## **Approximation Through Common and Differential Modes for Twist Wire Pair Crosstalk Model**

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Abstract — In this advanced paper, an approximation method is presented, which separates the twist wire pairs into common and differential modes using the multi-conductor transmission line model. The simulation storage and time cost has been significantly reduced and accuracy is better than traditional approximation methods. Analyses of dealing with the impedance and admittance and the separation procedure of common and differential modes are presented. In addition, the load dealing methods for terminal and source has also been specified in this paper. Numerical experiments validate the accuracy and listed the storage efficiency and time cost reduction of the proposed method.

*Index Terms* — Common mode, crosstalk, different mode, twist wire pair.

#### I. INTRODUCTION

In the recent years, Twist Wire Pair (TWP) has been well developed and has become a most widely used physical layer for a number of technologies in communication systems, such as controller-area-network buses and gigabit Ethernet cables. Crosstalk comes on them when these wires are used to transport signals, which affects the bandwidth and the transmission rate. A special structure of TWP is used to reduce crosstalk, which introduces non-uniformity into model and makes prediction more difficult [1-4].

In order to predict crosstalk, a wiring structure is proposed based on the simplified circuit modeling and interpretation of crosstalk [5-8]. The twisted pair is modeled as a cascade of normal transmission-line loops consisting of two-wire

sections with abrupt interchanges of wire positions between 2 different transmission line parts [5, 7]. This model is typically accurate for frequencies such that the total line length is no more than 1/10 of a wavelength [5]. According to this technique, one can easily add multiple TWPs into the normal Multi-Transmission Line Networks (MTLN). There are some advanced predicting techniques based on this model [9-14], such as the response of plane wave [9] and the effect of randomness of twist pitch lengths [10], but it needs more space and time cost to solve the transmission line parameters due to its model complexity and the parameters variation with distance, so some approximation is needed.

While introducing TWP or other cables into the transmission line model, one difficulty is the larger matrix size and the parameters stored in the MTLN model. In addition, the large-size matrices will also cause an extra simulation time.

Several approximation techniques have been proposed, such as the equivalent cable bundle method [15-19], which approximates the cable bundle size by calculating the common mode voltage and current along wires in the same group whose response is more critical than the differential-mode. Its accuracy depends on the ratio of the loading impedances on the terminal and the source sides to the characteristic impedance. For a model with TWP, this categorization may cause the two wires in TWP into two different groups. What is more, it will add the difficulties into the categorization since the characteristic impedance may vary with position.

An approximation model for TWP bundle was proposed using the finite-difference time-domain

Submitted On: March 3, 2014 Accepted On: July 25, 2014 method [20]. The abrupt section calculating was avoided for the approximation, so the procedure of solving the chain parameter matrix effect was much simplified, but this method does not have distinct reduction in the storage demand or the time cost.

In this paper, an advanced approximation method based on variation of the common and differential modes will be presented. Also, we will discuss the way of dealing the loading impedance in the advanced model and the usage of the simplified model in MTLN. The method will be proved to be accurate than the equivalent cable bundle method and will save much more space in storage and time.

### II. TWISTED WIRE PAIR CROSSTALK MODEL CASCADED THEORY OF PAUL AND MCKNIGHT

### A. Twisted wire pair crosstalk model cascaded theory of Paul and McKnight

Since TWP has a periodic structure, it can be cascaded. In the cascaded theory, the twisted wire pairs consist of loops of the half twists and each Half Twist (HT) is divided into a transmission line section and an abrupt transition section, as shown in Fig. 1. Wires 2 and 3 in Fig. 1 are twisted, while wire 1 is a single line.  $\Delta l$  is the length of the transmission line section, and the length of the abrupt transition section is assumed to be zero. This zero length will drive this section, acting an exchange of the voltages and currents on wires 2 and 3. In the transmission line section, the chain parameter matrix in  $\Phi_s$  can be written as the following format:

$$\mathbf{\Phi}_{s} = \begin{bmatrix} \cos(\beta \Delta l) \cdot \mathbf{I}_{3\times 3} & -j\sin(\beta \Delta l) \mathbf{Z}_{c} \\ -j\sin(\beta \Delta l) \mathbf{Y}_{c} & \cos(\beta \Delta l) \cdot \mathbf{I}_{3\times 3} \end{bmatrix}. \quad (2)$$

In the equations above,  $V_1 \sim V_3$  and  $I_1 \sim I_3$  are the voltages and currents on wires  $1 \sim 3$ .  $I_{3\times 3}$  is a  $3\times 3$  identity matrix.  $\Delta l$  is the total length of the

transmission line section in the half twist.  $\beta$  is the wave coefficient.  $Z_c$  and  $Y_c$  are the transmission line characteristic impedance matrix and characteristic admittance matrix, respectively [8].

For the abrupt transition section, the voltages and currents on the wires exchanging with each other, the transition matrix can be written as follows [5,8]:

$$\begin{bmatrix} V_{1}(x_{2}) \\ V_{2}(x_{2}) \\ V_{3}(x_{2}) \\ I_{1}(x_{2}) \\ I_{2}(x_{2}) \\ I_{3}(x_{2}) \end{bmatrix} = P \begin{bmatrix} V_{1}(x_{1}) \\ V_{2}(x_{1}) \\ V_{3}(x_{1}) \\ I_{1}(x_{1}) \\ I_{2}(x_{1}) \\ I_{3}(x_{1}) \end{bmatrix},$$
(3)

$$P = \begin{bmatrix} P_a & O_{3\times3} \\ O_{3\times3} & P_a \end{bmatrix},\tag{4}$$

where the  $O_{3\times3}$  represents a  $3\times3$  identity matrix of zeros, and  $P_a$  is equal to:

$$P_{a} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}. \tag{5}$$

Combining the equations (1)-(5), the total chain matrix of a full twist loop, which consists of two half twists, is multiplication of the matrices:

$$\begin{bmatrix} V_{1}(x_{4}) \\ V_{2}(x_{4}) \\ V_{3}(x_{4}) \\ I_{1}(x_{4}) \\ I_{2}(x_{4}) \\ I_{3}(x_{4}) \end{bmatrix} = \mathbf{P}\mathbf{\Phi}_{s} \mathbf{P}\mathbf{\Phi}_{s} \begin{bmatrix} V_{1}(x_{0}) \\ V_{2}(x_{0}) \\ V_{3}(x_{0}) \\ I_{1}(x_{0}) \\ I_{2}(x_{0}) \\ I_{3}(x_{0}) \end{bmatrix}. \tag{6}$$

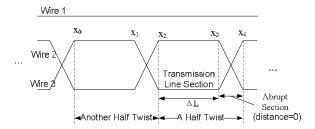


Fig. 1. Abrupt model of the TWP in the Paul and McKnight model.

### B. The approximation impedance and the admittance matrix of a full twist

Original model of the cascade theory separates a full twisted loop into 4 sections, which consists of 2 transmission line sections and 2 abrupt transition sections. The description of a full twisted loop is shown in Fig. 2. Notice that the ground effect has been brought into the model, this effect can be reflected through the changes of the characteristic impedance matrices and admittance matrices [5]. The sections from the point  $x_1$  to  $x_4$ can be regarded as an independent part. In this part, the transmission line chain parameter matrix can be defined as:

$$\begin{bmatrix} V_{1}(x_{4}) \\ V_{2}(x_{4}) \\ V_{3}(x_{4}) \\ I_{1}(x_{4}) \\ I_{2}(x_{4}) \\ I_{3}(x_{4}) \end{bmatrix} = \mathbf{\Phi}_{sP} \begin{bmatrix} V_{1}(x_{1}) \\ V_{2}(x_{1}) \\ V_{3}(x_{1}) \\ I_{1}(x_{1}) \\ I_{2}(x_{1}) \\ I_{3}(x_{1}) \end{bmatrix}, \qquad (7) \qquad \begin{bmatrix} P\Phi_{s}P\Phi_{s} = \Phi_{sp}\Phi_{s} \\ \cos(2\beta\Delta l)E_{3\times 3} & -j\sin(2\beta\Delta l)(\frac{Z_{c}+Z_{cP}}{2}) \\ -j\sin(2\beta\Delta l)(\frac{Y_{c}+Y_{cP}}{2}) & \cos(2\beta\Delta l)E_{3\times 3} \end{bmatrix}. \qquad (11)$$

$$\text{This can be true for several full twisted loops, if their total distance } (n\beta\Delta l) \text{ satisfies the condition } \sin(n\beta\Delta l) <<\cos(n\beta\Delta l).$$

$$\Phi_{cP} = P\Phi_{c}P$$

$$= \begin{bmatrix} \cos(\beta \Delta l) \cdot E_{3\times 3} & -j \sin(\beta \Delta l) Z_{cP} \\ -j \sin(\beta \Delta l) Y_{cP} & \cos(\beta \Delta l) \cdot E_{3\times 3} \end{bmatrix}.$$
 (8)

This part has an independent characteristic impedance matrix  $Z_{\text{cP}}$  and an independent characteristic admittance matrix Y<sub>cP</sub>. The matrixes  $Z_{cP}$  and  $Y_{cP}$  in equation (8) are expressed as:

$$\mathbf{Z}_{cP} = \mathbf{P}_{a} \mathbf{Z}_{c} \mathbf{P}_{a} = \begin{bmatrix} z_{11} & z_{13} & z_{12} \\ z_{31} & z_{33} & z_{32} \\ z_{21} & z_{23} & z_{22} \end{bmatrix}$$

$$\mathbf{Y}_{cP} = \mathbf{P}_{a} \mathbf{Y}_{c} \mathbf{P}_{a} = \begin{bmatrix} y_{11} & y_{13} & y_{12} \\ y_{31} & y_{33} & y_{32} \\ y_{21} & y_{23} & y_{22} \end{bmatrix}.$$

$$(9)$$

Substituting the equations (8) and (9) into (6), the matrix of a full twisted loop can be derived easily:

$$\mathbf{P}\mathbf{\Phi}_{S}\mathbf{P}\mathbf{\Phi}_{S} = \mathbf{\Phi}_{SP}\mathbf{\Phi}_{S} = \begin{bmatrix} \mathbf{\Phi}_{1} & \mathbf{\Phi}_{Z} \\ \mathbf{\Phi}_{Y} & \mathbf{\Phi}_{1} \end{bmatrix}$$

$$\mathbf{\Phi}_{1} = \cos^{2}(\beta \Delta l)\mathbf{I}_{3\times 3} - \sin^{2}(\beta \Delta l) \cdot \mathbf{Z}_{c}\mathbf{Y}_{cP} \quad (10)$$

$$\mathbf{\Phi}_{Z} = -j\sin(\beta \Delta l)\cos(\beta \Delta l)(\mathbf{Z}_{c} + \mathbf{Z}_{cP})$$

$$\mathbf{\Phi}_{Y} = -j\sin(\beta \Delta l)\cos(\beta \Delta l)(\mathbf{Y}_{c} + \mathbf{Y}_{cP}).$$

Considering the case:  $\sin(\beta \Delta l) \ll \cos(\beta \Delta l)$ , the matrices  $Z_c Y_{cP}$  and  $Y_c Z_{cP}$  after  $\sin^2(\beta \Delta l)$  in (10) can be replaced by an identification matrix  $E_{3\times 3}$ , then the equation (10) can be simplified into:

$$P\Phi_{s}P\Phi_{s} = \Phi_{sp}\Phi_{s}$$

$$\approx \begin{bmatrix} \cos(2\beta\Delta l)E_{3\times 3} & -j\sin(2\beta\Delta l)(\frac{Z_{c} + Z_{cP}}{2}) \\ -j\sin(2\beta\Delta l)(\frac{Y_{c} + Y_{cP}}{2}) & \cos(2\beta\Delta l)E_{3\times 3} \end{bmatrix}.$$
(11)

 $\sin(n\beta\Delta l) \ll \cos(n\beta\Delta l)$ .

The condition  $\sin(\beta \Delta l) \ll \cos(\beta \Delta l)$  requires that the  $(\beta \Delta l)$  is fewer than  $\pi/12$ , then the frequency should match  $f < c/(24\Delta l)$ , c is the speed of light of the material. For a TWP whose  $\Delta l$  is equal to 0.02 m, so the frequency should be no more than 625 MHz, it can match the condition  $\sin(\beta \Delta l) \ll \cos(\beta \Delta l)$  for the frequencies discussed in [5] (which is no more than  $10^7$  Hz); also, its accuracy will become a little worse when the frequency grows.

Equation (11) can be rewritten through the approximation way, the approximation impedance and admittance is:

$$Z_{eq} = (Z_c + Z_{cP})/2$$

$$Y_{eq} = (Y_c + Y_{cP})/2.$$
(12)

It is worthwhile to mention this pair of simplified approximation impedance admittance does not match the normal equation of the impedance and admittance  $ZY=YZ=E_{3x3}$ .

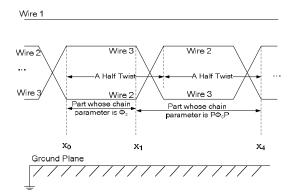


Fig. 2. Description of a full twist.

### C. The effectiveness of the approximated matrices

To show the validation of this approximation method, we simulated a TWP model and compared the result with equivalent cable bundle method. The TWP model is presented in Fig. 5, the radius r=0.406 mm, the height of TWP is h=2mm, the height of wire 1 is hg=4 mm; these heights are measured form the ground. The distance  $\Delta h$  is 0.432 mm, that is one half of the distance between the centers of wires 2 and 3. The load impedance on the terminal side and source side is showed in Fig. 6.

Figures 3 and 4 show the simulation results of the cascade theory matrix and the simplified matrix for length  $2\Delta l$  and matrix for a total length of  $8\Delta l$ . Figure 3 is the current on wire 1 at the near end (the current point which is nearest to the source), and Fig. 4 is the current at the far end (the current point which is nearest to the load).

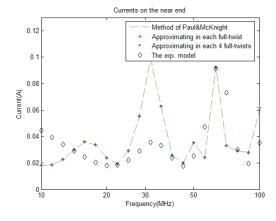


Fig. 3. Current variation with frequency on the source part on wire 1.

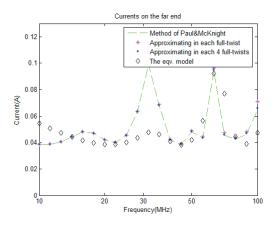


Fig. 4. Current variation with frequency on the load part on wire 1.

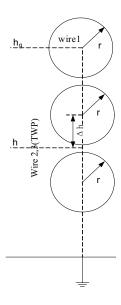


Fig. 5. Considered problem of one TWP and one signal line over earth.

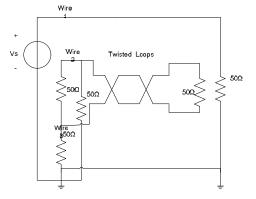


Fig. 6. The source side and the terminal side of the model.

In both Figs. 3 and 4, the result of using the method by Paul and McKnight was shown by the dashed line, and the result of the approximated by one full twist whose distance is  $2\Delta l$  and whose chain parameter matrix is eq. (11), is shown by the dots. The octangle shows a simple model of each 4 full-twists whose distance is  $8\Delta l$ , and the factor in the cosine and sine factor  $(2\beta\Delta l)$  in chain parameter matrix will be replaced by  $8\beta\Delta l$ . The diamond shows the result by using the equivalent cable bundle method.

Comparing the result of the current on wire 1 calculated by the different ways, we could find that their results vary little; it proves that the approximation method is valid when the distance satisfies the rule  $\sin(n\beta\Delta I) << \cos(n\beta\Delta I)$ .

### III. THE COMMON MODE AND THE DIFFERENTIAL MODE OF THE APPROXIMATION CHAIN PARAMETER MATRIX

### A. The approximation impedance and admittance matrices

The transforming of the voltages and currents on wire 2 and wire 3 to common mode and differential mode are usually expressed as equations below:

$$\begin{bmatrix} I_c \\ I_d \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \end{bmatrix}, \tag{13}$$

and

$$\begin{bmatrix} V_c \\ V_d \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix}. \tag{14}$$

Here we use another way of transforming, which is:

$$\begin{bmatrix} I_c \\ I_d \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \end{bmatrix}, \quad (15)$$

and

$$\begin{bmatrix} V_c \\ V_d \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix}. \tag{16}$$

This is because the transforming matrix T,

$$\mathbf{T} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix},\tag{17}$$

can match the condition T=T<sup>-1</sup>: for a three-wire

TWP system like Fig. 1 we get:

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}.$$
 (18)

For the MTLN theory, transforming the characteristic impedance matrix and admittance matrix into the common and differential mode can be expressed as:

$$\mathbf{Z}_{cd} = \mathbf{TZT}^{-1}$$

$$\mathbf{Y}_{cd} = \mathbf{TYT}^{-1}.$$
(19)

For the approximation impendence and admittance based on the  $\sin(\beta \Delta l) \ll \cos(\beta \Delta l)$  condition, which is shown in (12), these matrices will be transformed by (19), the result of the impendence transforming is:

$$\mathbf{Z}_{cd} = \begin{bmatrix} z_{11} & \frac{z_{13} + z_{12}}{2} & 0\\ \frac{z_{21} + z_{31}}{2} & \frac{z_{33} + z_{22} + 2z_{23}}{2} & 0\\ 0 & 0 & \frac{z_{33} + z_{22} - 2z_{23}}{2} \end{bmatrix}, \quad (20)$$

and the admittance  $\mathbf{Y}_{cd}$  has a similar form.

For the approximation equation (11) (whose original equation is (10)), expressing the chain parameter matrix in (11) in a common and differential mode, and considering **TIT**<sup>-1</sup>=**I**, for a full twist shown in Fig. 2 we get:

$$\begin{bmatrix} V_{1}(x_{4}) \\ V_{c}(x_{4}) \\ V_{d}(x_{4}) \\ I_{1}(x_{4}) \\ I_{c}(x_{4}) \\ I_{d}(x_{4}) \end{bmatrix} = \mathbf{\Phi}_{cd} \begin{bmatrix} V_{1}(x_{0}) \\ V_{c}(x_{0}) \\ V_{d}(x_{0}) \\ I_{1}(x_{0}) \\ I_{c}(x_{0}) \\ I_{d}(x_{0}) \end{bmatrix}, \tag{21}$$

where the chain parameter matrix is:

$$\Phi_{cd} = \begin{bmatrix} \cos(2\beta\Delta l)\mathbf{I}_{3\times3} & -j\sin(2\beta\Delta l)\mathbf{Z}_{cd} \\ -j\sin(2\beta\Delta l)\mathbf{Y}_{cd} & \cos(2\beta\Delta l)\mathbf{I}_{3\times3} \end{bmatrix}. (22)$$

There are 4 zero elements in above matrix (20), and these zero elements indicate that the voltages and currents of differential mode are irrelevant with the voltages and currents on other wires (wire 1) and the common mode, the voltage and current of the differential mode can be expressed as:

$$V_{d}(x_{4}) = \cos(\beta \Delta l) V_{d}(x_{0})$$

$$-j \sin(\beta \Delta l) \frac{Z_{33} + Z_{23} - Z_{23}}{2} I_{d}(x_{0})$$

$$I_{d}(x_{4}) = -j \sin(\beta \Delta l) \frac{Y_{33} + Y_{23} - Y_{23}}{2} I_{d}(x_{0})$$

$$+ \cos(\beta \Delta l) I_{d}(x_{0}).$$
(23)

So the differential mode can be calculated independently, the differential part of the approximation chain parameter matrix for length  $\Delta l$  (from x0 to x4) is:

$$\Phi_{sD} = \begin{bmatrix}
\cos(\beta \Delta l) & -j\sin(\beta \Delta l) \frac{z_{c33} + z_{c22} - 2z_{c23}}{2} \\
-j\sin(\beta \Delta l) \frac{y_{c33} + y_{c22} - 2y_{c23}}{2} & \cos(\beta \Delta l)
\end{bmatrix}$$
(24)

Also, since the voltages and currents on other wires (wire 1) and the common mode are irrelevant with that on the differential mode, which is calculated already, the approximation matrix of the common mode and wire 1 has no elements relevant with the differential mode. The matrix is proposed in (25):

$$\mathbf{\Phi}_{sC} = \begin{bmatrix} \cos(\beta \Delta l) \mathbf{I}_{2\times 2} & \sin(\beta \Delta l) \mathbf{Z}_{common 2\times 2} \\ \sin(\beta \Delta l) \mathbf{Y}_{common 2\times 2} & \cos(\beta \Delta l) \mathbf{I}_{2\times 2} \end{bmatrix} . (25)$$

Both  $Z_{common2\times2}$  and  $Y_{common2\times2}$  in (20) are 2×2 matrices with the equal elements of rows 1 and 2 and columns 1 and 2 in (22).

Above all, the total number of elements is 20 (see (24) and (25)), which are much fewer than the original number, 36.

#### B. The dealing of the load impedance

Since the voltages and currents have been changed into the common and differential modes, some transformations of the load and source impedance (or admittance) into the common and differential modes are required.

The definition of the load impedance is usually described as:

$$V(load) = \mathbf{Z}_{1}I(load). \tag{26}$$

In the equation above,  $Z_1$  represents the load impedance. While changing this equation into the common and differential modes by the way shown in part III in (14)-(16), one can get:

(23) 
$$V_{cd}(load) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} V(load), \quad (27)$$

$$I(load) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} I_{cd}(load). (28)$$

Substituting (27) and (28) into (26), the load impedance in the common and differential modes will be:

$$\mathbf{Z}_{l-cd} = \mathbf{T}^{-1} \mathbf{Z}_{l} \mathbf{T} . \tag{29}$$

The load impedance can be transformed by the common and differential approach. We can also transform the source impedance in a same way. The voltage source  $V_s$  is added at wire 1, so the voltage source matrix is:

$$V_{s} = \begin{bmatrix} v_{s} \\ 0 \\ 0 \end{bmatrix}, \tag{30}$$

and this  $V_s$  multiplies the transformation matrix, T will be  $TV_s=V_s$ .

So in the new model TWP was changed into a common mode line and a differential mode line that has fewer total elements in their matrices.

# IV. THE EFFECTIVENESS OF THE COMMON & DIFFERENTIAL MODE MODEL

### A. Numerical experiment of one TWP and one single wire

Now, we simulate the TWP model in Fig. 5 again in the common and differential mode approach. In this approach, the mode transform will happen on wire 2 and wire 3. Through this

way, the total number of the non-zero elements in the impedance (or admittance) matrix will be reduced from 9 to 5, this simplification will save the 4 spaces. The octangle in Fig. 7 shows the current at the near end (the current point which is nearest to the source) on wire 1 using the common and differential mode approach, and Fig. 8 shows the current at the far end (the current point which is nearest to the load). The dashed and dot lines represent the results using the Paul and McKnight method and the method shown in Section II B, respectively. We also calculated the current result by using the equivalent cable method for TWP [16, 17], which is shown by the diamond dot.

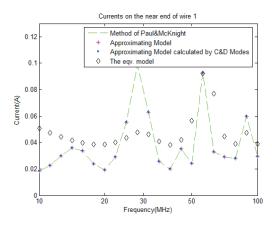


Fig. 7. Results of the near end current on wire 1 by different methods.

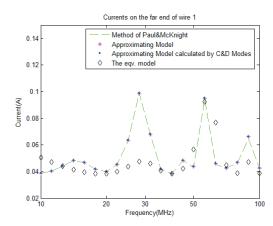


Fig. 8. Results of the far end current on wire 1 by different methods.

Figures 7 and 8 show that the variation in the original Paul and McKnight TWP model and the proposed approximated model is very little, the

total Root-Mean-Square (RMS) form 10 MHz to 100 MHz is  $3.1922 \times 10^{-5}$  at the near end and the one at the far end is  $3.8725 \times 10^{-4}$ , and the RMSs of the equivalent cable method are  $1.6046 \times 10^{-2}$  and  $2.1333 \times 10^{-2}$ . So it is obvious that the present method is more accurate. The time cost of the non-approximated Paul & McKnight method and the time cost of/the approximating model are 0.47s and 0.31s. The time cost of the approximating by common and differential mode and the equivalent model [16, 17] are 0.16s. The 2 models have both shortened the solving process through their own way, so the time cost has been reduced.

### **B.** Numerical experiment of two TWPs and one single wire

We have also calculated the situation of 2 TWPs. The equivalent bundle method's result has not been listed, this is because the model will approximate the TWPs by the value of the load and source impedance, and it was different from the model proposed. The model is shown in Fig. 9. and the lengths in Fig. 8 are selected to be h=2 mm,  $\Delta h$ =0.432 mm, d=3 mm, r=0.406 mm, h<sub>g</sub>=4 mm and d<sub>2</sub>=2 mm. In the common and differential mode we transform each wire pair twisted into their own common and differential modes. The results of the near end current and far end currents on the single wire are shown in Figs. 10 and 11. In this method, the mode transform will happen on each couple of the TWPs. There is 1 single wire and 2 common mode wires, and 2 differential mode wires in the total model. The differential modes are irrelevant with other wires, so non-zero elements number in the impedance or admittance matrixes of the reduced model will be 11 and the original number is 25. The total RMSs without mode transform are 1.1213×10<sup>-5</sup> at the near end and 2.3106×10<sup>-5</sup> at the far end. The ones with the transform are 7.0357×10<sup>-5</sup> at the near end and 6.7700×10<sup>-5</sup> at the far end. The RMSs grow a little because some errors occur in the calculation of the Z and Y in the mode transform, but it is still accurate for this method. When extending this method into N TWPs and M single wires, we could transform a  $(2N+M)^2$  non-zero elements' matrix into an (N+M)<sup>2</sup>+N elements' one. The total reducing number is N(2N+2M-1), and it will save much space for storing the matrix that will affect on the result little by using the approximation method.

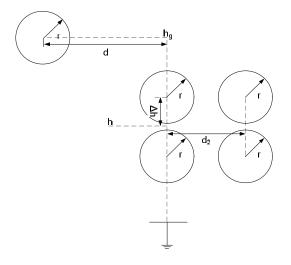


Fig. 9. Simulation model of 2 TWPs and one single wire.

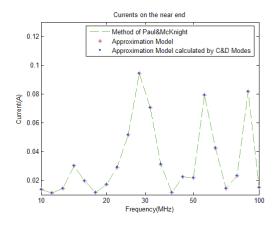


Fig. 10. Near end current results of the 2 TWPs and 1 single wire model.

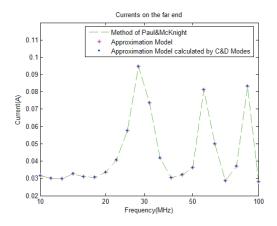


Fig. 11. Far end current results of the 2 TWPs and 1 single wire model.

#### V. CONCLUSION

In this paper, we proposed the approximation model of a twist wire pair into the common and differential mode, the derivation of the transforming has been provided. Due to the RMS result shown by the simulation, the approximation model varies little from the original model and it succeeded in the reduction in the total number of non-zero elements, and the method proposed has much reduction in storage and time cost.

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