Focusing of Electromagnetic Wave from Quartic Inhomogeneous Chiro-Slab

A. Ghaffar [1,4], M. Sharif [2], Q. A. Naqvi [3], M. A. S. Alkanhal [4], F. Khalid [1], and S. Shukurullah [1]

[1] Department of Physics
University of Agriculture, Faisalabad 3800, Pakistan
chabdulghaffar@yahoo.com, shizra_13@yahoo.com, zshukrullah@gmail.com

[2] Department of Physics
GC University, Faisalabad 3800, Pakistan
Msharif_gcuf@yahoo.com

[3] Department of Electronics
Quaid-i-Azam University, Islamabad, Pakistan
nqaisra@yahoo.com

[4] Department of Electrical Engineering
King Saud University, Riyadh, Saudi Arabia
majeed@ksu.edu.sa

Abstract **─** The focusing of electromagnetic waves from a quartic inhomogeneous chiro-slab is examined, using Maslov's method. Analytic field expressions are derived for the transmission coefficients at the interfaces and numerical computations are made to determine the field behavior around the caustic point of the chiro-slab. The effects of chirality parameter, permittivity and permeability on the intensity of the field around caustic point of the chiro-slab are also discussed. The results obtained through this method are shown to be in good agreement with those obtained from Huygens Kirchhoff's integral.

Index Terms **─** Caustic, chiral, Hamilton, intensity and Maslov's method.

I. INTRODUCTION

The discovery of chiral media and metamaterials during the last century has brought together the scientists and researchers from areas as diverse as physics, chemistry and biology [1]. This media offered a range of new millimeter, microwave and optical devices, in the area of electromagnetic and optics. For example, a chiral slab with negative refractive index has been shown to act as a perfect lens having sub-wavelength for circularly polarized waves [2-4]. Many other applications of the chiral metamaterials include but not limited to waveguides, antennas, polarization rotators and cloaking surfaces [5-14].

Two distinct properties of chiral media are the optical activity and circular dichroism. The irregular molecule arrangement enable the chiral material to rotate the plane of polarization of incident wave by an amount proportional to the width of the medium transverse relative to the wavelength of the wave. Moreover, the attenuation of the right-hand and left-hand circularly polarized wave is shown to be strongly affected by the chirality parameter of the medium [15,16]. Chiral scatterers are quite different from their dielectric or conducting counterparts, in that, the former produce both the co-polarized and cross-polarized scattered fields. The circular dichroism property of chiral media on the other hand, gives rise to different absorptivity of the right and left-circularly polarized waves inside a chiral medium.

Submitted On: January 26, 2013
Accepted On: March 8, 2014

In electromagnetics, to study the scattering and propagation, different numerical and analytical methods are available. Asymptotic Ray Theory (ART) is a simple and powerful tool for the evaluation of high frequency fields [17-19]. It is widely used to provide and explain the behavior of electromagnetic fields in both homogeneous and inhomogeneous media, but the field in caustics and shadow boundary has to be treated as a separate problem because ART shows singularities at these points. Kirchhoff integral is usually employed to calculate the fields at these points. There is a more convenient method based on Maslov's theory. Maslov's asymptotic theory is based on an idea that combines both the simplicity of ART and generality of Fourier transform. This method has been used by many authors [20-22] to find out the field behavior in the focal region of different geometries.

In this paper, we have treated a quartic inhomogeneous chiro slab to study the behavior of electromagnetic waves around caustic point using Maslov's method. We are extending our previous work from focusing of dielectric inhomogeneous slab and quadratic inhomogeneous slab composed of chiral medium to inhomogeneous chiral uniform thickness slab with quartic dependent permittivity on one transversal Cartesian coordinate using this technique [16,20]. We have determined the transmission coefficient of electromagnetic waves by an inhomogeneous quartic chiro slab for normal incident analytically. The numerical computations have been made to study the behavior of transmitted electromagnetic field from a quartic inhomogeneous slab composed of chiral medium.

II. FORMULATION

Consider an inhomogeneous chiral planar slab of uniform thickness d and placed in a dielectric medium as shown in Fig. 1. The dielectric medium is described by the constitutive parameters (μ_1,ε_1), whereas chiral medium filling the slab is described by the constitutive parameters $(\mu, \varepsilon, \gamma)$. It is assumed that inhomogeneity chiral medium slab is incorporated through permittivity parameter of the chiral medium. It is also assumed that distribution of permittivity is described by a fourth order polynomial equation (hereafter termed as quartic distribution) as given below[20]:

$$\varepsilon(x) = \varepsilon_c[1 - bx^2 - c(x^2)^2], \tag{1}$$

where ε_c is permittivity of corresponding homogeneous material, parameters b and c are associated with focal length of the chiro slab.

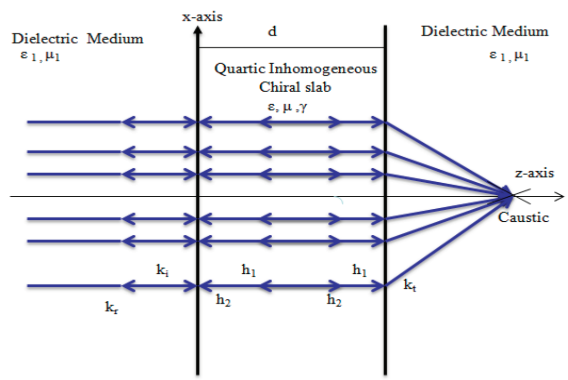

Fig. 1. Quartic inhomogeneous chiral slab.

The electromagnetic field in the chiral medium filling the slab region is described by [2-4]:

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} + j\gamma \boldsymbol{B}, \tag{2}$$

$$\boldsymbol{H} = j\gamma \boldsymbol{E} + \frac{1}{\mu}\boldsymbol{B}, \tag{3}$$

where ε and μ are the permittivity permeability of chiral medium and γ is the chirality parameter, which describes electromagnetic coupling. Two wave numbers h_1 and h_2 for eigenwaves propagating inside the chiral medium are:

$$h_1 = \frac{k}{1+k\gamma}, \tag{4}$$

$$h_2 = \frac{k}{1-k\gamma}, \tag{5}$$

with $k = \omega\sqrt{\mu_0 \epsilon_0}$. A set up of Cartesian coordinate system with unit vectors $\boldsymbol{a}_x$, $\boldsymbol{a}_y$ and $\boldsymbol{a}_z$ are introduced here to explain the problem of transmission through quartic inhomogeneous chiro slab. In the region $z \leq 0$, the incident plane electromagnetic wave is written as [16]:

$$\boldsymbol{E}_i = E_i \boldsymbol{a}_x\, e^{ik_i z}, \tag{6}$$

$$\boldsymbol{H}_r = \eta_1^{-1} E_r \boldsymbol{a}_y\, e^{ik_i z}. \tag{7}$$

The unknown reflected electromagnetic fields can be written as:

$$\boldsymbol{E}_r = E_r \boldsymbol{a}_x\, e^{-ik_i z}, \tag{8}$$

$$\boldsymbol{H}_r = -\eta_1^{-1} E_r \boldsymbol{a}_y\, e^{-ik_i z}. \tag{9}$$

The four electromagnetic waves are propagating inside the inhomogeneous chiro slab ($0 \leq z \leq d$), two propagating towards interface located at $z = d$ and the other two waves are propagating towards the interface $z = 0$, as shown in Fig. 1. The expressions for electromagnetic waves propagating towards the interface $z = d$ are represented as:

$$\boldsymbol{E}_{0c}^{+} = E_{01}^{+}(\boldsymbol{a}_x + i\boldsymbol{a}_y)e^{ih_1 z} + E_{02}^{+}(\boldsymbol{a}_x - i\boldsymbol{a}_y)\, e^{ih_2 z}, \quad (10)$$

$$\boldsymbol{H}_{0c}^{+} = -iZ^{-1}E_{01}^{+}(\boldsymbol{a}_x + i\boldsymbol{a}_y)e^{ih_1 z} + iZ^{-1}E_{02}^{+}(\boldsymbol{a}_x - i\boldsymbol{a}_y)\, e^{ih_2 z}. \quad (11)$$

The electromagnetic fields of the two waves propagating towards the interface $z = 0$ are represented as:

$$\boldsymbol{H}_{0c}^{-} = iZ^{-1}E_{01}^{+}(\boldsymbol{a}_x - i\boldsymbol{a}_y)e^{-ih_1 z} - iZ^{-1}E_{02}^{+}(\boldsymbol{a}_x + i\boldsymbol{a}_y)\, e^{-ih_2 z}, \quad (12)$$

$$\boldsymbol{E}_{0c}^{-} = -E_{01}^{-}(\boldsymbol{a}_x - i\boldsymbol{a}_y)e^{-ih_1 z} - E_{02}^{-}(\boldsymbol{a}_x + i\boldsymbol{a}_y)\, e^{-ih_2 z}, \quad (13)$$

where E_{01}^{+}, E_{02}^{+}, E_{01}^{-} and E_{02}^{-} are the amplitudes of the unknown coefficients. Z is wave impedance in the region $z \geq d$, the transmitted electromagnetic field in terms of unknown coefficient is written as:

$$\boldsymbol{E}_{0t} = E_t \boldsymbol{a}_x e^{ik_t z}, \quad (14)$$

$$\boldsymbol{H}_{0t} = \eta_1^{-1} E_t \boldsymbol{a}_y e^{ik_t z}, \quad (15)$$

where $k_t = k_i$. In above equations, unknown coefficients can be determined using the boundary conditions. The boundary conditions for the electromagnetic fields at interfaces $z = 0$ and $z = d$ are [4,15]:

$$(\boldsymbol{E_i} + \boldsymbol{E_r}) \times \boldsymbol{a}_z = (\boldsymbol{E}_c^{+} + \boldsymbol{E}_c^{-}) \times \boldsymbol{a}_z, \quad (16)$$

$$(\boldsymbol{H_i} + \boldsymbol{H_r}) \times \boldsymbol{a}_z = (\boldsymbol{H}_c^{+} + \boldsymbol{H}_c^{-}) \times \boldsymbol{a}_z, \quad (17)$$

$$(\boldsymbol{E}_c^{+} + \boldsymbol{E}_c^{-}) \times \boldsymbol{a}_z = \boldsymbol{E_t} \times \boldsymbol{a}_z, \quad (18)$$

$$(\boldsymbol{H}_c^{+} + \boldsymbol{H}_c^{-}) \times \boldsymbol{a}_z = \boldsymbol{H_t} \times \boldsymbol{a}_z, \quad (19)$$

application of above boundary conditions yield the following equations:

$$E_{01}^{+} + E_{02}^{+} - E_{01}^{-} - E_{02}^{-} - E_r = E_i, \quad (20)$$

$$E_{01}^{+} - E_{02}^{+} + E_{01}^{-} - E_{02}^{-} = 0, \quad (21)$$

$$E_{01}^{+} - E_{02}^{+} - E_{01}^{-} + E_{02}^{+} = 0, \quad (22)$$

$$gE_{01}^{+} + gE_{02}^{+} + gE_{01}^{-} + gE_{02}^{+} + E_r = E_i, \quad (23)$$

$$E_{01}^{+}e^{ih_1 d} + E_{02}^{+}e^{ih_2 d} - E_{01}^{-}e^{-ih_1 d} - E_{02}^{-}e^{-ih_2 d} - E_t e^{ik_i d} = 0, \quad (24)$$

$$E_{01}^{+}e^{ih_1 d} - E_{02}^{+}e^{ih_2 d} + E_{01}^{-}e^{-ih_1 d} - E_{02}^{-}e^{-ih_2 d} = 0, \quad (25)$$

$$E_{01}^{+}e^{ih_1 d} - E_{02}^{+}e^{ih_2 d} - E_{01}^{-}e^{-ih_1 d} + E_{02}^{+}e^{-ih_2 d} = 0, \quad (26)$$

$$gE_{01}^{+}e^{ih_1 d} + gE_{02}^{+}e^{ih_2 d} + gE_{01}^{-}e^{-ih_1 d} + gE_{02}^{+}e^{-ih_2 d} - E_t\, e^{ik_i d} = 0. \quad (27)$$

By solving set of equations (20-27), we obtain reflection and transmission coefficients. We consider transmission at the rear face of chiro slab, so transmission coefficient at this interface is given by [4]:

$$E_t = E_i \frac{2\mathrm{g}}{(1-\mathrm{g})^2} \frac{\mathrm{e}^{\mathrm{i}d(h_1 - k_i)} + \mathrm{e}^{\mathrm{i}d(h_2 - k_i)}}{[(1+\mathrm{g})/(1-\mathrm{g})]^2 - \mathrm{e}^{\mathrm{i}d(h_1 + h_2)}}, \quad (28)$$

where $\mathrm{g} = \sqrt{(\mu_1/\epsilon_1)\gamma^2 + (\epsilon\mu_1/\mu\epsilon_1)}$.

A. Solution of Hamilton's equations

The solution of Hamilton's equations for the inhomogeneous medium described by equation (1) are given as [20]:

$$\left.\begin{aligned} x &= \xi[(1+v)cos3\psi] \\ z &= \mathrm{p_z}\tau \\ p_x &= -\beta\xi[(1+v)sin\psi - 3vsin3\psi] \\ p_z &= \sqrt{\varepsilon - {p_x}^2} \end{aligned}\right\}, \quad (29)$$

where τ is the parameter along the ray length and $\mathrm{v} = \frac{\mathrm{c'b'^2\xi^2}}{4}$, $\mathrm{b'} = \lambda \mathrm{b}$ and $\mathrm{c'} = \mathrm{kc}$.

The Cartesian coordinates of refraction point at the rear face (ξ_1, η_1) and the components of associated wave vector are given by:

$$\left.\begin{aligned} \xi_1 &= \xi[(1+v)cos\psi_1 - vcos3\psi_1] \\ \zeta_1 &= d \\ p_{x0} &= -\beta\xi[(1+v)sin\psi_1 - 3vsin3\psi_1] \\ p_{z0} &= \sqrt{\varepsilon_c(1 - b^2\xi^2 + cb^4\xi^4)} \\ p_{z0}\tau_1 &= d, \qquad \psi_1 = \beta\tau_1 \end{aligned}\right\}, \quad (30)$$

where τ_1 is the arc length of the ray for $0 < z < d$. In above equations (ξ, η) are the Cartesian coordinates of refraction point of front face of the chiro slab. The coordinate of the ray after passing through the slab, there is $z > d$, are given by[18]:

$$x = \xi_1 + p_{x1}\,\tau,$$

$$z = \zeta_1 + p_{z1}\,\tau,$$

$$p_{x1} = -\beta\xi[(1+v)sin\psi_1 - 3vsin3\psi_1],$$

$$p_{z1} = \sqrt{1 - \beta^2\,\xi^2\,[(1+v)sin\psi_1 - 3vsin3\psi_1]^2}.$$

B. Focused field at caustic point

The geometrical optics solution is given by [20]:

$$\mathbf{E_t}(\mathrm{x,z}) = \mathbf{E}_{0\mathrm{t}}[\mathrm{J}(\tau)]^{-\frac{1}{2}}\exp[-\mathrm{jk}(\psi_0 + \tau)], \quad (31)$$

where $\mathrm{J}(\tau)$ is Jacobean and ψ_0 is the value of initial phase difference between front and rear faces of the chiro slab, which is given by [20]:

$$\psi_0 = \epsilon_c\left(1 - \frac{\beta^2\xi^2}{2}\right)\tau_1 - \frac{\beta\xi^2}{4}\sin 2)\,\psi_1.$$

Geometrical optics field contains singularity at the focal point. Our interest is to find the uniform field expression valid in focal region using Maslov's method. The uniform expression which is valid in the focal region is given by [20,21]:

$$\mathbf{E}(\mathrm{r}) = \sqrt{\frac{\mathrm{k}}{i2\pi}} \int_{-\infty}^{\infty} \boldsymbol{E}_{0t} \left[\frac{1}{\mathrm{D}(0)} \frac{\partial(\mathrm{p_{x1}},\mathrm{z})}{\partial(\xi,\mathrm{t})}\right]^{-\frac{1}{2}} \times \exp[-ik\,\psi_1(\mathrm{p_{x1}},\mathrm{z})]\mathrm{dp_{x1}} \quad (32)$$

Quantities in the square brackets and phase function $\psi_1(\mathrm{p_{x1}},\mathrm{z})$ are obtained by the procedure given in [20] and variable of integration are changed from wave vector domain to the ray domain. The field intensity expression in component form may be expressed as:

$$\mathrm{E_x} = \sqrt{\frac{\mathrm{k}}{2\pi i}} \int_0^{\mathrm{d}} \sqrt{\mathrm{S}}\, \mathrm{E_t}\, e^{-ik\psi_1(\mathrm{p_{x1}},\mathrm{z})} \mathrm{d}\xi, \quad (33)$$

$$\mathrm{E_z} = \sqrt{\frac{\mathrm{k}}{2\pi i}} \int_0^{\mathrm{d}} \Gamma\sqrt{\mathrm{S}}\, \mathrm{E_t}\, e^{-ik\psi_1(\mathrm{p_{x1}},\mathrm{z})} \mathrm{d}\xi. \quad (34)$$

The amplitude and phase function in simplified form can be written as:

$$\mathrm{S} = S_1 S_2$$

$$S_1 = \frac{\beta^4 \mathrm{d}\xi^2}{\mathrm{P}_{\mathrm{z}_0}^3} S_3 + \beta\left(9\mathrm{v}\sin 3\psi_1 - (1+3\mathrm{v})\sin\psi_1\right),$$

$$S_2 = \frac{\beta^3 \mathrm{d}\xi^2}{\mathrm{P}_{\mathrm{z}_0}^3} S_4 + (1+3\mathrm{v})\cos\psi_1 - 3\mathrm{v}\cos 3\psi_1,$$

$$S_3 = \left(2\mathrm{c}\xi^2 - 1 - \mathrm{v}\right)\cos\psi_1 - 9\mathrm{v}\cos 3\psi_1,$$

$$S_4 = \left(\left(2\mathrm{cb}^2\xi^2 - 1 - \mathrm{v}\right)\sin\psi_1 - 3\mathrm{v}\sin 3\psi_1\right),$$

$$\Gamma = \tan\left(\frac{\mathrm{d}(\mathrm{h}_2 - \mathrm{h}_1)}{2}\right).$$

The phase function is given as:

$$\psi_1(\mathrm{p_{x1}},\mathrm{z}) = \psi_1' + \mathrm{Y}(\mathrm{z}-\mathrm{d}) - \epsilon_c \mathrm{vb}^2\xi^2\ \psi_0',$$

$$\psi_0' = \tau_1 - \frac{\sin 4\psi_1}{4\beta},$$

$$\psi_1' = \psi_0 + \epsilon_c \mathrm{cb}^4\xi^4\psi_2' + \beta\xi^2\left(\vartheta\ \cos\psi_1 + \psi_3'\right),$$

$$\psi_2' = \frac{3}{8}\tau_1 + \frac{2}{16\beta}\sin 2\psi_1 + \frac{1}{32\beta}\sin 4\psi_1,$$

$$\psi_3' = \mathrm{v}\cos 3\psi_1\ \sin\psi_1 - 3\mathrm{v}\sin 3\psi_1\ \cos\psi_1,$$

$$\mathrm{Y} = \sqrt{1 - \beta^2\xi^2\left(\vartheta\sin\psi_1 - 6\mathrm{v}\ \sin\psi_1\sin 3\psi_1\right)^2},$$

$$\vartheta = (1+2\mathrm{v})\ \sin\psi_1.$$

III. HUYGENS KIRCHHOFF'S INTEGRAL

To check the accuracy of the results obtained in the Caustic region of quartic inhomogeneous chiro slab, we compare the computational results obtained by Kirchhoff's approximation. Using Green's theorem, we may show that transmitted field from quartic inhomogeneous chiro-slab is obtained by [16]:

$$\boldsymbol{E_t}(x,z) = \int_0^d \boldsymbol{E_{0t}} \sqrt{\frac{2}{\pi k \tau}}\, e^{-ik\left(\Psi_0 + \tau - \frac{\pi}{4}\right)} d\xi, \quad (35)$$

where $\tau = p_x(x-\xi) + p_z(z-d)$ and $\boldsymbol{E}_{0t}$ is defined in above section 4.

The field intensity expression in component form may be expressed as:

$$\mathrm{E_x} = \sqrt{\frac{\mathrm{k}}{2\pi i}} \int_0^{\mathrm{d}} \frac{\mathrm{E_t}}{\sqrt{\tau}}\, e^{-ik\Phi} d\xi, \quad (36)$$

$$\mathrm{E_z} = \sqrt{\frac{\mathrm{k}}{2\pi i}} \int_0^{\mathrm{d}} \frac{\mathrm{E_t}}{\sqrt{\tau}} \Gamma e^{-ik\Phi} d\xi, \quad (37)$$

$$\Phi = \psi_0 + \beta\,\xi^2 \sin\psi_1 \cos\psi_1 + \beta r\xi \sin\psi_1 + (z-d)\sqrt{1 - \beta^2\,\xi^2 \sin^2\psi_1} - \frac{\pi}{4}. \quad (38)$$

IV. RESULTS AND DISCUSSIONS

In this paper, we have solved equations (34), (35), (37) and (38) numerically to study the focusing behavior of quartic inhomogeneous chiro slab. The effect of chirality γ, permittivity ε and permeability μ on transmitted fields are observed by varying these parameters. The thickness of the slab is taken as $d = 0.7$, wave number is assumed as $k = 1000$, $b = 0.3$, $c = 0.1$ and $\beta = 0.3$ in this work. To check accuracy of our high frequency field expressions, we compare the results obtained using Maslov's method (solid line) and Huygens-Kirchhoff's integral (dashed line), which are in good agreement. These results are shown in Fig. 2.

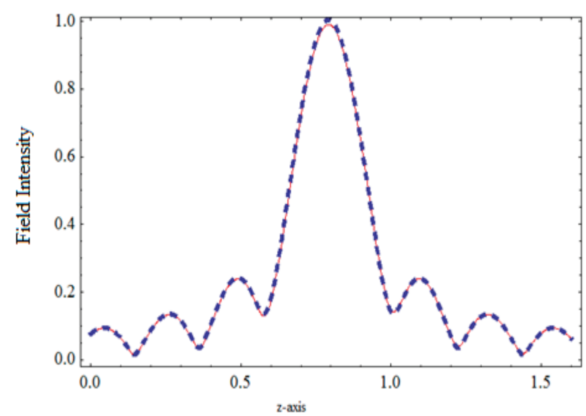

Fig. 2. Comparison of normalized field intensity of chiro slab around focal point along the z-axis using Maslov's method (solid line) and Huygens-Kirchhoff's integral (dashed line).

Figures 3 and 4 represent the comparison of responses for the case of normalized field intensity distribution around the focal region along the z-axis and x-axis, respectively, with the variation of chirality parameters. These comparisons show that the field intensity shifts to a smaller value as we decrease the chirality and vice versa. Figures 5 and

6 represent the comparison of normalized field intensity distribution around the focal region along the z-axis and x-axis, respectively, with the variation of permittivity. These comparisons show that the field intensity shifts to a smaller value as we increase the permittivity and vice versa. Figures 7 and 8 represent the comparison of normalized field intensity distribution around the focal region along the z-axis and x-axis, respectively, with the variation of permeability of chiral medium. These comparisons show that the field intensity shifts to a smaller value as we decrease the permeability and vice versa. A slab of such chiral medium may be used as a perfect lens, which provide sub wavelength resolution for circularly polarized waves.

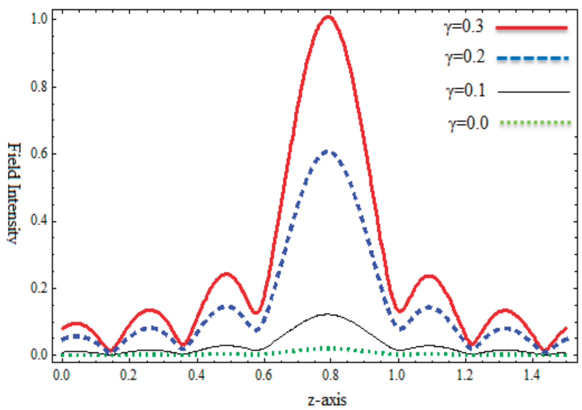

Fig. 3. Comparison of normalized intensity of slab around focal point along z-axis with w.r.t. values of chirality parameter.

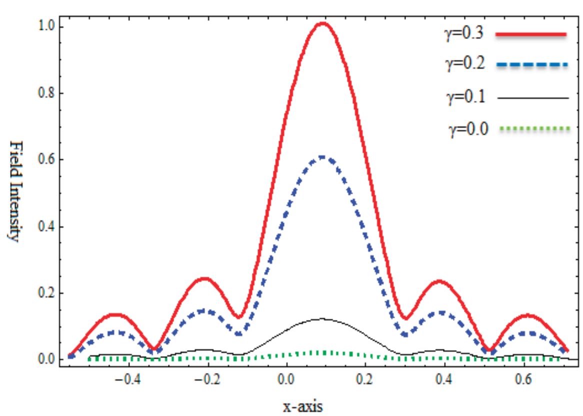

Fig. 4. Comparison of normalized intensity of slab around focal point along x-axis with w.r.t. values of chirality parameter.

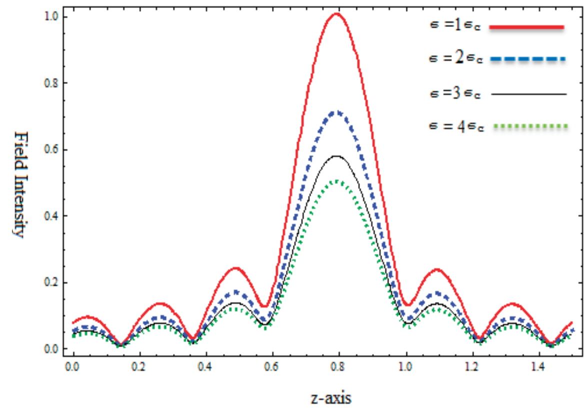

Fig. 5. Comparison of normalized intensity of slab around focal point along z-axis with w.r.t. values of permittivity.

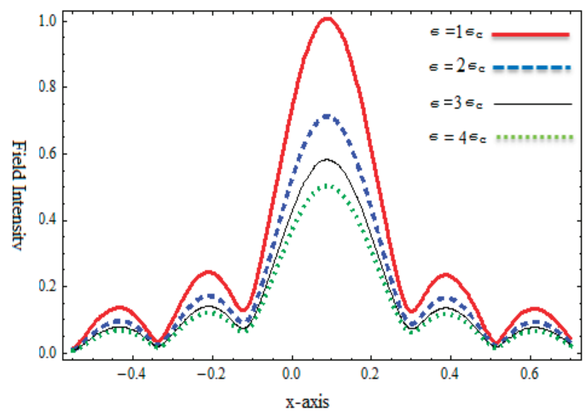

Fig. 6. Comparison of normalized intensity of slab around focal point along x-axis with w.r.t. values of permittivity.

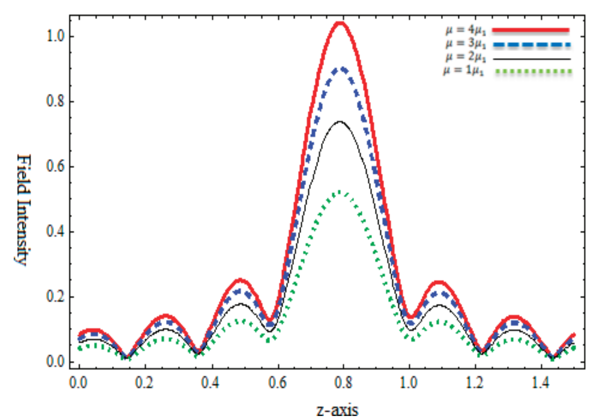

Fig. 7. Comparison of normalized intensity of slab around focal point along z-axis with w.r.t. values of permeability.

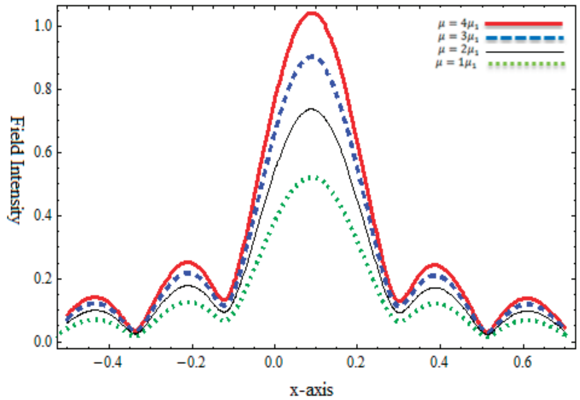

Fig. 8. Comparison of normalized intensity of slab around focal point along x-axis with w.r.t. values of permeability.

V. CONCLUSION

In this study, transmission of electromagnetic waves from a quartic inhomogeneous chiro slab is analyzed. We have used Maslov's method to calculate the field expressions for refracted field from the quartic inhomogeneous chiro slab. The effects of slab's parameters, i.e., chirality, permittivity and permeability on the amplitude of transmitted field in caustic region are shown and discussed. From the plots, we have observed that the field intensity in the caustic region of a quartic inhomogeneous chiral slab decreases by decreasing the chirality parameter. The results presented here will be helpful for potential applications in novel waveguide devices such as directional couplers, cloaking and polarization transformer.

ACKNOWLEDGMENT

Authors would like to thank Research Center, College of Engineering, Deanship of Scientific Research and King Saud University.

REFERENCES

[1] N. Engheta and D. L. Jaggard, "Electromagnetic chirality and its applications," *IEEE Antennas and Propag. Society Newsletter*, vol. 30, pp. 6-12, 1988.

[2] A. Lakhtakia, "Beltrami fields in chiral media," *World Scientific Series in Contemporary Chemical Physics*, 1994.

[3] J. Lekner, "Properties of a chiral slab waveguide," *Pure Applied Optics*, vol. 6, pp. 373-384, 1997.

[4] S. Bassiri, C. H. Papas, and N. Engheta, "Electromagnetic wave propagation through a dielectric-chiral interface and through a chiral slab," *J. Opt. Soc. Am. A.*, vol. 5, pp. 1450-1459, 1988.

[5] J. B. Pendry, "Negative refraction makes a perfect lens," *Phys. Rev. Lett.*, vol. 85, pp. 39-66, 2000.

[6] Y. Jin and S. He, "Focusing by a slab of chiral medium," *Optics Express*, vol. 30, pp. 49-74, 2005.

[7] A. Ghaffar, S. I. Ahmad, R. Fazal, S. Shukrullah, and Q. A. Naqvi, "Scattering of electromagnetic wave by perfect electromagnetic conductor (PEMC) sphere placed in chiral media," *Optik-Int. J. of Light and Electron Optics*, vol. 125, pp. 1589-1597, 2013.

[8] J. F. Dong and J. Li, "The reflection and transmission of electromagnetic waves by a uniaxial chiral slab," *Progress In Electromagnetic Research*, vol. 127, pp. 389-404, 2012.

[9] A. Ghaffar, M. Arif, Q. A. Naqvi, and M. Alkanhal, "Radiation properties of a uniaxial chiral quadratic inhomogeneous slab under oblique incidence," *Optik-Int. J. of Light and Electron Optics*, vol. 124, pp. 4947-4951, 2013.

[10] A. Ghaffar, "Focusing by a cylindrical plano convex lens of chiral medium," *J. Electromag. Wave Appli.*, vol. 25, pp. 833-844, 2011.

[11] A. K. Hamid, "Scattering by chiral lossy metamaterial elliptic cylinders," *Applied Computational Electromagnetics Society (ACES) Journal*, vol. 27, pp. 603-609, 2012.

[12] N. L. Tsitsas, "Analysis of electromagnetic wave propagation in frequency bands of nonlinear metamaterials," *Applied Computational Electromagnetics Society (ACES) Journal*, vol. 27, pp. 169-180, 2012.

[13] A. M. Attiya, "Coupled mode analysis of two-dimensional chiral grating," *Applied Computational Electromagnetics Society (ACES) Journal*, vol. 26, pp. 303-311, 2011.

[14] S. T. Imeci, F. Altunkilic, J. R. Mautz, and E. Arvas, "Transmission through an arbitrarily shaped aperture in a conducting plane separating air and a chiral medium," *Applied Computational Electromagnetics Society (ACES) Journal*, vol. 25, pp. 587-599, 2010.

[15] B. N. Khatir, M. Al-Kanhal, and A. Sebak, "Electromagnetic wave scattering by elliptic chiral cylinder," *J. Electromag. Wave Applic.*, vol. 20, pp. 1377-1390, 2006.

[16] A. Ghaffar, H. M. Shahbaz, and Q. A. Naqvi, "Analysis of electromagnetic field transmitted by a quadratic inhomogeneous slab composed of chiral medium," *Int. J. of Applied Electromagnetics and Mechanics*, vol. 42, pp. 613-622, 2013.

[17] V. P. Maslov, "Thtorie des perturbations et mtthodes asymptotiques," *Dunod*, Paris, France, 1972.

[18] V. P. Maslov, “Perturbations theory and asymptotiques method, *Moskov. Gos. Univ.*, Moscow, 1965.
[19] R. W. Ziolkowski and G. A. Deshamps, “Asymptotic evaluation of high frequency field near a caustic: an introduction to maslov’s method,” *Radio Sci.*, vol. 19, pp. 1001-1025, 1984.
[20] A. Ghaffar Hussain, Q. A. Naqvi, and K. Hongo, “Radiation characteristics of an inhomogeneous slab using maslov’s method,” *J. Electromag. Wave Applic.*, vol. 22, pp. 301-312, 2008.
[21] A. Ghaffar, S. Ahmed, and Q. A. Naqvi, “Electromagnetic field in the caustic region of a spherical reflector under oblique incidence using maslov’s method,” *Int. J. of Applied Electromagnetics and Mechanics*, vol. 38, pp. 1-17, 2012.
[22] A. Ghaffar, M. Shoaib, N. Mehmood, M. Y. Naz, M. Azam, and Q. A. Naqvi, “Electromagnetic field diffraction of a paraboloidal mirror under oblique incidence,” *Int. J. of Applied Electromagnetics and Mechanics*, vol. 41, pp. 579-494, 2013.

Abdul Ghaffar was born in Mitha Tiwana district Khushab Pakistan, Punjab (Pakistan) on January 1, 1976. He received his M.Sc. (Physics) degree from Uninversity of Punjab in 2001. He received his Ph.D. degree from Quaid-i-Azam University, Islamabad, Pakistan in 2009. He is Assistant Professor at Agriculture University, Faisalabad (Pakistan). Currently he is working towards his research at King Saud University, Saudi Arabia. His research interests are chiral media, high frequency electromagnetic scattering from slabs and antennas. He has published over 35 papers in international journals. He has successfully supervised more than 10 M.Sc. and 15 M. Phil students after his Ph.D.

M. Sharif was born in Vehari Pkistan. He received his Ph.D. degree from Quaid-i-Azam in 2007. He is an Assistant Professor at GCUF. His research intrest is scattering Plasm structure**.**

Qaisar Abbas Naqvi was born in the village of District Narowal (Pakistan). He received his M.Sc., M.Phil and Ph.D. degrees from Quaid-i-Azam University, Islamabad (Pakistan). He has published over one hundred and fifty (150) papers in international journals. He’s also author of a book and chapters in different books. He has successfully supervised eight Ph.D.’s and more than 80 M.Phil students after his Ph.D. in 1998. His research ineterst are Kobyashi potential, Maslov’s method, GTD, fractional electromagnetics, buried cylinders, waveguides, slits, strips, antennas and numerical electromagnetics.

Majeed A. S. Alkanhal obtained his B.Sc. and M.Sc. degrees in Electrical Engineering from King Saud University (Saudi Arabia) in 1984 and 1986, respectively. He received his Ph.D. degree in Electrical Engineering from Syracuse University, Syracuse, New York in 1994. Alkanhal is currently a Full Professor in the Electrical Engineering Department at King Saud University. His research interests include RF and microwave/millimeter-wave communications and radar systems, electromagnetic scattering from complex bodies and material objects**,** antenna design and optimization for communication and radar systems, application of modern estimation and optimization techniques to signal processing and digital communications problems, numerical solutions to electromagnetics and electronic/optical device problems.

Farayal Khalid was born in Faisalabad (Pakistan). She received her B.S. (honour) from GC Faisalabad and M.Phil degree in Physics from University of Agriculture Faisalabad in 2012. Her current research interest include high frequency techniques used electromagnetic.

Shazia Shukurullah was born in Faisalabad. She received her M.Sc. degree from the University of Punjab. She is currently a Ph.D student. Her research intrests are numerical techniques in electromagnetics and Nano technology**.**