An Improved Combination of IE-ODDM and MLFMA

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Abstract ─ Domain decomposition methods are efficient for analyzing scattering problems with large-scale structures. In the present paper, an improved combination of Overlapped Domain Decomposition Method of Integral Equations (IE-ODDM) and Multilevel Fast Multi-pole Algorithm (MLFMA) is developed. Amount of independent MLFMA progresses of sub-domains are departed and re-integrated, such that the total CPU time of coupled effects in IE-ODDM can be saved. The proposed method developed minimal-completed sub-trees of sub-domains to reduce redundant aggregations of the MLFMA process blended in IE-ODDM. Numerical results and comparisons with the original method are provided, which suggest that the proposed combination integrates MLFMA with IE-ODDM better than the original combined method, and it can greatly improve the computational efficiency of coupled effects in IE-ODDM.

Index Terms ─ Improved combined method, method of moments, multilevel fast multipole algorithm, overlapped domain decomposition method of integral equations.

I. INTRODUCTION

Method of Moments (MoM) for surface integral equations has been widely used [1], but the dense matrix makes it difficult for analyzing large-scale objects. In the past two decades, this problem has been circumvented by developments of sparse matrix methods and matrix partition iterative schemes. Sparse matrix method utilizes analytical or numerical low-rank matrix decompositions to accelerate matrix-vector multiplication, like fast multipole method [2] and Multilevel Fast Multipole Algorithm (MLFMA) [3-11] based on plane-wave spectral approximation. Matrix partition iterative schemes divide and solve large problems, like forward-backward iterative scheme [12,13] and Overlapped Domain Decomposition Method for Integral Equations (IE-ODDM) [14,15]. Moreover, sparse matrix methods and matrix partition iterative schemes can be combined for higher performance, like the overlapped domain decomposition method based on MLFMA (IE-MLFMA-ODDM) [16-18] which was newly reported by many researchers [19-22].

In the original IE-MLFMA-ODDM, MLFMA processes are simply embedded in IE-ODDM iterative scheme to accelerate calculations of many matrix-vector multiplications in self-effects and coupled effects of sub-domains. Although the combined method is successful, the above combination of MLFMA and IE-ODDM is still primitive. In the original combined routine, there are many different and independent MLFMA processes of sub-domains, which consume most of CPU time and memory resources. All those MLFMA processes in the combined method are completed processes like the one without IE-ODDM. That is to say, MLFMA processes and IE-ODDM are isolated, and every MLFMA process utilizes too little information from IE-ODDM and other MLFMA processes. If decompositions consist of many sub-domains and all sub-domains are with amount of exterior unknowns, amount of MLFMA processes are still time-consuming.

In the present paper, an improved combination

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is proposed. Compared with the original combined method, the proposed combination integrates MLFMA with IE-ODDM much better, and it optimizes MLFMA processes of sub-domains, which can greatly accelerate the calculation of coupled effects. It can be called IE-O-MLFMA-ODDM. The proposed IE-O-MLFMA-ODDM divides many completed and independent MLFMA processes consisting of aggregations, translations and disaggregations into segments, extracts and reuses their common information. Redundancies of many MLFMA processes are reduced, and those independent MLFMA processes and IE-ODDM are blended. Localized aggregations of sub-domains are introduced, and radiation patterns of coupled effects are saved and shared in IE-ODDM loops. The above localization/sharing mechanism can greatly improve the efficiency of IE-ODDM. However, everything has double folds. To support saving and sharing of radiation patterns, the proposed method needs more memory. In order to manage local radiation patterns efficiently, one data structure called minimal-completed sub-tree is introduced.

II. THE ORIGINAL COMBINED METHOD

In order to find the distribution of current on perfect electrical conductors by MoM, one should build the following electric field integral equation [23]:

$$-\left[\int_{\Omega}\bar{\bar{G}}_{\mathrm{E,J}}(r,r')\vec{J}(r')ds'\right]\Big|_{\tan}=\vec{E}^{\mathrm{INC}}(r)\Big|_{\tan}, \qquad (1)$$

and magnetic field integral equation:

$$-\left[\int_{\Omega}\bar{\bar{G}}_{\mathrm{H,J}}(r,r')\vec{J}(r')ds'\right]\Big|_{\tan}=\vec{H}^{\mathrm{INC}}(r)\Big|_{\tan}, \qquad (2)$$

where Ω represents the surface of objects and $\bar{\bar{G}}$ represents the dyadic Green's function, $\vec{J}$ is the surface current, and $\vec{E}^{\mathrm{INC}}$ and $\vec{H}^{\mathrm{INC}}$ are incident fields. The MoM matrices derived from the above integral equations or their combinations are usually dense, and the corresponding equations from large scale problems cannot be easily solved. Matrix partition iterative methods are useful methods for solving matrix equations with large scales. The domain decomposition methods for integral equations, one kind of matrix partition iterative method, decomposed the whole surface into many completed covering sub-domains $\Omega=\cup\Omega_i$, and solve Eq. (1) or Eq. (2) or their combined equation on sub-domains recursively to update the entire current. This principle of domain decomposition methods is the simplest, but it isn't practicable as edge effects. Edge effects would make the iterative process of non-overlapped domain decomposition method unstable and divergent. So, modifications of the original domain decomposition method are necessary. One feasible approach is extending buffer domains like the method in IE-ODDM [14]. In IE-ODDM, Eq. (3) or Eq. (4) or their combined equation are solved on extend region $\bar{\Omega}_i$:

$$-\left[\int_{\bar{\Omega}_i}\bar{\bar{G}}_{\mathrm{E,J}}(r,r')\vec{J}(r')ds'\right]\Big|_{\tan}=$$
$$\vec{E}^{\mathrm{INC}}(r)\Big|_{\tan}+\left[\int_{\bar{\Omega}_i^C}\bar{\bar{G}}_{\mathrm{E,J}}(r,r')\vec{J}(r')ds'\right]\Big|_{\tan}, \qquad (3)$$
$$-\left[\int_{\bar{\Omega}_i}\bar{\bar{G}}_{\mathrm{H,J}}(r,r')\vec{J}(r')ds'\right]\Big|_{\tan}=$$
$$\vec{H}^{\mathrm{INC}}(r)\Big|_{\tan}+\left[\int_{\bar{\Omega}_i^C}\bar{\bar{G}}_{\mathrm{H,J}}(r,r')\vec{J}(r')ds'\right]\Big|_{\tan}, \qquad (4)$$

where extension includes original sub-domains and buffer domains. Parts of their results restricted on original sub-domains are saved to update the entire current. Compared with the unstable iterative scheme of non-overlapped domain decomposition method, this scheme of IE-ODDM has a great convergence.

Corresponding to the above physical principle, IE-ODDM has another matrix interpretation. After discretizations of Eq. (3) or Eq. (4) or their combination utilizing basis functions and test functions like Galerkin procedures in [24], one can obtain the following linear equations on $\bar{\Omega}_i$:

$$\sum_{j=1}^{N_{\bar{\Omega}_k}}Z_{ij}^{\bar{\Omega}_k}I_j^{\bar{\Omega}_k}=V_i^{\bar{\Omega}_k}-\sum_{j'=1}^{N-N_{\bar{\Omega}_k}}Z_{ij'}^{\bar{\Omega}_k^C}I_{j'}^{\bar{\Omega}_k^C}, \qquad (5)$$

where

$$Z_{ij}^{\bar{\Omega}_k}=Z_{g_i^{\bar{\Omega}_k}g_j^{\bar{\Omega}_k}},\quad Z_{ij'}^{\bar{\Omega}_k^C}=Z_{g_i^{\bar{\Omega}_k}g_{j'}^{\bar{\Omega}_k^C}},$$

$g_i^{\bar{\Omega}_k}$ and $g_{j'}^{\bar{\Omega}_k^C}$ are global indexes of the i-th and the j-th basis functions in the iterative and coupled domain, respectively. Matrices $Z^{\bar{\Omega}_k}$ and Z are the local and global impedance matrices, and $I^{\bar{\Omega}_k}$ is the local current coefficients. The first right-hand term represents the excitation from incident waves, and the second right-hand term represents the coupled effect from other sub-domains. Equation (5) suggests that IE-ODDM is one kind of matrix partition iterative methods.

In IE-ODDM, a loop like block Gauss-Seidel iteration method repeatedly scans all sub-domains and updates local current until the whole current becomes stable. For convenience, the IE-ODDM should classify all basis functions as "interior", "edge", "buffer" or "exterior" basis functions, where the interior, edge and buffer basis functions are iterative basis functions, and the exterior basis functions are coupled basis functions. The effects of coupled basis functions on the iterative basis functions are called coupled effects, and the effects of the iterative basis functions on the iterative basis functions are called self-effects. Coupled effects take on calculating right-hand terms of sub-problems, and self-effects take on solving the local current.

In order to accelerate IE-ODDM, MLFMA could be combined and utilized in matrix-vector multiplications of coupled effects and self-effects. The direct combinations are that original combined method IE-MLFMA-ODDM [16-18]. When the MLFMA is utilized in matrix-vector multiplication, the coupled effect in Eq. (5) is divided into the following two terms:

$$\sum_{j'=1}^{N-N_{\tilde{\Omega}_k}} Z_{ij'}^{\tilde{\Omega}_k^C} I_{j'}^{\tilde{\Omega}_k^C} = \sum_{n^L \in \mathrm{Neigh}(m_{g_i^L}^L)} \sum_{g_{j'}^{\tilde{\Omega}_k^C} \in n^L} Z_{ij'}^{\tilde{\Omega}_k^C} I_{j'}^{\tilde{\Omega}_k^C} + \sum_{n^L \notin \mathrm{Neigh}(m_{g_i^L}^L)} \sum_{g_{j'}^{\tilde{\Omega}_k^C} \in n^L} Z_{ij'}^{\tilde{\Omega}_k^C} I_{j'}^{\tilde{\Omega}_k^C}, \tag{6}$$

where

$$\sum_{n^L \notin \mathrm{Neigh}(m_{g_i^L}^L)} \sum_{g_{j'}^{\tilde{\Omega}_k^C} \in n^L} Z_{ij'}^{\tilde{\Omega}_k^C} I_{j'}^{\tilde{\Omega}_k^C} \approx \sum_p \vec{V}_{g_i^{\tilde{\Omega}_k}, m_{g_i^L}^L}^{\mathrm{F}}(\hat{k}_p^L) \vec{W}_{m_{g_i^L}^L}^{(\mathrm{DA,C})}(\hat{k}_p^L),$$

and

$$\vec{W}_{m_{g_i^{l+1}}^{l+1}}^{(\mathrm{DA,C})}(\hat{k}_p^{l+1}) = \vec{W}_{m_{g_i^{l+1}}^{l+1}}^{(\mathrm{T,C})}(\hat{k}_p^{l+1}) + \sum_t \left(C_{l+1}^p(t) e^{-jk\hat{k}_t^l \cdot \vec{r}_{m_{g_i^{l+1}}^{l+1}, m_{g_i^l}^l}} \vec{W}_{m_{g_i^l}^l}^{(\mathrm{DA,C})}(\hat{k}_t^l) \right),$$

and

$$\vec{W}_{m_{g_i^l}^l}^{(\mathrm{T,C})}(\hat{k}_s^l) = \sum_{n^l \in \mathrm{HypoNeigh}(m_{g_i^l}^l)} \alpha_{m_{g_i^l}^l, n^l}^s (\hat{k}_s^l \hat{r}_{m_{g_i^l}^l, n^l}) \vec{W}_{n^l}^{(\mathrm{A,C})}(\hat{k}_s^l),$$

and

$$\vec{W}_{n^l}^{(\mathrm{A,C})}(\hat{k}_p^l) = \sum_{n^{l+1} \in \mathrm{Child}(n^l)} \left(e^{-jk\hat{k}_p^l \cdot \vec{r}_{n^l, n^{l+1}}} \sum_t C_{l+1}^t(p) \vec{W}_{n^{l+1}}^{(\mathrm{A,C})}(\hat{k}_t^{l+1}) \right).$$

In Eq. (6), m_k^l is the index of the father node of the k-th basis function in l-th level. $\vec{V}$ is the receiver pattern. $\vec{W}_m^{(\mathrm{A,C})}$, $\vec{W}_m^{(\mathrm{T,C})}$, and $\vec{W}_m^{(\mathrm{DA,C})}$ are process patterns of aggregation, translation and disaggregation, respectively. Matrix C is the interpolating matrix of different level. Herein, we just list some critical equations of MLFMA. More details, please refer to [16] and references therein.

According to the above description, every MLFMA process in the original combination has an exclusive and completed oct-tree for the aggregation, translation and disaggregation. According to Eq. (6), leaves/nodes of those completed oct-trees should be marked and classified as iterative leaves/nodes and coupled leaves/nodes. Like in Fig. 1, the ones marked by horizontal lines are iterative leaves/nodes, and the ones marked by vertical lines are coupled leaves/nodes.

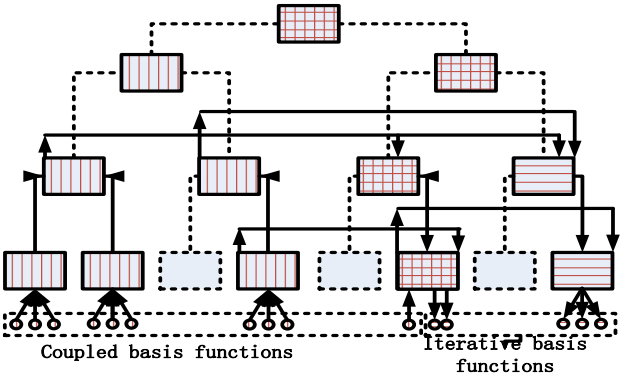

Fig. 1. The sketch of the MLFMA process in the original IE-MLFMA-ODDM. Keeping the critical processes, radiation patterns in coupled domain are aggregated from leaves to nodes, and translated to iterative nodes in the same level, and disaggregated in iterative domains from nodes to leaves. Since some nodes consist of coupled and iterative basis functions, they are both coupled and iterative nodes.

III. THE IMPROVED COMBINED METHOD

In the MLFMA processes utilized for coupled effects, radiation patterns of coupled leaves/nodes are aggregated from coupled leaves to coupled nodes, and disaggregated from iterative nodes to

iterative leaves. The whole processes have many redundant operations. In the aggregation, radiation patterns of useless iterative leaves are set zero and are also aggregated; while in the disaggregation process, radiation patterns of coupled leaves/nodes are not useful but are still disaggregated. For large-scale problems with many segments, these redundant processes waste a great of CPU time. That is to say, the original combined method just simply embeds MLFMA into IE-ODDM, which can be optimized.

In fact, there is another hidden redundancy. When the IE-ODDM loop serially scans sub-domains, the current coefficients of two steps are partly different. It is because that every step in the loop only renews the current on its sub-domain and keeps the current on other sub-domains. It is a redundancy that unchanged radiation patterns are aggregated again and again in the same IE-ODDM loop. For example, one can run an IE-ODDM loop for decomposition in Fig. 2, where Domain A, B and C are three different sub-domains. When the loop starts, only the current on Domain A is updated. After that, the loop comes to Domain B and calculates the coupled effect. Meanwhile, the unchanged radiation patterns on Domain C are aggregated again, which has been done when the loop in Domain A. The same phenomenon occurs when the loop comes to Domain C.

In order to reduce the above hidden redundancy, one can modify the aggregation processes and design an optimized routine. The optimized combination is simply called IE-O-MLFMA-ODDM. Compared with the original combined method, completed MLFMA processes in IE-ODDM loops are replaced by simplified processes which consist of some simplified aggregation, translation and disaggregation processes. Moreover, different from the classification in the original combination, basis functions are grouped according to the decomposition, and oct-trees for different sub-domains are also replaced by oct-trees for different group, where aggregation processes of all groups has their exclusive oct-trees like in Fig. 2. More details are shown as Fig. 3. Because of these optimized techniques, the CPU time can be greatly saved, and the efficiency for solving coupled effects can be improved.

Compared with the original combined method, the proposed scheme is more efficient, but it needs extra memory to support this acceleration. There is a balance between the CPU time and the memory requirement. Although, extra memory requirement may not be a big problem for hardware, we can still save some memory from the proposed method by using some special localized data structures such as minimal-completed sub-trees. Like in Fig. 2, the sub-tree with leaves/nodes marked by left diagonal lines is a minimal-completed sub-tree of Group A. Since the aggregation process of Group A only updates radiation patterns of its group, the redundant leaves/nodes can be cut-down. As most of memory is occupied by those leaves/nodes in higher levels and most of redundant leaves have been cut-down by sub-trees, the application of this technique can efficiently save memory.

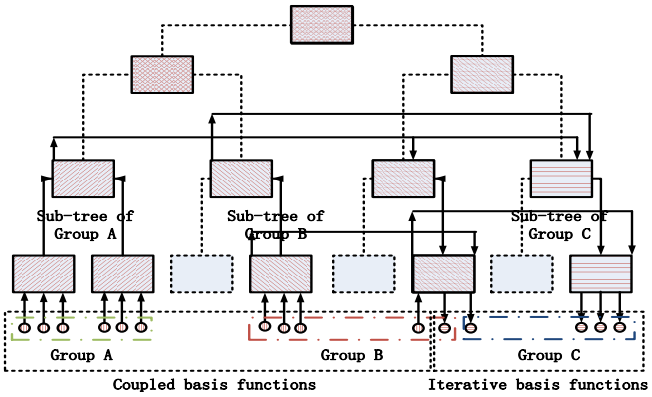

Fig. 2. Groups and their minimal-completed sub-trees. Different from the original MLFMA, leaves and nodes are grouped according to the decomposition. Sub-trees of groups are completed trees without a single node in one level. Because of the same original oct-tree, some nodes are of different sub-trees. Compared with direct aggregations, radiation patterns of groups are saved and reused.

1. Grouping basis functions as interior groups or edge group according to sub-domains;
2. Classifying oct-tree nodes as iterative or coupled nodes of each group;
3. IE-ODDM loop for scanning sub-domains and updating current repeatedly:
   - 3.1 Solving coupled-effect by the modified MLFMA:
     - 3.1.1 Aggregating radiation patterns of the edge group;
     - 3.1.2 Translating radiation patterns of other sub-domains to the iterative sub-domain;
     - 3.1.3 Aggregating, conversing and translating the radiation patterns of the buffer functions;
   - 3.2 Solving the self effect matrix equation by MLFMA;
   - 3.3 Updating the local current;
   - 3.4 Aggregating the radiation patterns of the interior function group of the current sub-domain;
4. Calculating the relative residual of the obtained result and the previous result;
5. Repeating Step 3 till the relative residual is satisfied.

Fig. 3. Detailed description of the proposed method.

IV. NUMERICAL RESULTS

In order to validate the correction and efficiency of the proposed IE-O-MLFMA-ODDM, the following three examples are considered.

Example I:

A PEC spherical object with a radius of 5λ in Fig. 4 is considered, and the number of unknowns is 119,544. Its surface is divided into eight congruent sub-domains. The numbers of unknowns in iterative domains and coupled domains are 16,511 and 103,033, respectively. The CPU times for the calculation of coupled effects are listed in Table 1. It suggests that the CPU time utilized in the proposed method is about 25.0% of the time utilized in the original IE-MLFMA-ODDM. The extra memories for the radiation patterns of different groups are listed in Table 2. It suggests that the extra memory is about 1.2 times more than the memory utilized in the original IE-MLFMA-ODDM. The RCS of two different methods are also shown as Fig. 4. The result of the proposed method agrees with the result of the original method, which validates the correction of the proposed method.

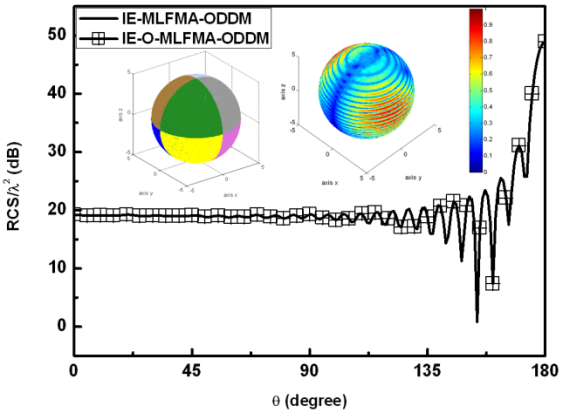

Fig. 4. The RCS and current of a spherical PEC object with a radius 5λ.

Table 1: CPU time consumptions of coupled effects of a spherical PEC object

Sub-Domains	Unknowns		CPU Time of Coupled Effects (minutes)	
	Iterative	Coupled	Original Method	Proposed Method
D1	16,511	103,033	6.8530	1.6336
D2	16,511	103,033	6.6780	1.6198
D3	16,511	103,033	6.6892	1.6625
D4	16,511	103,033	6.6787	1.6233
D5	16,511	103,033	6.6990	1.6380
D6	16,511	103,033	6.7074	1.6408
D7	16,511	103,033	6.6962	1.6611
D8	16,511	103,033	6.6878	1.6520

Table 2: Memory requirements of coupled effects

Example	Unknowns	Memory Requirement of Coupled Effects (megabytes)	
		Original Method	Proposed Method
I	119,544	35.7432	43.7757
II	117,450	52.5482	85.4319
III	188,202	86.7601	145.506

Example II:

A PEC airplane-type object in Fig. 5 is considered, and the number of unknowns is 117,450. Its surface is divided into six sub-domains. The numbers of unknowns in iterative domains vary from 13,274 to 32,450, and the numbers of unknowns in coupled domains vary from 104,176 to 85,000. Table 3 lists the CPU time for the calculation of coupled effects, which suggests that the CPU time utilized in the proposed method is about 29.0% (varying from 23.7% to 37.3%) of the time utilized in the original IE-MLFMA-ODDM. Table 2 suggests that the extra memory is about 1.6 times more than the memory utilized in the original IE-MLFMA-ODDM. Figure 5 shows that two RCS from different methods can well agree with each other.

Example III:

A PEC missile-type object in Fig. 6 is considered, and the number of unknowns is 188,202. Its surface is divided into five sub-domains. The numbers of unknowns in iterative domains vary from 11,521 to 55,978, and the numbers of unknowns in coupled domains vary from 176,681 to 132,224. Table 4 lists the CPU time for the calculation of coupled effects, which suggests that the CPU time utilized in the proposed method is about 50.6% (varying from 31.5% to 62.8%) of the time utilized in the original IE-MLFMA-ODDM. Table 2 suggests that the extra memory is about 1.6 times more than the memory utilized in the original IE-MLFMA-ODDM. Figure 6 shows that two RCS from different methods can well agree with each other.

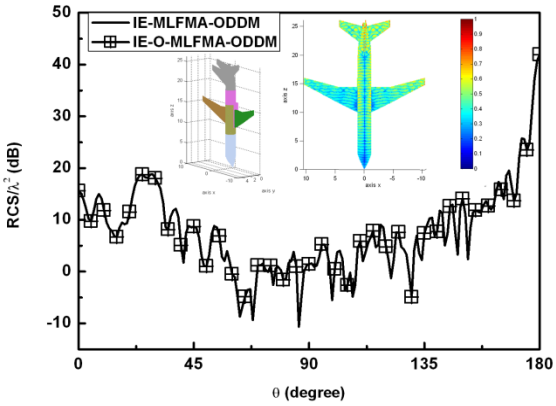

Fig. 5. The RCS and current of an airplane-type PEC object.

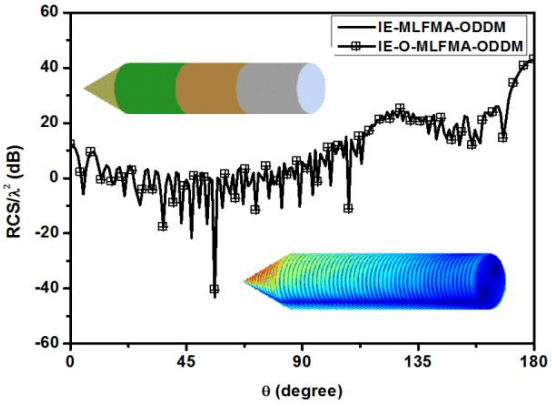

Fig. 6. The RCS and current of a missile-type PEC object.

Table 3: CPU time consumptions of coupled effects of an airplane-type PEC object

Sub-Domains	Unknowns		CPU Time of Coupled Effects (minutes)	
	Iterative	Coupled	Original Method	Proposed Method
D1	19,140	98,310	6.7515	1.8571
D2	32,450	85,000	7.2072	2.6894
D3	19,189	98,261	6.7760	1.9460
D4	19,495	97,955	6.6570	1.8158
D5	13,274	104,176	6.5534	1.5547
D6	20,941	96,509	6.9181	2.0461

Table 4: CPU time consumptions of coupled effects of a missile-type PEC object

Sub-Domains	Unknowns		CPU Time of Coupled Effects (minutes)	
	Iterative	Coupled	Original Method	Proposed Method
D1	11,521	176,681	3.5104	1.1057
D2	55,978	132,224	3.6260	1.9685
D3	52,986	135,216	3.6794	2.3107
D4	54,881	133,321	3.6226	2.2333
D5	22,615	165,587	3.5117	1.5016

The above numerical results and comparisons suggest that the proposed method can greatly improve the efficiency of combined method, and the improvement depends on the decomposition. More fine decomposition, more high performance. Moreover, we can find that the memory required to save the radiation patterns grows with the problem size, and the extra memory of saving radiation patterns is about 1.5 times more than the memory required in the original completed MLFMA process. Considering the good performance of MLFMA and the development of hardware, we think this extra memory requirement is meaningful and can be acceptable to improve a high computational efficiency.

V. CONCLUSION

In the present paper, we improve the combined method of overlapped domain decomposition method and multilevel fast multi-pole algorithm. In the proposed method, amount of independent MLFMA processes for different sub-domains are departed, reduced redundancies and re-integrated with IE-ODDM loops, so that aggregations of sub-domains can be simplified and radiation patterns can be reused. Meanwhile, localized data structures of minimal-completed sub-trees are used to reduce memory redundancies. Numerical results show that the proposed method can improve the efficiency of combined method with acceptable extra memory.

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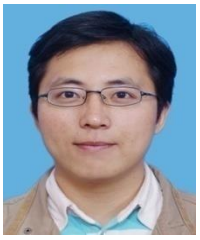

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