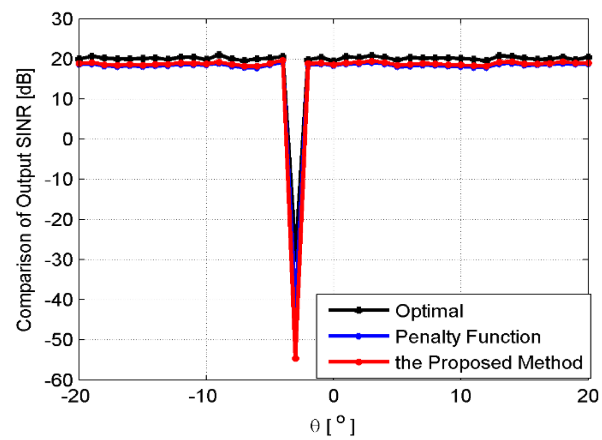

Fig. 4. SINR comparison of the three methods.

It can be seen from Table 1 that the proposed method has higher SINR compared with penalty function while performing better in sidelobe controlling. Both adaptive subarray tapering suffer a small SINR loss. This is the price paid to get much lower sidelobe.

Table 1: SINR comparison [dB].

Methods	SINR	SINR Loss
Optimal	20.25	--
Penalty Function	18.53	-1.72
Proposed method	18.88	-1.37

V. DISCUSSIONS AND CONCLUSION

A. Discussions

Compared with quiescent Chebyshev tapering pattern, the other two adaptive patterns have evident sidelobe jitter. This is caused mainly by subarray number and size. More subarrays allow optimized weights closer to reference tapering. Equally division will lead to grating lobe. How to choose a suitable subarray partition is complicated and usually needs compromise. Further study is under gone. Meanwhile, the proposed method suffers a small SINR loss, which is the cost of proposed method. Thus, a tradeoff should be considered in practical situations.

B. Conclusion

In this paper, a convex optimization based method is proposed to form adaptive sum beam with sidelobe control at subarray level. Compared with penalty function method, the proposed method has the merits of considerably reducing sidelobe adjacent to mainlobe, and producing better shaped mainlobe. Moreover, our optimization problem is formulated without parameters predetermining. The aforementioned merits of the proposed method have been verified by computer simulations.

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