# Uniaxial Dielectric Waveguide Filter Design Accounting for Losses Using Mode Matching Technique

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Abstract – Dielectric filters can provide compact solutions for filter design problems. However, most dielectrics exhibit uniaxial properties, as well as, losses that will undoubtedly affect performance if not accounted for. This paper derives dispersion relations for lossy uniaxial media in dielectric waveguides and also accounts for lossy conducting walls. The waveguide discontinuity problem in the presence of lossy uniaxial media and finite conductivity waveguide walls, is calculated by mode matching technique and the results are applied to a Ka band filter. The design specifications for the proposed filter are a 32.5 GHz center frequency with 6%. Good agreement between simulated and measured results are shown.

*Index Terms* — Band-Pass Filter (BPF), Computer-Aided Design (CAD), dielectric waveguide filters, microwave filters, Mode-Matching Technique (MMT) and uniaxial media.

### **I. INTRODUCTION**

Waveguide filters offer far superior performance to microstrip filters and dielectric filled waveguide filters significantly reduce waveguide filter size without sacrificing filter performance [1]. In addition to high performance, low manufacturing cost and small size, dielectric waveguide filters allow for flip-chip bonding; which makes for easy integration in millimeterwave systems [2-3]. At millimeter-wave frequencies, dielectric waveguide filters also

circumvent radiation loss; which is common in planar filters [4]. Recently, [5-6] use mode matching/hybrid method in the analysis of waveguide class filter problems. Wexler [7] first mentioned the advantages of choosing the Mode Matching Technique (MMT) versus all other methods when solving a class of waveguide problems. In [8-10], the MMT is applied to analyze dielectric waveguide and SIW filters as well as couplers. However, the MMT has never been applied to a lossy uniaxial dielectric waveguide filter with non-PEC walls. This paper will use the MMT to design and simulate a dielectric waveguide filter that is manufactured on a lossy uniaxial media with finite conductivity in the waveguide walls. The filter is designed in the Ka band, due to interest expressed by the manufacturing company that graciously manufactured and measured the filter free of charge.

## II. DISPERSION RELATION IN LOSSY UNIAXIAL MEDIA

In order to account for a lossy dielectric in the MMT dispersion relations, Maxwell's equations must be modified. From [11], one finds Maxwell's equations for a conducting media. In this paper, it is assumed the relative permittivity tensor  $\overline{\overline{e_r}}$  takes the form of:

$$\overline{\overline{\varepsilon}_r} = \begin{bmatrix} \varepsilon & 0 & 0\\ 0 & \varepsilon_y & 0\\ 0 & 0 & \varepsilon \end{bmatrix}.$$
 (1)

Submitted On: July 3, 2013 Accepted On: July 25, 2014 Replacing the relative permittivity tensor in (1) with that of the relative permittivity tensor for a lossy uniaxial media:

$$\overline{\overline{\varepsilon_{cd}}} = \begin{bmatrix} \varepsilon - j\tau\varepsilon & 0 & 0\\ 0 & \varepsilon_y - j\tau\varepsilon_y & 0\\ 0 & 0 & \varepsilon - j\tau\varepsilon \end{bmatrix}.$$
 (2)

Putting (2) into Maxwell's equation, multiplying by the dot product of  $(\bar{\bar{\varepsilon}}_{cd}^{-1})^{-1}$  and then taking the curl, we obtain:

$$\nabla \times \left( \bar{\bar{\varepsilon}}_{cd}^{-1} \cdot (\nabla \times H) \right) = j\omega\varepsilon_0 (\nabla \times E). \quad (3)$$

Substituting the  $\nabla \times E$  term of Maxwell's equation into (3) gives:

$$\nabla \times (\bar{\alpha}_{cd} \cdot (\nabla \times H)) = j\omega\varepsilon_0(-j\omega\mu_0 H),$$
 (4)  
where  $\bar{\alpha}_{cd} = \bar{\varepsilon}_{cd}^{-1}$ . Assuming propagation is in  
the  $\hat{z}$  direction and solving for the  $k_z$  of the  $H_z$   
components provide the dispersion relation. After  
some tedious but straight forward calculations,  
the  $\hat{z}$ -components of (4) are given by:

$$\begin{pmatrix} \alpha_{22} \frac{\partial^2}{\partial x \partial z} - \alpha_{23} \frac{\partial^2}{\partial x \partial y} - \alpha_{12} \frac{\partial^2}{\partial y \partial z} + \alpha_{13} \frac{\partial^2}{\partial y^2} \end{pmatrix} H_x + \\ \begin{pmatrix} -\alpha_{12} \frac{\partial^2}{\partial x \partial z} + \alpha_{23} \frac{\partial^2}{\partial x^2} + \alpha_{11} \frac{\partial^2}{\partial y \partial z} - \alpha_{13} \frac{\partial^2}{\partial x \partial y} \end{pmatrix} H_y \\ + \begin{pmatrix} \alpha_{12} \frac{\partial^2}{\partial x \partial y} - \alpha_{22} \frac{\partial^2}{\partial x^2} - \alpha_{11} \frac{\partial^2}{\partial y^2} + \alpha_{12} \frac{\partial^2}{\partial x \partial y} \end{pmatrix} H_z \\ = k_0^2 H_z.$$
(5)

It can be seen that the coupling effect of  $H_x$  and  $H_y$  makes the calculation of  $k_z$  difficult for uniaxial media. Later in this paper, MMT is carried out to analyze the TE<sub>10</sub> mode. This assumption enforces the conditions on (5) such that  $H_y = E_x = k_y = 0$ . Under these conditions, using Gauss's law and knowing from calculations that  $\alpha_{23} = \alpha_{12} = \alpha_{12} = 0$ , simplifies (5) to:

$$-\alpha_{22}\frac{\partial^2}{\partial z^2}H_z - \alpha_{22}\frac{\partial^2}{\partial x^2}H_z - \alpha_{11}\frac{\partial^2}{\partial y^2}H_z = k_o^2H_z.$$
 (6)

Under the assumption in a waveguide,  $H_z$  is of the form  $\cos(k_x x) \cos(k_y y) e^{-jk_z z}$ , solving for  $k_z$  in (6) provides the dispersion relation for lossy media:

$$k_{z} = j \sqrt{k_{x}^{2} - \frac{k_{0}^{2}}{\alpha_{22}}} = j \sqrt{k_{x}^{2} - (\varepsilon_{y} - j\tau\varepsilon_{y})k_{0}^{2}},$$
(7)

where  $k_x = m\pi/a$  and *a* is the *x* dimension of the dielectric waveguide in Fig. 1.



Fig. 1. Rectangular waveguide dimension reference.

#### **III. LOSS IN CONDUCTOR WALLS**

Most MMT calculations of the dielectric waveguides assume perfect conducting walls. In practice however, metallic walls exhibit a finite conductivity,  $\sigma_c$  and therefore cause attenuation of the signal. Under the assumption of TE<sub>10</sub> mode of propagation, Kong's [12] perturbation method is used to calculate the attenuation for the waveguide wall dimensions that are defined in Fig. 1:

$$\alpha_c = \frac{P_w}{2P_f},\tag{8}$$

where  $P_w$  represents the time-average power loss in the walls and  $P_f$  is the time-average power flowing through a cross section of the waveguide.  $P_f$  and  $P_w$  are defined as:

$$P_f = \frac{1}{2} Re\{\iint \mathbf{E} \times \mathbf{H}^* \, dx \, dy\},\tag{9}$$

$$P_w = 2P_{w,x=0} + 2P_{w,y=0},$$
 (10)

where  $P_{w,x=0}$  and  $P_{w,y=0}$  is the power loss on the conducting walls at x=0 and y=0, respectively. The factor of two arises because all four walls must be considered.  $P_{w,x=0}$  and  $P_{w,y=0}$  are calculated by integrating the square of the current density on the waveguide wall multiplied by the surface resistance over the length of the wall, namely:

$$P_{w,x=0} = \int_0^b J_s(x=0) \cdot J_s^*(x=0) R_s \, dy, \quad (11)$$
  

$$P_{w,y=0} = \int_0^a J_s(y=0) \cdot J_s^*(y=0) R_s \, dx, \quad (12)$$
  
where  $J_s = \hat{n} \times H$  and

$$R_s = \sqrt{\frac{\omega\mu_0}{2\sigma_c}}.$$
 (13)

The assumption of only  $TE_{10}$  of mode propagation states only the existence of:

$$E_y = E_0 \sin \frac{\pi x}{a} e^{-jk_z z},\tag{14}$$

$$H_x = Y_0 E_0 \sin \frac{\pi x}{a} e^{-jk_z z},\tag{15}$$

$$H_z = Y_0 E_0 \cos\frac{\pi x}{a} e^{-jk_z z},\tag{16}$$

in the waveguide, where  $E_0$  is a wave amplitude coefficient and  $Y_0$  is the admittance that relates the H-field amplitude to the E-field amplitude. Using (14)-(16) in (9) and (10) provides the attenuation constant, due to finite conductivity in the waveguide walls as:

$$\alpha_c = \frac{4Y_0 R_s(a+b)}{ab}.$$
 (17)

It is important to note again that this attenuation constant is only valid for dominant  $TE_{10}$  mode analysis.

# IV. MODE MATCHING FORMULATION

Figure 2 shows a waveguide step discontinuity. Assuming only  $TE_{m0}$  modes propagating in the waveguide, one can express the electric and magnetic fields in region A as a sum of the incident and reflected waves from the junction at z=0:

$$E_y^A = \sum_{m=1}^M A_m^+ \sin \frac{m\pi x}{a} e^{-\gamma_{am} z} + A_m^- \sin \frac{m\pi x}{a} e^{\gamma_{am} z},$$
(18)

$$H_x^A = \sum_{m=1}^M Y_{am} A_m^- \sin \frac{m\pi x}{a} e^{\gamma_{am} z} -Y_{am} A_m^+ \sin \frac{m\pi x}{a} e^{-\gamma_{am} z}.$$
 (19)

In region B, the fields are expressed as:

 $E_{\gamma}^{B} = \sum_{n=1}^{K} B_{n}^{+} \sin \frac{n\pi x}{c} e^{-\gamma_{bn} z}$ 

$$+B_n^- \sin\frac{n\pi x}{c} e^{\gamma_{bn} z}, \qquad (20)$$

$$H_x^B = \sum_{n=1}^K Y_{bn} B_n^- \sin \frac{n\pi x}{c} e^{\gamma_{bn} z} -Y_{bn} B_n^+ \sin \frac{n\pi x}{c} e^{-\gamma_{bn} z},$$
(21)

where in Fig. 2,  $A_m^+, B_m^-$  are unknown incident wave amplitude coefficients and  $A_m^-, B_m^+$  are the reflected wave amplitudes in their respective region. In (14)-(16), the propagation constant for a respective region is:

$$\gamma_z = \alpha_c + jk_z, \tag{22}$$

where  $\alpha_c$  and  $k_z$  are defined by (17) and (7), respectively, and

$$Y_i = \frac{k_z}{j\omega\mu_0}.$$
 (23)

The propagation constant  $(k_z)$  and admittance  $(Y_i)$  are the two calculated values used in determining the unknown wave amplitude coefficients of the TE<sub>10</sub> mode, because the MMT equations are only valid at z=0 and  $0 \le x \le c$ , they account for dielectric and conductor loss of wave amplitudes at this finite location. The propagation constant of (22) is also used later on to account for losses in the connecting sections of waveguides for the TE<sub>10</sub> mode as well. Setting the tangential fields at z=0 equal to each other, assuming TE10 excitation and making use of mode orthonogality, one can calculate the transmission coefficients:

$$S_{21} = \left(\sum_{\nu=1}^{K} \left(\sum_{m=1}^{M} \left(\frac{2}{a} Y_{am} H_{m\nu} H_{mp}\right) + Y_{b\nu} \frac{c}{2} \delta_{\nu p}\right)\right)^{-1} (2Y_{a1} H_{1\nu}) \text{ for } p = 1, 2, \dots K.$$
(24)

The reflection coefficients are given by:

$$S_{11} = \frac{2}{a} \sum_{p=1}^{K} (H_{mp} S_{21}) - \delta_{m1}$$
  
for m = 1,2,...M, (25)

where

$$H_{ij} = \int_0^c \sin\frac{i\pi x}{a} \sin\frac{j\pi x}{c} dx, \qquad (26)$$

where  $S_{21}$  are the transmission coefficients of the discontinuity,  $S_{11}$  are the reflection coefficients of the discontinuity, *M* are the number of modes in region A and *K* are the number of modes in region B. The reader is invited to explore [13] on MMT, in order to gain a full understanding on MMT formulation.



Fig. 2. Waveguide step discontinuity between region A and Region B.

#### V. FILTER DESIGN

A three pole filter with 6% bandwidth in the Ka band is designed using a dielectric with  $\varepsilon$ =67 and  $\varepsilon_y$ =61, due to expressed interest from the filter manufacturer. The filter is designed using Hong and Lancaster's [14] method to calculate filter coupling coefficients (k<sub>ij</sub>) from g values.

This method provides coupling coefficients  $k_{12}=k_{23}=0.107$ . Using the resonant peak method of [14], coupling coefficients are then calculated and compared with ideal coupling coefficients to provide initial filter dimensions. Typically, the inductive coupling irises between resonators are kept thin with respect to waveguide dimensions, as shown by [15]. Bearing this in mind, Fig. 3 shows calculated coupling coefficients assuming the thickness (d) of the coupling irises is set to the minimum that manufacturing tolerances allow. One can see from Fig. 3, that if the dielectric is not treated as uniaxial, the coupling coefficients vary drastically. Using the results of Fig. 3 as a starting point and S-parameter theory to cascade

waveguide sections, the filter dimensions are given in Fig. 4 after the first and third resonator lengths were optimized for return loss. +Uniaxial



the discontinuities with respective connecting

Fig. 3. Coupling coefficients for isotropic and uniaxial media for varying ratios of c/a and d=0.127 mm. Insert displays the junction and coordinate system under analysis.



Fig. 4. Dimensions of initial filter.

# **VI. RESULTS**

A narrowband waveguide filter is designed

and simulated at 32.5 GHz. The dielectric is assumed to have a thickness of 0.254 mm,  $\tau = 3e$ -4 and  $\sigma_c$ =3.5e7. The waveguide is excited with a TE<sub>10</sub> mode using an optimized microstrip-towaveguide transition, as was done in [16]. The exact dimensions of the transition are omitted at the request of the manufacture. The postoptimization results provided by the MMT are in Fig. 5, with measured filter performance in solid circles and results from a commercial FEM solver are also given for comparison (solid blue). The measured results of Fig. 5 show a VSWR of 2:1 or better from 31.6-33.8 GHz, with a center frequency of 32.68 GHz. The insertion loss is 6 dB or better in this band, with the passband ripple better than 1 dB peak-to-peak. It should be noted that according to the manufacturer 6 dB of insertion loss is acceptable. Figure 6 shows the manufactured filter above the word liberty on a US penny for size comparison. One can see good agreement between MMT and measured results. After analysis, it was discovered that the dimensions of the filter varied slightly from those in Fig. 4. These dimensional changes are noted in Fig. 6. It was also discovered that the donated substrate had higher dielectric loss and lower conductivity than initially estimated. However, when the adjusted losses and dimensional differences are accounted for in the MMT analysis, one can notice that there is now excellent agreement in Fig. 7 between the MMT and measured results.



Fig. 5. Measured filter performance vs. simulated performance in black triangles.

0.9

0.8



Fig. 6. Manufactured filter next to the word LIBERTY on a US penny for size comparison.



Fig. 7. Measured filter performance (solid circles) vs. simulated performance with adjusted dimension and loss numbers (black triangles).

#### **VII. CONCLUSION**

Derivations are presented for the propagation constant of a dielectric waveguide when the dielectric is assumed to exhibit uniaxial properties. Using this propagation constant with the MMT, a filter is designed, analyzed and manufactured. The MMT accounts for dielectric losses and conductor wall losses. It is shown that accounting for uniaxial media and losses in the analysis of dielectric filters using the MMT allows for accurate prediction of filter performance. The manufactured dielectric filter shows excellent agreement with simulated results.

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