A Hybrid MoM-PO Method Combining ACA Technique for Electromagnetic Scattering from Target above a Rough Surface

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Abstract **─** In this paper, an efficient hybrid method of moments (MoM)-physical optics (PO) method combining adaptive cross approximation (ACA) technique is applied to calculate the electromagnetic scattering from three-dimensional (3-D) target and rough surface composite model. The current on the rough surface is obtained through the PO approximation, while the current on the target surface is obtained through the MoM. Furthermore, an ACA technique is used to accelerate the coupling interaction between the target and the rough surface. Numerical results demonstrate that the memory and time cost can be substantially reduced without losing precision by applying the hybrid method, and which can be used to analyze large scale target/rough surface scattering problems.

Index Terms **–** Adaptive cross approximation (ACA), electromagnetic scattering, and MoM-PO, rough surface.

I. INTRODUCTION

The electromagnetic scattering calculation of target and rough surface composite model has been applied in the fields of radar surveillance, microwave remote sensing, target recognition and target tracking extensively [1-6]. The solutions of the composite scattering problems are complicated but practical.

Some numerical methods have been developed for three-dimensional (3-D) target/rough surface scattering problems, e.g., the finite-difference time-domain (FDTD) algorithm [7-8], a hybrid Kirchhoff approximation (KA)-method of moments (MoM) algorithm [9], multilevel UV method [10-11], the MoM using higher order basis functions [12], the hybrid MoM-physical optics (PO) method [13], most of which are based on the MoM.

The conventional MoM yields a dense complex linear system, which is a serious handicap especially for electrically large scattering problems. Some hybrid methods such as MoM-PO [13-14] are applied to reduce the computation time and memory requirement substantially, while the results are in reasonable agreement with those based on an application of the MoM alone.

In [15], an adaptive cross approximation (ACA) algorithm is used to accelerate MoM computations of electromagnetic compatibility (EMC) problems. It takes advantage of the rank-deficient character of the coupling matrix blocks representing well-separated MoM interactions [16-18]. The ACA algorithm has several important advantages over the multilevel fast multipole algorithm (MLFMA) [19-25]. The beauty of the ACA algorithm is its purely algebraic characteristic. Thus, the development and implementation of ACA algorithm do not depend on the complete knowledge of the integral

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equation kernel, basis functions or the integral equation formulation itself. Moreover, due to its algebraic characteristic, ACA can be modular and very easily integrated into various MoM codes.

In this paper, an efficient hybrid MoM-PO method combining ACA technique is applied to calculate the electromagnetic scattering from 3-D target and rough surface composite model. Both the target and rough surface are assumed to be perfect electric conductor (PEC). Numerical results demonstrate that the memory and time cost can be substantially reduced without losing precision by applying the hybrid method, and which can be used to analyze large scale target/rough surface scattering problems.

II. FORMULATIONS

A. MoM-PO formulation

According to Fig. 1, the surface of the scattering body is split into a MoM-region and a PO-region, which correspond to a target and a rough surface, respectively. In principle, this subdivision can be performed in an arbitrary manner. We divide the scattering body in the manner aiming at making a tradeoff between solution accuracy and efficiency.

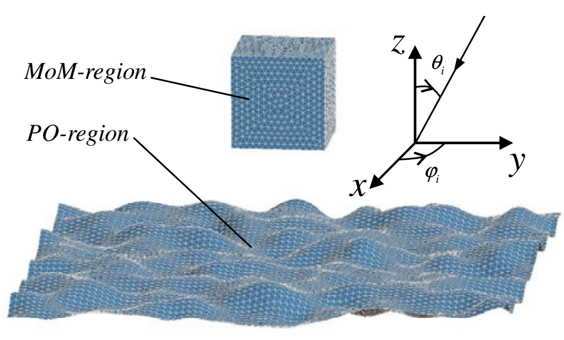

Fig. 1. Composite scattering model of target above a rough surface.

The surface currents of MoM-region and PO-region can be expanded by RWG basis function, written as,

$$\mathbf{J}^{\mathrm{MoM}} = \sum_{n=1}^{N^{MoM}} \alpha_n \mathbf{f}_n \tag{1}$$

$$\mathbf{J}^{\mathrm{PO}} = \sum_{k=1}^{N^{PO}} \beta_k \mathbf{f}_k \, , \tag{2}$$

where N^{MoM} and N^{PO} denote the number of unknowns in MoM-region and PO-region respectively, α_n and β_k are expansion coefficients of $\mathbf{f}_n$ and $\mathbf{f}_k$, both of which are RWG basis functions [26].

In the hybrid MoM-PO method, the relationship between the current in PO-region, the incident field, the current in MoM-region could be expressed as,

$$\mathbf{J}^{\mathrm{PO}}(\mathbf{r}) = 2\hat{\mathbf{n}}\times\mathbf{H}^{inc}(\mathbf{r}) + \sum_{n=1}^{N^{MoM}} 2\alpha_n \hat{\mathbf{n}}\times L^H \mathbf{f}_n \tag{3}$$

where $\mathbf{H}^{inc}(\mathbf{r})$ denotes the incident magnetic field, L^H is the magnetic field integral operator and $L^H\mathbf{f}_\mathbf{n} = \nabla\times\iint_{S'} \mathbf{f}_\mathbf{n}(\mathbf{r}')\cdot g(\mathbf{r},\mathbf{r}')dS'$, here $g(\mathbf{r},\mathbf{r}') = e^{-jk|\mathbf{r}-\mathbf{r}'|}/4\pi|\mathbf{r}-\mathbf{r}'|$, the Green's function of free space, $\mathbf{r}'$ and $\mathbf{r}$ denote the locations of source and observation point, respectively, $\hat{\mathbf{n}}$ denotes the unit outward normal vector of the conductor surface.

In order to get the expansion coefficient β_k , the two unit vectors $\hat{\mathbf{t}}_k^{\pm}$ are introduced in the middle of the kth edge. The $\hat{\mathbf{t}}_k^{\pm}$ are respectively lying in the plane of the triangles $\mathrm{T}_k^{\pm}$ defined by the kth edge, and perpendicular to the kth edge. As shown in Fig. 2, $\mathbf{f}_k(\mathbf{r}_k)\cdot\hat{\mathbf{t}}_k^{\pm} = 1$ is valid when the point $\mathbf{r}_k$ is in the middle of the kth edge.

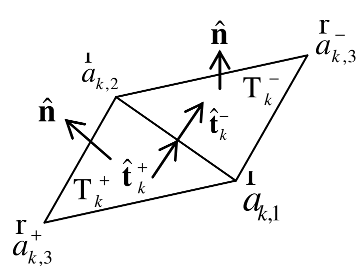

Fig. 2. The kth edge with two adjacent triangles T_k^{+} and T_k^{-}.

Multiplying both sides of equation (2) with $\frac{1}{2}(\hat{\mathbf{t}}_k^{+} + \hat{\mathbf{t}}_k^{-})$ and inserting equation (3) in the resulting equation lead to,

$$\beta_k = \tau_k + \sum_{n=1}^{N^{MoM}} \alpha_n \cdot \tau_{n,k} \, , \tag{4}$$

where $\tau_k = (\hat{\mathbf{t}}_k^- + \hat{\mathbf{t}}_k^+) \cdot (\hat{\mathbf{n}} \times \mathbf{H}^{inc}(\mathbf{r}))$ and $\tau_{n,k} = (\hat{\mathbf{t}}_k^- + \hat{\mathbf{t}}_k^+) \cdot (\hat{\mathbf{n}} \times L^H \mathbf{f}_n)$.

For the MoM-region, the electric field integral equation (EFIE) could be written as,

$$(L^E \mathbf{J}^{\mathrm{MoM}})_{\tan} + (L^E \mathbf{J}^{\mathrm{PO}})_{\tan} = -\mathbf{E}_{\tan}^{inc} \quad (5)$$

where L^E is electric field integral operator and $L^E \mathbf{J} = jk_0 \eta_0 \iint_{S'} \left(\bar{\mathbf{I}} + \frac{\nabla\nabla}{k_0^2} \right) g(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J} dS'$, here k_0 and η_0 are the wave number and the wave impedance of free space, $\mathbf{E}^{inc}$ is the incident electric field.

Finally, inserting equations (1), (2), and (4) into equation (5) results in,

$$\sum_{n=1}^{N^{MoM}} \alpha_n \left[L^E \mathbf{f}_n + \sum_{k=1}^{N^{PO}} \tau_{n,k} \cdot L^E \mathbf{f}_k \right]_{\tan} = -\mathbf{E}_{\tan}^{inc} - \sum_{k=1}^{N^{PO}} \tau_k \cdot (L^E \mathbf{f}_k)_{\tan} \quad . \quad (6)$$

Testing equation (6) with RWG basis functions in MoM-region, we can achieve the matrix equation expressed as,

$$(Z^{MoM} + Z^{MoM,PO} \cdot \tau') I^{MoM} = V - Z^{MoM,PO} \cdot \tau \quad (7)$$

where the Z^{MoM}, $Z^{MoM,PO}$, τ' are $N^{MoM} \times N^{MoM}$, $N^{MoM} \times N^{PO}$, $N^{PO} \times N^{MoM}$ complex matrix respectively, I^{MoM} and V are vectors of size N^{MoM}, τ is vector of size N^{PO}. The matrix elements are written as,

$$Z_{mn}^{MoM} = \left\langle \mathbf{f}_m, L^E \mathbf{f}_n \right\rangle \quad (8)$$

$$I_n^{MoM} = a_n , \quad (9)$$

$$V_m = -\left\langle \mathbf{f}_m, \mathbf{E}_{\tan}^{inc} \right\rangle , \quad (10)$$

$$Z_{mk}^{MoM,PO} = \left\langle \mathbf{f}_m, L^E \mathbf{f}_k \right\rangle , \quad (11)$$

$$\tau'_{kn} = (\hat{\mathbf{t}}_k^+ + \hat{\mathbf{t}}_k^-) g (\hat{\mathbf{n}} \times L^H \mathbf{f}_n) , \quad (12)$$

$$\tau_k = (\hat{\mathbf{t}}_k^+ + \hat{\mathbf{t}}_k^-) g (\hat{\mathbf{n}} \times \mathbf{H}^{inc}(\mathbf{r})) . \quad (13)$$

B. The application of ACA algorithm in MoM-PO

For the composite scattering problems of target above rough surface, the matrix $Z^{MM,PO}$ and τ' for interaction between MoM-region and PO-region have rank-deficient characters because the distance between source point and observation point is relatively far. The ACA algorithm fast achieves the low-rank decomposition form by using the rank-deficient character of matrix [15, 17]. The basic principle of ACA algorithm is as follow. The low-rank representation of a matrix could be got by the elements of partial rows and columns but not all the matrix elements. It means that by selecting right rows and columns, we can get the singular value decomposition form of the matrix approximately, so as to achieve the purpose of improving the computational efficiency.

Let the $m \times n$ rectangular matrix $Z^{m\times n}$ represent the interaction between two well-separated cubes. The ACA algorithm aims to approximate $Z^{m\times n}$ by $\tilde{Z}^{m\times n}$ in the following form,

$$\tilde{Z}^{m\times n} = U^{m\times r} V^{r\times n} = \sum_{i=1}^{r} u_i^{m\times 1} v_i^{1\times n} \quad (14)$$

where r is the effective rank of the matrix $Z^{m\times n}$. The goal of ACA is to achieve,

$$\| R^{m\times n} \|_F = \| Z^{m\times n} - \tilde{Z}^{m\times n} \|_F \le \varepsilon_{ACA} \| Z^{m\times n} \|_F \quad (15)$$

for a given tolerance ε_{ACA}, where R is termed as the error matrix, $\|\square\|_F$ denotes the matrix Frobenius norm. If $r \ll \min(m,n)$, the memory requirement will be reduced significantly from $m \times n$ to $(m+n) \times r$.

Selecting the value of the ε_{ACA} is very important. If the ε_{ACA} is too small, the computational cost will be high, while if the ε_{ACA} is too big, the computational accuracy will be low. Therefore, it is necessary to make a tradeoff. The more details of the ACA algorithm can be found in [15].

III. NUMERICAL EXAMPLES

In this section, several numerical examples are presented to illustrate the validity and efficiency of the proposed method. In these examples, the composite models are illuminated by tapered wave [27], which is employed to avoid rough surface edge scattering effects. The tapered wave is expressed as,

$$\mathbf{E}^{inc}(x, y, z) = \exp\left[-jk_0(z\cos\theta_i + x\sin\theta_i\cos\phi_i + y\sin\theta_i\sin\phi_i)(1+\omega)\right]\exp(-t_x - t_y) \quad (16)$$

where

$$t_x = \frac{(x\cos\theta_i\cos\phi_i + y\cos\theta_i\sin\phi_i + z\sin\theta_i)^2}{g^2\cos^2\theta_i} , \quad (17)$$

$$t_y = \frac{(-x\sin\phi_i + y\cos\phi_i)^2}{g^2} \quad , \tag{18}$$

$$\omega = \frac{1}{k^2}\left(\frac{2t_x - 1}{g^2\cos^2\theta_i} + \frac{2t_y - 1}{g^2}\right) . \tag{19}$$

Here, θ_i and ϕ_i are the elevation angle and azimuth angle of the incident wave, while g is the parameter to control the width of the tapered wave. All the computations are performed on a PC with Intel Dual-core 3.1 GHz CPU and 8 GB RAM in double precision. The terminating tolerances of the ACA is set as $\varepsilon_{ACA} = 0.001$.

The relative residual error at the kth iteration is used for monitoring the convergence of the proposed method, which is defined as,

$$\varepsilon(V,k) = \frac{\| V - ZI^{(k)} \|_2}{\| V \|_2} \tag{20}$$

where $\|\square\|_2$ denotes the 2-norm of the complex vector. The iteration stops when the $\varepsilon\,(V, k)$ is less than 0.001.

As the first example, the composite scattering model of a PEC sphere above a PM spectrum rough surface is considered to test the validity of the proposed method. The mesh sizes of the sphere and the rough surface are set as 0.1λ and 0.15λ, respectively. The radius of the sphere is 0.5λ and the rough surface size is 24λ × 24λ, whose corresponding numbers of the unknowns are 930 and 76480. The height of the sphere center from rough surface is 2.0λ. The width of the tapered wave is 6.0λ. The incident wave is from Theta = 30° and Phi = 0°. A total number of 77410 unknowns are involved in this example. Figure 3 shows the RCS results (VV-Polarization) at Phi=0° computed by the proposed method and the conventional MoM-PO. It can be seen that both results are in good agreement. The computational cost of the first example is shown in Table I. By applying the proposed hybrid method, the memory requirement and the total CPU time are dramatically reduced compared to the conventional MoM-PO without losing precision.

Table I: Computational cost of the first example.

	Memory Requirement (GB)	CPU Time (s)
MoM-PO	1.165	1091
MoM-PO-ACA	0.171	885

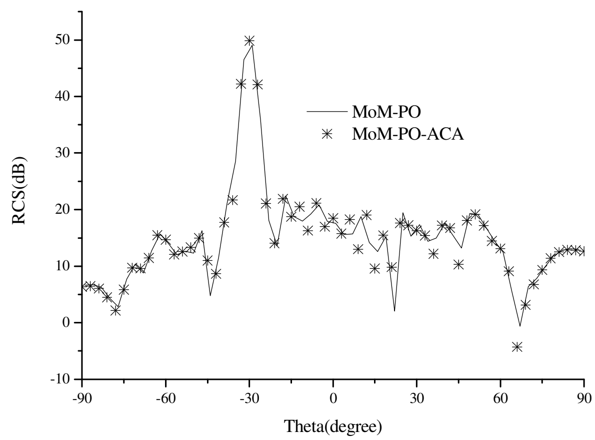

Fig. 3. Bistatic RCS of a PEC sphere above a PM spectrum rough surface.

The second example is a composite model of a missile above a Gaussian rough surface. This missile is a lying cylinder. The sizes of the missile and rough surface are 10.5λ×2.0λ×2.0λ and 20λ×20λ, respectively, and their distance is 10λ. The mesh sizes of the missile and the rough surface are set as 0.2λ and 0.125λ, and the corresponding numbers of the unknowns are 5760 and 76480. The width of the tapered wave is 5.0λ. The incident wave is from Theta=30° and Phi=0°. The root-mean-square height and correlation length of the rough surface are 0.1λ and $l_x = l_y =$ 1.0λ, respectively. Figure 4 shows the RCS results (VV-Polarization) at Phi=0° computed by the proposed method, which agree well with the results computed by the conventional MoM-PO. Table II shows that the computational cost can be reduced significantly compared to the conventional MoM-PO.

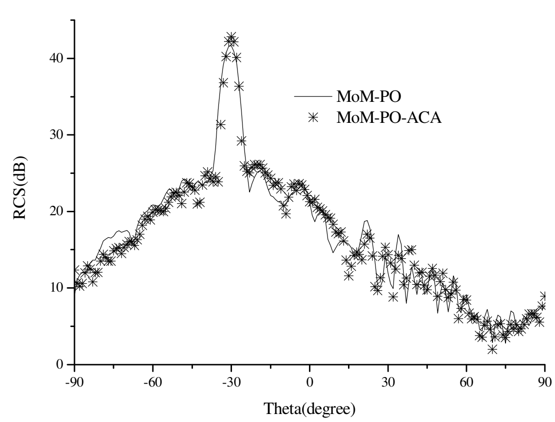

Fig. 4. Bistatic RCS of a missile above a Gaussian rough surface.

Table II: Computational cost of the second example.

	Memory Requirement(GB)	CPU Time
MoM-PO	6.817	>3 days
MoM-PO-ACA	0.74	1.5 hours

IV. NUMERICAL EXAMPLES

In this paper, a hybrid MoM-PO method combining ACA algorithm is applied to solving scattering from composite model of target and rough surface. Numerical examples have demonstrated that the memory requirement and CPU time can be significantly reduced without losing precision by applying the proposed method, and which can be used to analyze large scale target/rough surface scattering problems.

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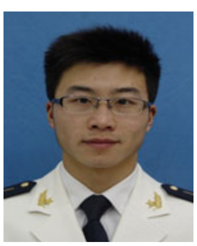

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