Time-Domain Integral Equation Solver Using Variable-Order Temporal Interpolators

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Abstract **─** A novel, efficient, and simple modification to standard marching-on-in-time (MOT)–based time-domain integral equation (TDIE) solvers is presented. It allows for the use of high-order temporal interpolators without the need to extrapolate and predict future unknowns. The order of these temporal interpolators is increased as the distance of source and testing quadrature points increases. The proposed TDIE solver significantly increases the accuracy of solutions by exploiting high-order temporal interpolation at no significant extra computational cost. Numerical examples are presented to validate the proposed method.

Index Terms - Marching-on-in-time (MOT), temporal interpolator, and time-domain integral equation (TDIE).

I. INTRODUCTION

MOT–based TDIE solvers represent an increasingly mature technology for analyzing transient electromagnetic wave interactions with perfect electrically conducting (PEC) surfaces. To allow for the solution of large-scale scattering problems, these solvers often are accelerated by multilevel plane wave time domain (PWTD) [1] or time-domain adaptive integral methods (TD-AIM) [2]. Their stability and accuracy has been observed to be closely related to the method used for discretizing the surface current in both space and time as well as the method used for evaluation of MOT matrix elements [3-5]. To increase the accuracy of the spatial discretization, high-order interpolatory [6, 7] or hierarchical [8] spatial basis functions are often used. To enhance the solver's stability, smooth temporal basis functions are preferred [9, 10]. Two basis functions often used for this purpose are Lagrange [11] and Quadratic B-Spline (QBS) interpolants [10]. Their frequency spectra decay as $1/f^{2}$ and $1/f^{3}$, respectively; this renders the QBS slightly preferable.

In this paper a new MOT scheme that allows for the use of different temporal interpolators depending on the distance between source and test points, is presented. The advantages of the proposed method are threefold: (i) It increases the accuracy of a TDIE solver without sacrificing its stability or computational efficiency. (ii) It alleviates the introduction of spurious high-frequency modes into the solution without the need to extrapolate and/or predict future unknowns. (iii) When applied to TDIE solvers based on time domain Green's functions (TDGFs) of layered media [12-15], in addition to the above-mentioned advantages, which lead to more stable TDIE solvers, the proposed method significantly decreases the computational expense of taking the convolution of TDGFs with temporal interpolators by increasing the temporal smoothness.

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II. STANDARD TIME DOMAIN EFIE

Consider a perfect electrically conducting (PEC) surface S with unit normal vector $\hat{\mathbf{n}}$ that resides in a homogeneous medium with permittivity ε and permeability μ. The incident electric field $\mathbf{E}^{inc}(\mathbf{r},t)$ induces a current $\mathbf{J}(\mathbf{r},t)$ on S, which generates the scattered electric field $\mathbf{E}^{\text{sca}}(\mathbf{r},t)$. The total electric field $\mathbf{E}^{inc}(\mathbf{r},t)+\mathbf{E}^{\text{sca}}(\mathbf{r},t)$ tangential to S vanishes. The same therefore is true for its time derivative,

$$\hat{\mathbf{n}}\times\left[\dot{\mathbf{E}}^{\text{inc}}(\mathbf{r},t)+\dot{\mathbf{E}}^{\text{sca}}(\mathbf{r},t)\right]=0 \qquad \mathbf{r}\in S. \tag{1}$$

Here, a dot on a symbol implies temporal differentiation, and

$$\dot{\mathbf{E}}^{\text{sca}}(\mathbf{r},t)=-\mu\ddot{\mathbf{A}}(\mathbf{r},t)+\frac{1}{\varepsilon}\nabla\nabla\cdot\mathbf{A}(\mathbf{r},t) \tag{2}$$

with

$$\mathbf{A}(\mathbf{r},t)=\iint_S \frac{\mathbf{J}(\mathbf{r}',t-\frac{R}{c})}{4\pi R}d\,\mathbf{r}', \tag{3}$$

$c=1/\sqrt{\mu\varepsilon}$ is the speed of light, and $R=|\mathbf{r}-\mathbf{r}'|$. To solve equation (1), $\mathbf{J}(\mathbf{r},t)$ is spatially discretized as,

$$\mathbf{J}(\mathbf{r},t)=\sum_{n=1}^{N_s}\mathbf{S}_n(\mathbf{r})I_n(t)\,; \tag{4}$$

$I_n(t)$ is the temporal signature of the n^{th} spatial basis function $\mathbf{S}_n(\mathbf{r})$. Assuming that $\mathbf{E}^{inc}(\mathbf{r},t)$ is temporally quasi-bandlimited to frequency $f_{\max}$ and vanishingly small for $\mathbf{r}\in S$ and $t<0$, $I_n(t)$ can be reconstructed from its samples,

$$I_n(j\,\Delta t)=I_{nj}\ ,\quad j=1,2,...,N_t \tag{5}$$

as

$$I_n(t)=\sum_{j=1}^{N_t}I_{nj}T(t-j\,\Delta t) \tag{6}$$

where the time step $\Delta t=1/(2\beta f_{\max})$, β is a temporal oversampling factor typically chosen in the range $3<\beta<20$, and $T(t-j\,\Delta t)$ is a suitably chosen interpolator. Equation (6) and (4) imply the following space-time discretization of $\mathbf{J}(\mathbf{r},t)$,

$$\mathbf{J}(\mathbf{r},t)=\sum_{n=1}^{N_s}\sum_{j=1}^{N_t}I_{nj}\mathbf{S}_n(\mathbf{r})T(t-j\,\Delta t). \tag{7}$$

Substituting equation (7) into equations (1-3) and enforcing the resulting equation by Galerkin testing in space and point matching in time yields [1, 2,16-18],

$$\sum_{j=1}^{N_t}\mathbf{Z}_{i-j}\mathbf{I}_j=\mathbf{V}_i\,,\quad i=1,2,...,N_t \tag{8}$$

where

$$[V_i]_m=\iint_S \mathbf{S}_m(\mathbf{r})\cdot\dot{\mathbf{E}}^{inc}(\mathbf{r},i\,\Delta t)ds\,,\quad [I_j]_m=I_{mj}\,. \tag{9}$$

and

$$[Z_k]_{mn}=$$
$$\frac{\mu}{4\pi}\iint_S\iint_{S'}\frac{\mathbf{S}_m(\mathbf{r})\cdot\mathbf{S}_n(\mathbf{r}')}{R}\ddot{T}(k\,\Delta t-\frac{R}{c})\,ds'ds$$
$$+\frac{1}{4\pi\varepsilon}\iint_S\iint_{S'}\frac{\nabla\cdot\mathbf{S}_m(\mathbf{r})\nabla'\cdot\mathbf{S}_n(\mathbf{r}')}{R}T(k\,\Delta t-\frac{R}{c})\,ds'ds \,. \tag{10}$$

A temporal interpolator $T(t)$ satisfying,

$$T(t)=0,\quad t<-\Delta t \tag{11}$$

is said to be causal. For causal interpolators,

$$\mathbf{Z}_{-k}=0,\quad k=1,2,3,... \tag{12}$$

and equation (8) reduces to the standard MOT equations from which the expansion coefficients I_{nj} can be retrieved, one time step at a time,

$$\mathbf{Z}_0\mathbf{I}_i=\mathbf{V}_i-\sum_{j=1}^{i-1}\mathbf{Z}_{i-j}\mathbf{I}_j\,,\quad i=1,2,...,N_t\,. \tag{13}$$

Next we demonstrate that condition (11) can be relaxed without relinquishing the MOT form of equation (13).

III. DISTANCE-DEPENDENT TEMPORAL INTERPOLATORS

A. Concept

By discretizing the spatial integrations in equation (10) by N_{tqp} test quadrature points and N_{sqp} source quadrature points, equation (8) can be rewritten as,

$$\begin{aligned}[V_i]_m&=\sum_{j=1}^{N_t}\sum_{n=1}^{N_s}[Z_{i-j}]_{mn}[I_j]_n\\&=\sum_{n=1}^{N_s}\sum_{q=1}^{N_{tqp}}\sum_{q'=1}^{N_{sqp}}w_{mq}w_{nq'}\\&\quad\left[a_{mq,nq'}\sum_{j=1}^{N_t}I_{nj}\ddot{T}((i-j)\Delta t-\frac{R_{mq,nq'}}{c})\right.\\&\quad\left.+b_{mq,nq'}\sum_{j=1}^{N_t}I_{nj}T((i-j)\Delta t-\frac{R_{mq,nq'}}{c})\right]\end{aligned} \tag{14}$$

where

$$\begin{aligned}a_{mq,nq'}&=\frac{\mu\mathbf{S}_m(\mathbf{r}_{mq})\cdot\mathbf{S}_n(\mathbf{r}'_{nq'})}{4\pi R_{mq,nq'}},\\ b_{mq,nq'}&=\frac{\nabla\cdot\mathbf{S}_m(\mathbf{r}_{mq})\nabla'\cdot\mathbf{S}_n(\mathbf{r}'_{nq'})}{4\pi\varepsilon R_{mq,nq'}}\end{aligned} \tag{15}$$

with $R_{mq,nq'} = \left|\mathbf{r}_{mq} - \mathbf{r}'_{nq'}\right|$, $\mathbf{r}_{mq}$ and $\mathbf{r}'_{nq'}$ are position vectors of q^{th} quadrature point of m$^{\text{th}}$ spatial basis function and q'^{th} quadrature point of n$^{\text{th}}$ spatial basis functions, respectively, w_{mq} and $w_{nq'}$ are the corresponding quadrature weights.

The two summations inside brackets of equation (14) are nothing but the temporal interpolation of $\ddot{I}_n(t)$ and $I_n(t)$, respectively at time $t = i\Delta t - R_{mq,nq'}/c$. More clearly from equation (6) we have,

$$I_n(i\Delta t - \frac{R_{mq,nq'}}{c}) = \sum_{j=1}^{N_t} I_{nj}\, T((i-j)\Delta t - \frac{R_{mq,nq'}}{c})$$
$$\ddot{I}_n(i\Delta t - \frac{R_{mq,nq'}}{c}) = \sum_{j=1}^{N_t} I_{nj}\, \ddot{T}((i-j)\Delta t - \frac{R_{mq,nq'}}{c}) \tag{16}$$

.

It is easy to see that by replacing *T(t)* by any other interpolatory function, equations (14)-(16) still remain valid. Moreover, this replacement can be done based on the position of source and test quadrature points. Therefore, a more flexible restriction on *T(t)* to satisfy equation (12) is,

$$T(t) = 0, \quad t < -\Delta t - \frac{R_{mq,nq'}}{c}. \tag{17}$$

This condition means that as the distance of source and test quadrature points increases, it is allowed to choose wider temporal interpolator $T(t)$ that contradicts equation (11) and still use the MOT scheme of equation (13).

In the next section B-Spline functions of arbitrary order, which are used in numerical results as distance-dependent interpolators are defined.

B. B-Spline functions

B-Spline functions of order m are defined as,

$$b^{(m)}(t) = \frac{1}{\Delta t} b^{(0)}(t) * b^{(m-1)}(t), \quad m = 1,2,3,\ldots \tag{18}$$

where '*' denotes temporal convolution and

$$b^{(0)}(t) = rect(\frac{t}{\Delta t}) = \begin{cases} 1 & , \left|\dfrac{t}{\Delta t}\right| < 1 \\ 0 & , \text{otherwise} \end{cases} . \tag{19}$$

Consider the shifted B-Spline functions defined as,

$$b'^{(m)}(t) = b^m(t - \Delta t/2), \qquad m = 2,3,4,\ldots \tag{20}$$

Given the definition in equation (18), the spectrum of shifted B-Spline functions of equation (20) is,

$$\tilde{b}'^{(m)}(f) = \Delta t \ \mathrm{Sinc}^{m+1}(f\ \Delta t) e^{-j\pi f \Delta t} = \frac{1}{(\Delta t)^m}\left(\frac{\sin(\pi f\,\Delta t)}{\pi f}\right)^{m+1} e^{-j\pi f \Delta t} \tag{21}$$

which indicates that the spectrum of B-Spline function of order m, decays as $1/f^{(m+1)}$.

Shifted B-Spline functions of different orders are depicted in Fig. 1. An arbitrary function $s(t)$ can be expanded in terms of B-Spline functions of equations (18) and (19) as,

$$s(t) \simeq \sum_{n=-\infty}^{+\infty} s(n\Delta t) b^{(m)}(t - n\Delta t), \; m \in \{0,1,2,\ldots\} \tag{22}$$

Note that for the special case of m=1, equation (18) is the standard triangular (hat) function and equation (22) is nothing but a piecewise linear interpolation of $s(t)$. Considering equations (20)and (22), it is obvious that an arbitrary function $s(t)$ can also be expanded in terms of shifted B-Spline functions of equation (20) as,

$$s(t - \Delta t/2) \simeq \sum_{n=-\infty}^{+\infty} s(n\Delta t) b'^{(m)}(t - n\Delta t), \quad m \in \{2,3,4,\ldots\} \tag{23}$$

By applying the restriction in equation (17) it is seen that $b'^{(m)}(t)$ can be safely used in TDIE solvers for distances R satisfying,

$$R > \frac{(m-2)}{2} c\Delta t. \tag{24}$$

Note that $b'^{(2)}(t)$ is the only shifted B-Spline temporal interpolator defined in equation (20) that satisfies equation (17) for the worst case of $R_{mq,nq'} = 0$ and therefore is selected as temporal interpolator for near distances of source and test quadrature points in numerical results.

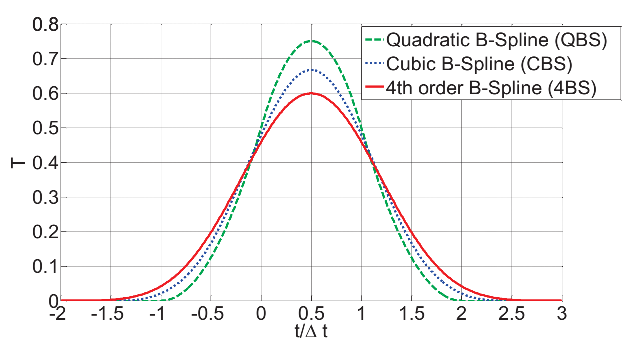


Fig. 1. Shifted B-Spline functions of different order.

IV. NUMERICAL RESULTS

In parts A and B of this section, the above solver is applied to the analysis of scattering from a sphere and cube illuminated by the modulated Gaussian plane wave,

$$\mathbf{E}^{inc}(\mathrm{r},t) = \hat{\mathbf{x}} e^{-[(\tau - t_p)/\sqrt{2}\sigma]^2} \cos(2\pi f_0 \tau). \quad (25)$$

with $\tau = t - \mathbf{r} \cdot \hat{\mathbf{z}} / c$, the center frequency $f_0 = 40$ MHz, the delay $t_p = 0.5 \times 10^{-6}$ s, and $\sigma = 6/(2\pi f_{BW})$ with the nominal bandwidth $f_{BW} = 20$ MHz. The MOT time step is $\Delta t = 6.25 \times 10^{-10}$ s and the number of time steps $N_t = 1600$. In these examples, shifted B-Splines of order $m = 2, 3,$ and 4 are used for $R < 0.0938$ m, $0.1875 \text{ m} > R \geq 0.0938$ m, and $R \geq 0.1875$ m, respectively, in agreement with the condition of equation (24). Frequency-domain results attributed to the solver were obtained by Fourier transforming time-domain data while accounting for the spectral content of the incident field.

In part C of this section the proposed distance-dependent temporal interpolation scheme is applied to a recently developed TDIE solver for analyzing planar structures in layered media.

A. Sphere

The surface of a PEC sphere of radius 1 m (centered about the origin) is discretized using 48 curvilinear patches, resulting in $N_s = 72$ spatial RWG basis functions [19]. Each patch is obtained by means of an exact mapping from a reference RWG patch onto the sphere surface.

Fig. 2 (a) shows the bistatic radar cross section (RCS) for $\varphi = 0$ and $-180 \leq \theta \leq 0$ for frequency $f = 43$ MHz and different choices of temporal interpolating functions.

Fig. 2 (b) shows the relative error of the computed RCS with respect to Mie series solution. The norm of current vector $\mathbf{I}_j$ is plotted in

Fig. 2 (c). Clearly the use of a shifted QBS temporal interpolator results in more accurate results compared to Lagrange interpolators [11, 20]. Moreover, as expected, using distance-dependent variable order B-Splines of Fig. 1 as temporal interpolators, significantly increases the accuracy without affecting the stability of solutions. By exploiting variable order B-Splines as temporal interpolators, the worst case relative error in RCS is decreased by 48 % with respect to the case where only QBS is used as temporal interpolator.

B. Cube

The surface of a PEC cube with side length of 1 m (centered about the origin and with cube edges aligned with the major coordinate axes) is discretized using 256 flat patches, resulting in $N_s = 384$ spatial RWG basis functions. Since there is no analytical solution for the cube example, the results of the TDIE solver when the surface current of the cube is densely discretized using 1773 RWG spatial basis functions are considered as reference solution for comparison. The results for frequency $f = 50$ MHz and different choices of temporal interpolating functions are plotted in

Fig. 3. Accuracy improvements on par with those observed in the previous example when using distance-dependent high-order B-Splines are obtained here leading to 45% decrease in worst case relative error in RCS with respect to the case where only QBS is used as temporal interpolator.

C. Microstrip patch antenna array

As the last example, to show the ability of the proposed variable-order and distance-dependent temporal interpolator scheme in increasing the accuracy and therefore the stability of the TDIE solvers, this scheme is incorporated into a recently developed TDIE solver based on the TDGFs of the layered media [15]. In this solver the direct convolution of the TDGFs with temporal interpolators are computed using a novel and highly efficient 2D finite difference scheme.

Consider a 2 by 1 array of microstrip patch antenna as shown in

Fig. **4** (a). The units in this figure are in millimeter. The patch antenna is located over a PEC backed dielectric substrate with relative permittivity of $\varepsilon_r = 2.2$ and thickness of h=1.524 mm. The antenna is fed by a modulated Gaussian voltage signal of,

$$V(t) = e^{-[(t - t_p)/\sqrt{2}\sigma]^2} \cos(2\pi f_0 t). \quad (26)$$

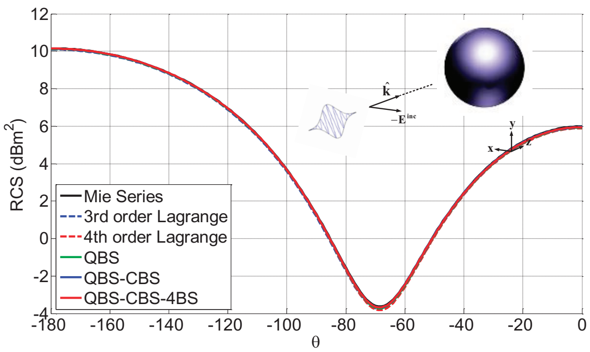
With the center frequency f_0 = 4.5 GHz, the delay $t_p = 0.5 \times 10^{-8}$ s, and $\sigma = 6/(2\pi f_{BW})$ with the nominal bandwidth f_{BW} = 2 GHz. The surface of the antenna array is discretized using 468 triangular patches, resulting in N_s=614 spatial RWG basis functions. The MOT time step is set to $\Delta t = 5 \times 10^{-12}$ s.

First, we only use QBS as temporal interpolator for all distances of the source to test quadrature points as is used in standard TDIE solvers.

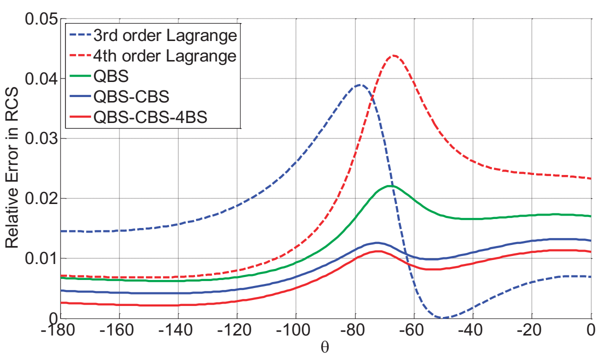
For comparison we also run the solver when shifted B-Splines of order $m = 2, 3,$ and 4 are used for $R < 1$ mm, $1 \text{ mm} \leq R < 1.6$ mm, and

$R \geq 1.6\,\text{mm}$, respectively as temporal interpolators in agreement with the condition of equation (24). The TDIE solver runs for N_t=10000 time steps. The stability of the TDIE solver is shown in

Fig. **4** (b) where the norm of current vector is plotted. As can be seen from this figure, the standard TDIE solver is instable while the proposed TDIE solver based on variable order and distance-dependent temporal interpolators gives stable results.



(a)



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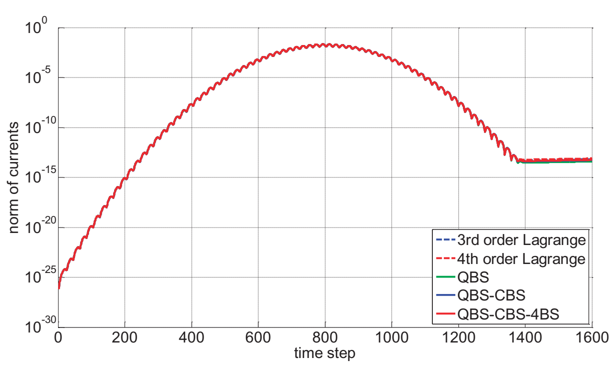
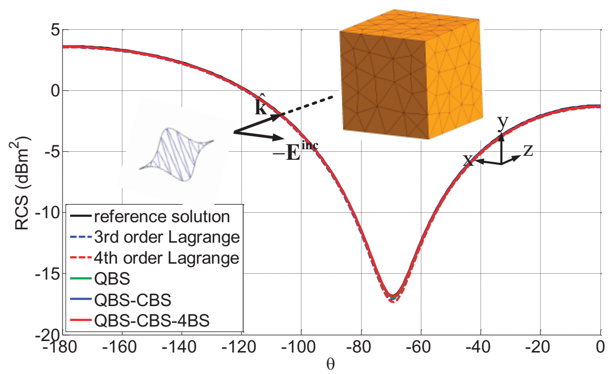
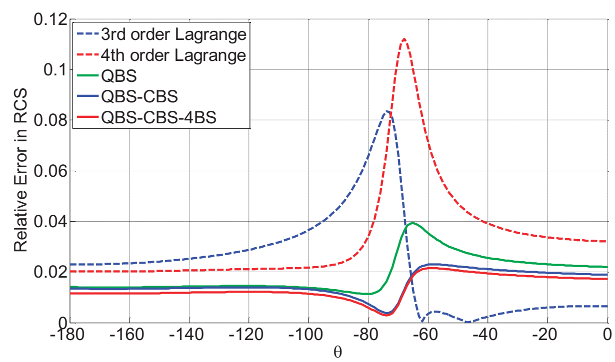


(c)

Fig. 2. Bistatic RCS of a unit PEC sphere at 43 MHz for different choices of temporal interpolators. The surface of the sphere is modeled using 48 curvilinear triangular patches. (a) Bistatic RCS, (b) relative error in the RCS with respect to Mei's series solution, and (c) norm of current vector at each time step.



(a)



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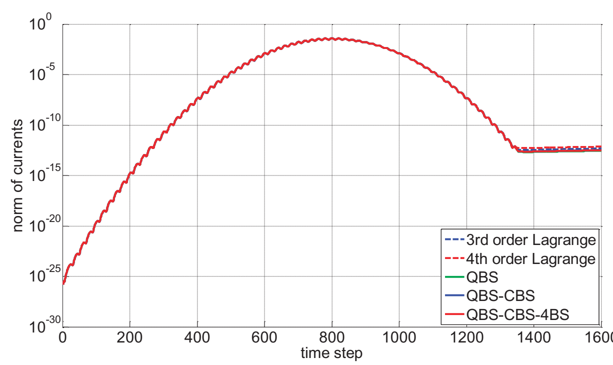


(c)

Fig. 3. Bistatic RCS of a PEC cube with side length of 1 m at 50 MHz for different choices of temporal interpolators. The surface of the cube is modeled using 256 flat triangular patches. (a) Bistatic RCS, (b) relative error in the RCS with respect to

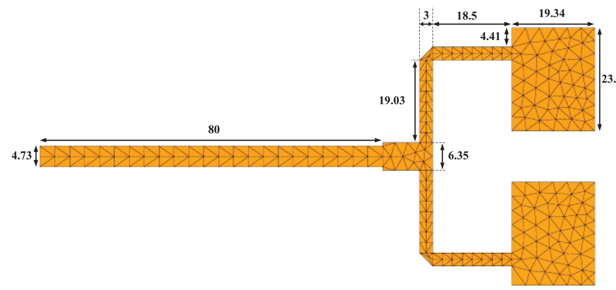
reference solution, and (c) norm of current vector at each time step.

It is worth mentioning that in the second run the only change with respect to the first run is that a fixed temporal interpolator i.e., QBS is replaced with the high-order and distance-dependent temporal interpolators. This replacement not only may increase the accuracy of the solver but also significantly decreases the cost of computing the convolution of temporal interpolators with the TDGFs of layered media by using much more smooth temporal interpolators for non-near pair of source-testquadrature points.

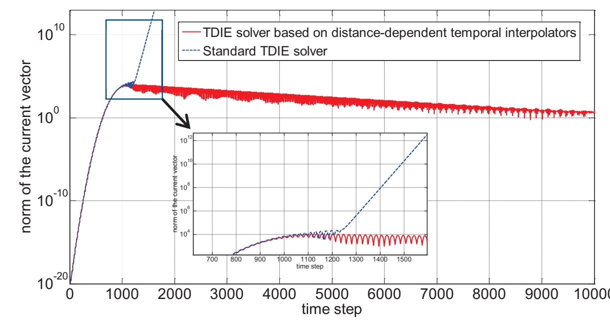
Fig. **4** (c) shows the reflection amplitude $|S_{11}|$ of the patch antenna achieved by using a proper post processing technique applied to the stable time domain surface currentoutput of the proposed TDIE solver based on distance-dependent temporal interpolators. The results are being compared with that of the commercial software ADS-Momentum, which is based on frequency domain method of moments (MoM). A good agreement between the results of two methods is observed.

V. CONCLUSION

A new MOT-TDIE solver that uses distance-dependent high-order temporal interpolators was introduced. The solver tunes the basis functions' temporal support to the distance between source and observer points, maximizing temporal smoothness and avoiding non-causal excitations along the way. When compared to classically formulated MOT-TDIE schemes, the new method can markedly improve solution accuracy by suppressing high-frequency, out-of-band spurious solution components stemming from the use of temporal interpolators with spectral support far exceeding that of the excitation. The proposed distance-dependent interpolation scheme is also very advantageous in TDIE solvers for analyzing electromagnetic interactions with structures residing in layered media. In this case the proposed method not only may increase the solution accuracy but also can significantly decrease the cost of computing the convolution of temporal interpolators with the TDGFs.



(a)



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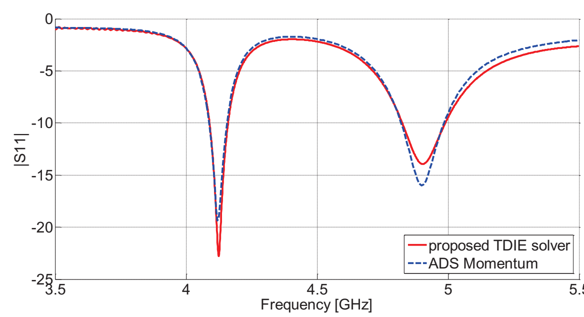

(c)

Fig. 4. TDIE analysis of the patch antenna array (a) antenna layout (the units are in millimeter), (b) norm of current vector at each time step using proposed TDIE solver and standard TDIE solver, and (c) the amplitude of the reflection coefficient $|S_{11}|$ achieved using the proposed TDIE solver and its comparison with the results of ADS-Momentum.

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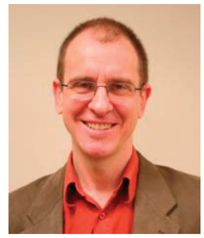

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