Radiation from Slotted Cylinder Embedded in Cylindrical Capped Corner Reflector

Hassan A. Ragheb and Essam E. Hassan

Department of Electrical Engineering
King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia
hragheb@kfupm.edu.sa, ehassan@kfupm.edu.sa

Abstract **─** An integral equation is formulated for the current distribution on slotted cylinder placed in a cylindrical capped corner reflector. The moment method is used to obtain the current distribution on the antenna conducting surfaces. The radiation pattern of the antenna is then calculated for different corner angles. Interesting results are obtained for different corner angle. The advantage of this geometry over the traditional corner reflector antenna is that it can be part of a ship or aircraft, in which the slotted cylinder is embedded in a conducting corner.

Index Terms - Corner reflector antenna, integral equation formulation and slotted cylinder.

I. INTRODUCTION

Radiation from axial slot on a circular conducing cylinder is the subject of considerable investigations, for its numerous applications in the communication and airspace industry. The residue series and the geometrical optics representation [1], the Green's function formulation [2] and the Fourier integral representation [3] have been employed for analytical treatment of different slots on a circular conducting cylinder. The dielectric coated cylindrical antennas were also investigated using similar methods [4-5]. The concentric case of a dielectric coated slotted conducting cylinder in a ground plane has been also tackled in [6]. Further, radiation from a dielectric coated slotted elliptic cylinder has been also the subject of many investigations [7]-[12]. In all of the previous work, the effect of mounting the antenna on any communication system has not been considered. The present work is generalizing the problem by considering the metallic slotted cylinder embedded in a caped conducting corner. This arrangement can be used to enhance the antenna characteristics and to optimize its radiation pattern. The ground plane can be used to support the slotted conducting circular cylindrical antenna. This plane could be the body of an air craft, a ship or any other mobile system. The integral equation formulation along with the moment method is employed here to obtain the radiation pattern of this antenna. It should be mentioned that an exact solution to this problem has been published by the first author [13]. The difference is in the exact solution the reflector is considered infinitely long, while in the present numerical solution a finite reflector is considered, which is more practical.

II. INTEGRAL EQUATION FORMULATION

A two dimensional cross section of the geometry of the problem is illustrated in Fig. 1. A slotted conducting cylinder of radius "*a*" is embedded in a cylindrical capped corner reflector of corner angle θ extending from $\phi = \theta/2$ to $\phi = -\theta/2$ and cap radius "*b*". The conducting corner planes have finite length "*R*". The axial slot is centered at $\phi = \phi_o$ and has an angular width equals to 2α.

To formulate an integral equation for electric and magnetic current distribution on the surface of the antenna, Green's second identity is employed, i.e.:

$$\oint\!\!\!\oint_s \left[G\frac{\partial\psi}{\partial n} - \psi\frac{\partial G}{\partial n}\right] ds = \iiint_{vol.} \left[\psi\nabla^2 G - G\nabla^2\psi\right] dv \quad . \quad (1)$$

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The wave equation can be written as:

$$\nabla^2 \psi(\bar{r}) + k^2 \psi(\bar{r}) = 0 \,. \qquad (2)$$

It is assumed that there is no variation of $\psi(\bar{r})$ along z-direction. Therefore, only one is concerned with a two dimensional Laplcian operator and a two dimensional space (x-y) plane.

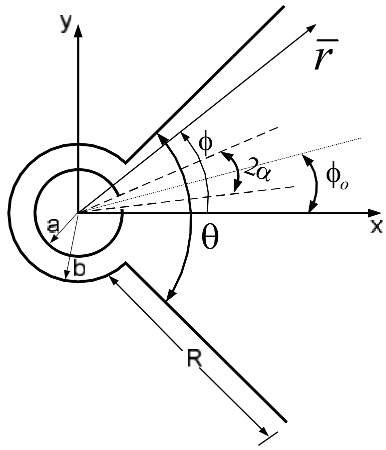

Fig. 1. Geometry of the problem.

The wave equation of an infinite line source of unit density in terms of the Green's function is:

$$\nabla^2 G(\bar{r},\bar{r}') + k^2 G(\bar{r},\bar{r}') = \delta(\bar{r}-\bar{r}') \,. \qquad (3)$$

Multiplying equation (2) by G and equation (3) by $\psi(\bar{r})$ and subtract then using it in (1), consider the two dimensional contour in Fig. 2, it results in:

$$\psi(\bar{r}) = \oint_C \psi(\bar{r}) \frac{\partial G(\bar{r},\bar{r}')}{\partial n} d\ell' - \oint_C G(\bar{r},\bar{r}') \frac{\partial \psi(\bar{r})}{\partial n} d\ell' \,. \qquad (4)$$

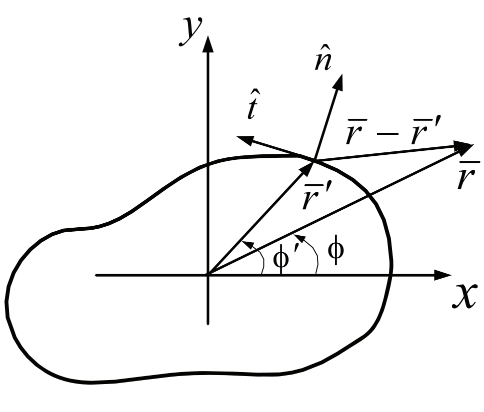

Fig. 2. Contour of a conducting surface.

The scalar function for TM case is associated with the E_z component of the field, i.e.:

$$\psi(\bar{r}) = E_z \,, \qquad (5)$$

$$\bar{H}_T = \frac{-j}{\omega\mu} \frac{\partial \psi}{\partial n} \hat{t} \qquad \text{tangential } \bar{H} \,. \qquad (6)$$

These field components may then be related to the equivalent surface current by:

$$\bar{M} = \bar{E} \times \hat{n} \,, \qquad (7)$$

$$\bar{J} = \hat{n} \times \bar{H} \,. \qquad (8)$$

Using (5) and (6) in (7) and (8), one obtains:

$$\bar{M} = \psi(\bar{r}) \, \hat{t} \,, \qquad (9)$$

$$\bar{J} = \frac{-j}{\omega\mu} \frac{\partial \psi(\bar{r})}{\partial n} \hat{z} \,. \qquad (10)$$

Upon substituting these definitions into the boundary integral equation (4), we obtain:

$$E_z(\bar{r}) = \oint_C M(\bar{r}') \frac{\partial G(\bar{r},\bar{r}')}{\partial n} d\ell' - j\omega\mu \oint_C \bar{J}(\bar{r}') G(\bar{r},\bar{r}') d\ell' \,. \qquad (11)$$

Equation (11) is the general integral equation for the field component E_z due to equivalent magnetic and electric current sources. On the perfectly conducting surface the total tangential field E_z must vanish. Upon applying this boundary condition, one may get:

$$\int_{\substack{slots\\only}} E_z(\bar{r}') \frac{\partial G(\bar{r},\bar{r}')}{\partial n} d\ell' - j\omega\mu \oint_{\substack{C\\except\\on\ slots}} \bar{J}(\bar{r}') G(\bar{r},\bar{r}') d\ell' = \begin{cases} E_z(\varphi) & on\ slots \\ 0 & otherwise \end{cases} \qquad (12)$$

where

$$G(\bar{r},\bar{r}') = \frac{1}{4j} H_0(k\,|\bar{r}-\bar{r}'|) \,. \qquad (13)$$

Equation (12) can be manipulated for the geometry illustrated in Fig. 1, to obtain:

$$-\frac{k}{4j} \int_{slot} E_z(\bar{r}') \frac{a - r\cos(\varphi-\varphi')}{\sqrt{r^2 + a^2 - 2ra\cos(\varphi-\varphi')}} H_1^{(2)}(k\,|\bar{r}-\bar{r}'|) d\ell' = j\omega\mu \oint_{\substack{C\\except\\on\ slot}} \bar{J}(\bar{r}') G(\bar{r},\bar{r}') d\ell' + \begin{cases} E_z(\varphi) & on\ slots \\ 0 & otherwise \end{cases} \,. \qquad (14)$$

To solve for the current density $J(\bar{r}')$, the conducting surface denoted by the arc length C is divided into N segments ΔC_n and the current density $J(\bar{r}')$ may then be represented by:

$$J = \sum_{n=1}^{N} \alpha_n f_n , \quad (15)$$

where f_n is the basis function defined by:

$$f_n(r) = \begin{cases} 1 & over \ \Delta C_n \\ 0 & otherwise \end{cases} .$$

In this case, equation (14) can be re-written as:

$$-\frac{k}{4j}\int_{slot} E_z(\bar{r}') \frac{a - r\cos(\phi-\phi')}{\sqrt{r^2 + a^2 - 2ra\cos(\phi-\phi')}} H_1^{(2)}(k|\bar{r}-\bar{r}'|)d\ell'_n = \frac{\omega\mu}{4} . \quad (16)$$

$$\sum_{n=1}^{N}\int_{\Delta C} \alpha_n H_0^{(2)}(k|\bar{r}-\bar{r}'_n|)d\ell'_n + \begin{cases} E_z(\phi) & on\ slots \\ 0 & otherwise \end{cases}$$

In order to calculate the unknown coefficients α_n in the above equation, one may discrete the above equation on the conducting surface, i.e. for the m[th] segment:

$$\frac{\omega\mu}{4}\sum_{n=1}^{N}\int_{\Delta C_n} \alpha_n H_0^{(2)}(k|\bar{r}_m-\bar{r}'_n|)d\ell'_n = \frac{-k}{4j}\int_{slot} E_z(\bar{r}') \frac{a - r_m\cos(\phi_m-\phi')}{\sqrt{r_m^{\ 2} + a^2 - 2r_m a\cos(\phi_m-\phi')}} H_1^{(2)}(k|\bar{r}_m-\bar{r}'|)d\ell'_n , \quad (17)$$

where m in equation (17) can take values from 1 to N (on the conducting surface only). Equation (17) can be written as:

$$[L_{mn}][\alpha_n] = [G_m], \quad (18)$$

where

$$L_{mn} = -j\eta_o \Delta C_n H_0^{(2)}\left(k\sqrt{(x_m-x_n)^2 + (y_m-y_n)^2}\right), \quad (19)$$

and for $n=m$

$$L_{mn} = -j\eta_o \Delta C_n \left[1 - j\frac{2}{\pi}\left(\gamma + \ln\left(\frac{\Delta C_n k}{4e}\right)\right)\right], \quad (20)$$

and M is the number of segments on the slot

$$G_m = \sum_{l=1}^{M} E_z(\phi_l) \frac{a - r_m\cos(\phi_m-\phi_l)}{\sqrt{r_m^{\ 2} + a^2 - 2r_m a\cos(\phi_m-\phi_l)}} H_1^{(2)}(k|\bar{r}_m-\bar{r}_l|)\Delta C_l . \quad (21)$$

Upon solving the matrix equation (18), one can obtain the current distribution on the conducting surface and radiation pattern can then be obtained from (11) as:

$$E_z(\bar{r}) = \frac{-k}{4j}\int_{slot} E_z(\bar{r}') \frac{a - r_m\cos(\phi_m-\phi')}{\sqrt{r_m^{\ 2} + a^2 - 2r_m a\cos(\phi_m-\phi')}} H_1^{(2)}(k|\bar{r}_m-\bar{r}'|)d\ell' - \frac{\omega\mu}{4}\oint_{\substack{except\\slot}} J(\bar{r}') H_0^{(2)}(k|\bar{r}_m-\bar{r}'_n|)d\ell'_n . \quad (22)$$

The far field Hankel function can be replaced by its asymptotic expression for large argument, i.e.:

$$H_n^{(2)}(x) = j^n\sqrt{\frac{2}{\pi x}}\, e^{-j(x-\pi/4)} .$$

Also, the approximation $|\bar{r}-\bar{r}'| = r - r'\cos(\phi-\phi')$ can be used. This gives:

$$E_z(\bar{r}) = \frac{-k}{4j}\int_{slot} E_z(\bar{r}') \frac{\frac{a}{r} - \cos(\phi-\phi')}{\sqrt{1 + \left(\frac{a}{r}\right)^2 - 2\frac{a}{r}\cos(\phi-\phi')}}\, j\sqrt{\frac{2}{\pi k|\bar{r}-\bar{r}'|}} e^{-jk|\bar{r}-\bar{r}'|} e^{j\pi/4} d\ell' - \frac{\omega\mu}{4}\oint_{\substack{except\\slot}} J(\bar{r}')\sqrt{\frac{2}{\pi k|\bar{r}-\bar{r}'|}}\, e^{-jk|\bar{r}-\bar{r}'|} e^{j\pi/4} d\ell' . \quad (23)$$

Since $r >> a$, the term a/r may be neglected. Employing this approximation one can get:

$$E_z(\bar{r}) = \frac{k}{4}\sqrt{\frac{2}{\pi kr}} e^{j\pi/4} e^{-jkr} \left[\int_{slot} E_z(\bar{r}')\cos(\phi-\phi')\, e^{jkr'\cos(\phi-\phi')} d\ell' - \eta_o \oint_{\substack{except\\slot}} J(\bar{r}')\ e^{jkr'\cos(\phi-\phi')|} d\ell'\right], \quad (24)$$

which can be written as:

$$E_z(\bar{r}) = \sqrt{\frac{k}{8\pi r}} e^{j\pi/4} e^{-jkr} f(\phi) , \quad (25)$$

where

$$f(\phi) = \sum_{l=1}^{M} E_z(\phi_l)\cos(\phi-\phi_l)\, e^{jkr_l\cos(\phi-\phi_l)} \Delta C_l - \eta_o \sum_{n=1}^{N} \alpha_n e^{jkr_n\cos(\phi-\phi_n)|}\Delta C_n . \quad (26)$$

The electric field on the slot may be assumed [6] as:

$$E_z(\phi_1) = E_o \cos\left(\frac{\pi(\phi_1 - \phi_o)}{2\theta}\right). \qquad (27)$$

The circular waveguide can be excited by a probe such that it will propagate the mode, which produces field distribution on the slot given in (27).

III. NUMERICAL RESULTS

To check the accuracy of our computation, comparison between radiation patterns using the numerical solution is presented here and the exact solution in [13] will be presented. Throughout all examples, the slot angle is taken as $2\alpha = 10^o$ centered at $\phi_o = 0$. The geometrical parameters for the following two examples are $a = 0.5\lambda$, $b = 0.6\lambda$ and $R = 3\lambda$.

In Fig. 3, the radiation patterns corresponding to the numerical and the exact solutions for corner angle $\theta = 180^o$ is presented. It is clear that they are in good agreement. The discrepancy after $|\phi| > 65^o$, is due to the fact that in numerical solution the reflector surface is considered finite while in the exact solution it is considered infinite.

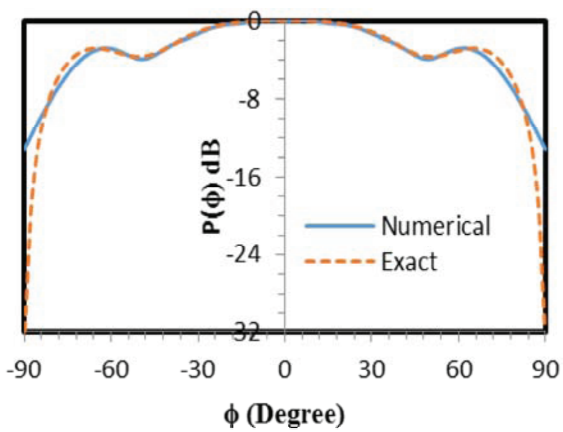

Fig. 3. Comparison between radiation pattern using numerical and exact solutions for corner angle $\theta = 180^o$.

Another example is illustrated in Fig. 4 for the same geometrical parameters of Fig. 3, except that the corner angle here is $\theta = 90^o$. Comparison between numerical and exact solutions is also excellent, except for $|\phi| > 40^o$, due to finite reflector length considered in the numerical solution.

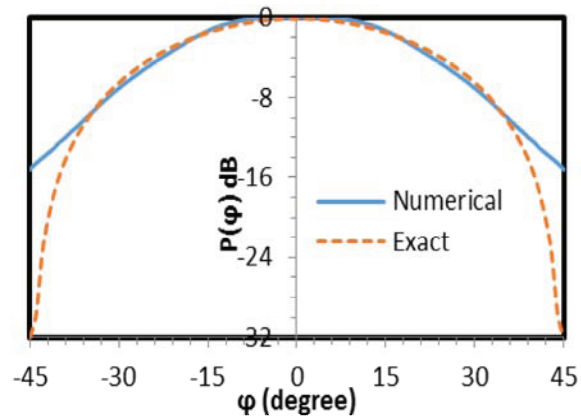

Fig. 4. Comparison between radiation pattern using numerical and exact solutions for corner angle $\theta = 90^o$.

Figure 5 shows the radiation patterns for corner angle $\theta = 180^o$ and geometrical parameters $b = 0.8\lambda$ and $R = 3\lambda$ at different values of slotted cylinder radius.

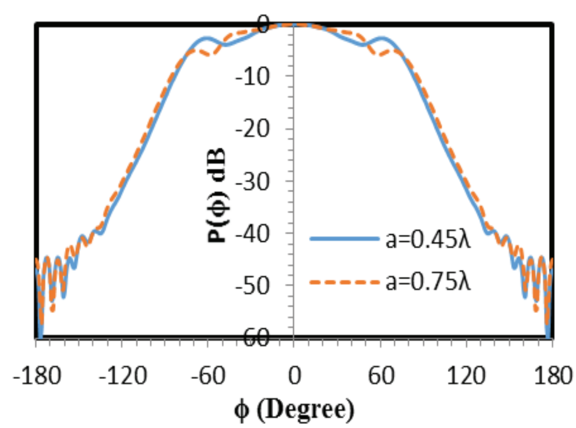

Fig. 5. Radiation patterns for corner angle $\theta = 180^o$ and different slotted cylinder radius.

As can be seen from Fig. 5, the change in radiation pattern is minimal, but as one decreases the corner angle to $\theta = 120^o$ for the same geometrical parameters, the radiation pattern gets narrower as the slotted cylinder radius gets smaller. This is shown in Fig. 6.

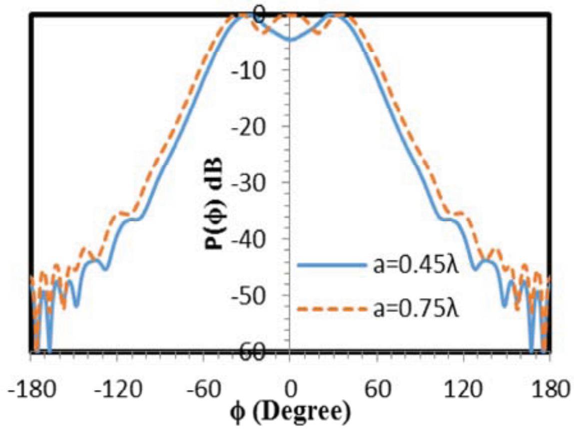

Fig. 6. Radiation patterns for corner angle $\theta = 120^o$ and different slotted cylinder radius.

The radiation patterns corresponding to different slotted cylinder radii, are illustrated in Fig. 7 with the same geometrical parameters as presented earlier, but the corner angle is reduced to $\theta = 90^o$. As one can see from Fig. 7, the smaller the radius of the slotted cylinder, the narrower the radiation pattern is. The deviation between radiation patterns in this case corresponding to lower and upper slotted cylinder radii is the largest in this case.

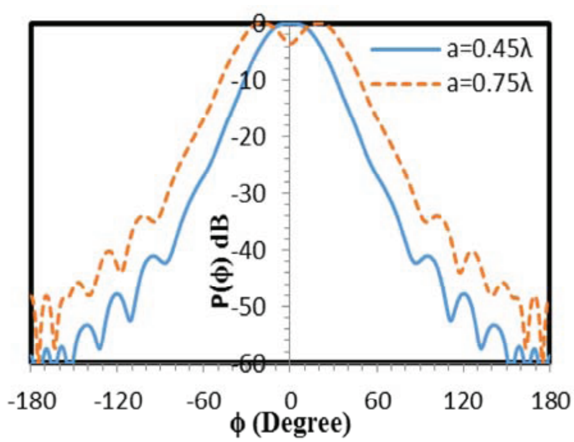

Fig. 7. Radiation patterns for corner angle $\theta = 90^o$ and different slotted cylinder radius.

In the next example shown in Fig. 8, the corner angle is considered as $\theta = 60^o$. Radiation patterns corresponding to different slotted cylinder radii are considered. As shown in this figure, the deviation in radiation patterns in this case is minimal. Accordingly, the above results show that for corner angles between 120^o and 90^o, the radiation pattern gets narrower as the slotted cylinder radius gets smaller.

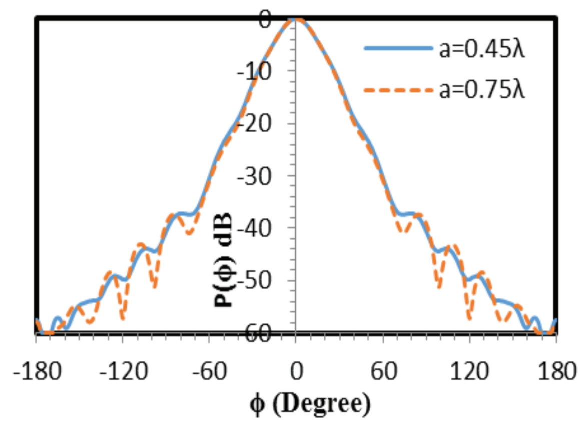

Fig. 8. Radiation patterns for corner angle $\theta = 60^o$ and different slotted cylinder radius.

The final example is for the geometrical parameters $a = 0.45\lambda$, $b = 0.8\lambda$ and $\theta = 90^o$. The radiation patterns corresponding to different reflector length are shown in Fig. 9. The radiation pattern in this case is the same for $-30^o < \phi < 30^o$, while for larger angles the level of the radiation gets higher as the reflector length gets lower.

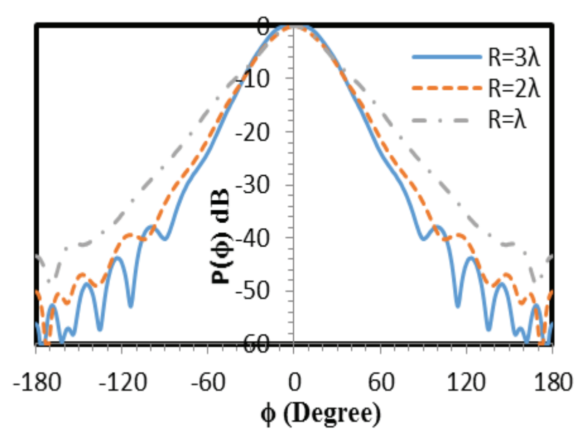

Fig. 9. Radiation patterns for corner angle $\theta = 90^o$ and different reflector length.

CONCLUSION

The axial slot on a conducting circular cylinder embedded in a capped corner reflector has been analyzed. Results corresponding to different geometrical parameters are presented. The geometry is supporting the antenna instead of using the mechanical mounting supporting system. The results show that one can shape the antenna pattern by changing the slotted cylinder radius or the reflector angle.

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Hassan Ragheb was born in Port-Said, Egypt in 1953. He received his B.Sc. degree in Electrical Engineering from Cairo University, Egypt in 1977. He received his M.Sc. and Ph.D. degrees in Electrical Engineering from the University of Manitoba, Winnipeg, Canada, in 1984 and 1987, respectively. From 1987 to 1989, he was a Research Assistant in the Department of Electrical Engineering, University of Manitoba. In 1989, he joined the Department of Electrical Engineering at the King Fahd University of Petroleum and Minerals, where he is now a Professor of Electrical Engineering. His research interests include electromagnetic scattering by multiple and coated objects, micro strip antennas, phased arrays and slot and open ended waveguide antennas.

Essam E. Hassan obtained his B.Sc. from Alexandria University, Egypt in 1970 and his M.Sc. and Ph.D. from the University of Manitoba in 1974 and 1978, respectively. Hassan worked with Northern Telecom Canada in the Digital Switching Division from

1977 to 1979. Since then, he joined the Electrical Engineering Department at King Fahd University of Petroleum and Minerals at Dhahran, Saudi Arabia, where he is now a Professor in the EE Department. He is an author or coauthor of over 50 papers in this field or closely related fields. Hassan's research interests is primarily in the microwave filters and devices and electromagnetic field and anomalous propagation.