A Parallel Two-Level Spectral Preconditioner for Fast Monostatic Radar Cross-Section Calculation

Zi He

Department of Communication Engineering
Nanjing University of Science and Technology, Nanjing, 210094, China
eechenrs@njust.edu.cn

Abstract **─** Although the Multilevel Fast Multipole Method (MLFMM) and the parallel technology can accelerate the matrix-vector product operation, the iteration number does not reduce at all in the iterative solution. A new proposed two-level spectral preconditioning technique is developed for the generalized minimal residual iterative method, in which the MLFMM is used to accelerate the calculation. The Multifrontal Massively Parallel Solver (MUMPS) is used to damp the high frequencies of the error, and the low frequencies of the error are eliminated by a spectral preconditioner in a two-level manner. This technique is a combination of MUMPS and a low-rank updated spectral preconditioner, in which the restarted deflated Generalized Minimal Residual (GMRES) with the newly constructed spectral two-level preconditioner is considered as the iterative method for solving subsequent systems. Numerical experiments indicate that the proposed preconditioner is efficient for the MLFMM and can significantly reduce both the iteration number and computational time.

Index Terms **─** MLFMM, MUMPS, preconditioning, scattering problems, spectral.

I. INTRODUCTION

The Method of Moments (MoM) is widely used to solve the Electric Field Integral Equation (EFIE) in RCS calculations [1]-[3]. There are two means to accelerate the computing of large-scale objects scattering problems. One is to accelerate the construction of the impedance matrix and the other is to fast solve the linear equations [4]-[6]. It is meaningless to construct impedance matrix efficiently if the linear equations can not be solved quickly. Therefore, it plays a very important role in fast solve linear equations which the MLFMM formed. Direct method has memory requirement of O(N^2) and computational complexity of O(N^3), where N is the number of unknowns. Therefore, iterative solution has become a successful application in recent years for electrically large objects. More improvements are needed for the iterative solution because of the slow convergence or the misconvergence.

It is well known that EFIE provides ill-conditioned linear system. Therefore, it is natural to use preconditioning techniques to improve the condition number of the system. Many scholars have done a lot of research on improving the efficiency of the iterative solution in the past few decades [7]-[14]. Diagonally perturbed incomplete factorization preconditioned CG algorithms is used in [7], incomplete LU preconditioner is applied for FMM implementation in [8]-[9]. And a sparse approximate inverse preconditioner is used for nonsymmetrical linear systems [10]-[13]. However, every preconditioner has its merits and demerits. Diagonal preconditioner (Diag) and Symmetrical Successive Over-Relaxation preconditioner (SSOR) are simple to construct, but can not improve the convergence rate of the iterative algorithm greatly because of the bad approximation of the inverse matrix. Incomplete LU decomposition preconditioner (ILU) and Sparse Inverse preconditioner (SAI) can improve convergence speed greatly, but needs long construction time and large structure complexity. ILU is unstable under many circumstances to destroy the convergence of the iterative algorithm.

Submitted On: April 26, 2013
Accepted On: August 25, 2014

In most of the cases, a single preconditioner can improve the iteration convergence speed to a certain extent. We can get more obvious convergence improvements when combining different preconditioners. A spectral two-level preconditioning was presented for electromagnetic problems [15]-[18]. The two-level spectral preconditioning technique proposed in [17] obtained a good performance. However, the SAI preconditioned two-level spectral method may result in bad convergence for some structures, because the SAI preconditioner can only obtain the approximate inversion of the near-field impedance matrix. In this paper, the Multifrontal Massively Parallel Solver (MUMPS) is used together with a spectral preconditioner in a two-level manner that results in a faster convergence rate.

This paper is organized as follows. Section 2 gives an introduction to the proposed two-level spectral preconditioner in detail. Numerical experiments with a few electromagnetic scattering problems are presented in Section 3 to show the efficiency of the spectral two-step preconditioner. Section 4 gives some conclusions and comments.

II. THEORY AND FORMULATION

The problem we focus on in this paper is the monostatic RCS calculation of an object. The procedure consists of considering a set of waves with the same wavelength but different incident angles that illuminate the object. For each of these waves, we compute the electromagnetic field backscattered in the direction of the incident wave. This requires solution of one linear system per incident wave. Therefore, a sequence of linear systems with the same coefficient matrix but different right-hand sides is derived,

$$Z(I_1, I_2 ..., I_p) = (V_1, V_2 ..., V_p) . \quad (1)$$

Where Z, I_i and V_i are the EFIE impedance matrix, the induced current vector and the excitation vector with respect to p different incident waves, respectively.

The MLFMM is applied to reduce the memory requirement and the computational complexity. In MLFMM, the impedance matrix Z can be split into two parts as:

$$Z = Z_{NF} + Z_{FF} . \quad (2)$$

Where Z_{NF} denotes the sparse matrix that corresponds to near-filed interactions, while Z_{FF} denotes the matrix that corresponds to far-field interactions. The near-filed interactions can be calculated directly by MoM, while the far-field interactions can be computed by MLFMM.

The matrix based on EFIE is usually ill-conditioned and requires a large number of iterations to reach convergence. In order to speed up the convergence rate, the preconditioning techniques are often used. To this end, we first consider a MUMPS preconditioner based on a multifrontal approach. MUMPS is a package [19] for solving systems of linear equations. MUMPS implements a direct method based on a multifrontal approach which performs a direct LU factorization. And the sparse matrix can be either unsymmetric, symmetric positive definite, or general symmetric. MUMPS exploits both parallelism arising from sparsity in the matrix and from dense factorizations kernels. MUMPS distributes the work tasks among the processors, but an identified processor (the host) is required to perform most of the analysis phase, to distribute the incoming matrix to the other processors (slaves) in the case where the matrix is centralized, and to collect the solution. The parallel version of MUMPS requires MPI for message passing and makes use of the BLAS, BLACS, and ScaLAPACK libraries [20]-[22].

Since Z_{FF} is not readily available, it is customary to construct the preconditioner from Z_{NF}. Z_{NF} is assumed to be a good approximation to Z. In this paper, we chose the preconditioning matrix $M_1 = Z_{NF}$. Through the MUMPS package, the preconditioning matrix can be factorized efficiently,

$$M_1 = Z_{NF} = L_{NF} U_{NF} . \quad (3)$$

Where L_{NF} is a lower triangular matrix and U_{NF} is an upper triangular matrix. Then M_1^{-1} can be obtained by the MUMPS package with high efficiency. And M_1^{-1} is stored in sparse storage format.

To accelerate iterative solvers, the linear equation (1) is always converted to:

$$M_1^{-1} Z I_i = M_1^{-1} V_i \quad (i = 1, 2 ..., p) . \quad (4)$$

Where M_1^{-1} is a matrix for preconditioning the matrix Z from the left. The purpose of preconditioner is to make the preconditioned matrix $M_1^{-1}Z$ as close to the identity matrix I as possible.

Although, the MUMPS preconditioner described above is very effective as shown in the following numerical results; the construction of it is inherently local. When the exact inverse of the original matrix is globally coupled, this lack of global information may have a severe impact on the quality of the preconditioner. We can get more obvious convergence improvements if recovering global information. In this case, some suitable mechanism has to be considered to recover global information.

We firstly let the most of eigenvalues of the near-field interactions concentrate on the unit by using the parallel MUMPS preconditioner, which eliminates the high frequency component of iteration process and accelerates the iteration convergence speed. A spectral preconditioner proposed in [23] can be introduced and used in a two-level manner for the above parallel MUMPS preconditioned system. The purpose here is to recover global information by removing the effect of some smallest eigenvalues in magnitude in the MUMPS preconditioned matrix, which potentially can slow down the convergence of Krylov solvers [24]. In this paper, the first right-hand side system is solved particularly with the MUMPS preconditioned GMRES-DR algorithm, which also generates approximations to eigenvectors as a byproduct.

Suppose $\lambda_1, \lambda_2, ..., \lambda_n$ be the eigenvalues of the MUMPS preconditioned matrix $M_1^{-1}Z$ from small to large, where n represents the number of unknowns. And suppose U be a set of eigenvectors of dimension k associated with the smallest eigenvalues of $M_1^{-1}Z$. It will take a long time to extract the eigenvalues if k is large. On the other hand, it will obtain small improvement if k is small.

Define the second spectral preconditioner as:

$$M_2^{-1} = I_n + U(1/|\lambda_n| T - I_k)U^H . \quad (5)$$

Where $T = U^H(M_1^{-1}Z)U$, I_n and I_k are unit matrix of dimension n and k, respectively.

From the above analysis, we can convert the k smallest eigenvalues of the coefficient matrix $M_1^{-1}Z$'s characteristic spectrum, which is based on parallel MUMPS preconditioner to k arithmetic numbers whose values are $|\lambda_n|$. This process can eliminate negative influences of the k smallest eigenvalues. Combining the second preconditioner with the previously preconditioner in a two-level manner, a new two-level preconditioner is derived and has the form of:

$$M_2^{-1}M_1^{-1}ZI_i = M_2^{-1}M_1^{-1}V_i \quad (i = 1, 2..., p) . \quad (6)$$

Supposing that M_1^{-1} is a preconditioner of Z, M_2^{-1} is a preconditioner of $M_1^{-1}Z$. Therefore, a new two-level spectral preconditioning for multilevel fast multipole method is presented, which is a combination of a parallel MUMPS preconditioner and a parallel spectral preconditioner. The procedure can be concluded as follows:

(1) Construct the MUMPS preconditioner M_1^{-1} by using the near-field matrix element of the impedance matrix Z_{NF};

(2) Solve the k smallest eigenvalues of the matrix $M_1^{-1}Z$ and construct the second spectrum preconditioner M_2^{-1} by using the information of eigenvectors;

(3) Solve the linear equations (6) by the iterative method.

III. NUMERICAL RESULTS

In this section, we show some numerical results that illustrate the effectiveness of the proposed method for the solution of linear systems with multiple right-hand sides arising from the discretization of EFIE formulation in monostatic RCS computation. The first system (system with the first right-hand side) is solved with the MUMPS preconditioned GMRES-DR algorithm, and at the same time eigenvector information is extracted to construct the spectral two-level preconditioner. In our experiments, the restarted version of GMRES(m) [25] algorithm is used to solve subsequently left systems, where m is the dimension size of Krylov subspace for GMRES. All cases are tested on HP server with Intel Xeon CPU X5550 (2.67 GHz). The operating system is Red Hat Enterprise Linux Server release 5.3. The environment of compiling is Intel Visual Fortran 9. Additional details and comments on the implementation are given below:

- Choose m=100 as the maximum size of the subspace and k=80 as the desired number of approximate eigenvectors in the GMRES-DR(m,k).
- The maximum number of iterations is limited to be 5000.
- Zero vector is taken as initial approximate solution for all examples and all systems in each example.
- The iteration process is terminated when the normwise backward error is reduced by 10^{-3} for the first two examples and by $5*10^{-3}$ for the last war craft example.
- The third example is performed on 4-node cluster connected with an Infiniband network. Each node includes 8 cores and 48 GB of RAM. One node is used in the first two examples with 8 cores.

First of all, a comparison is made among the SAI preconditioned two-level spectral preconditioner [17], the MUMPS preconditioned two-level spectral preconditioner, and the traditional MLFMM for the hypervelocity vehicles X43. X43 is an open structure with the size of 3.67 m×1.42 m×0.62 m. There are 128,458 triangles and 192,562 unknowns after discretization. The incident plane wave direction is fixed at $\theta^{inc}=0^{\circ},\phi^{inc}=0^{\circ}$, the frequency is 2.2 GHz, and the scattering angle is fixed at $\theta_s=0^{\circ}-180^{\circ},\phi_s=90^{\circ}$.

As shown in Fig. 1, the comparison is made for the bistatic RCS of parallel polarization. It can be found that there is an excellent agreement between them and this demonstrates the validation of the proposed algorithm. The convergence history is given in Fig. 2. Since a good preconditioner depends not only on the convergence effect, but also on its construction and iteration time. As shown in Table 1, the construction time, the iteration time and the number of iterations are listed with different preconditioners, where ＊ refers to no need to take. The construction time is for constructing both M_1 and M_2. It can be observed that the proposed new two-level spectral preconditioner decreases the number of iterations greatly when compared with the SAI preconditioned two-level spectral method.

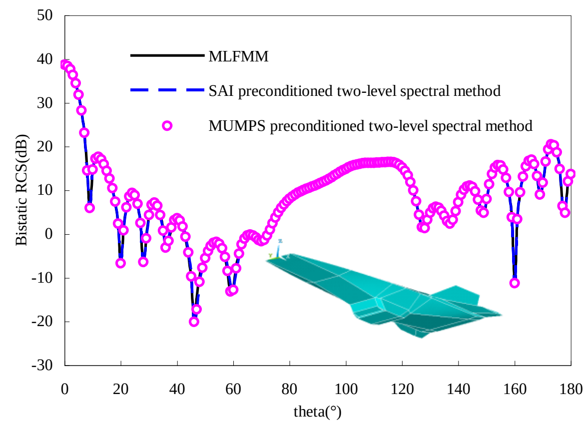


Fig. 1. Bistatic RCS of the X43 at 2.2 GHz.

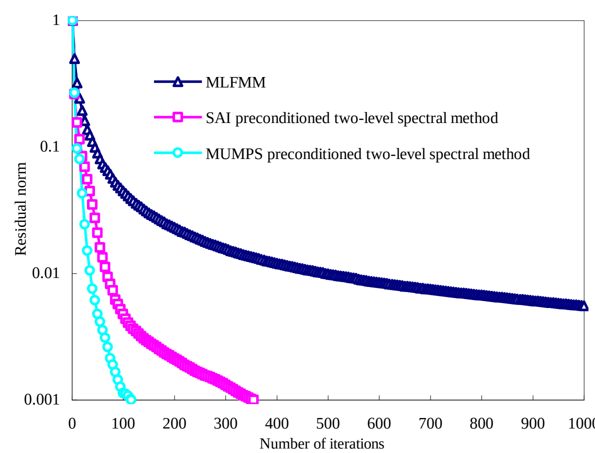


Fig. 2. Convergence history of GMRES algorithms for the X43 at 2.2 GHz.

Table 1: Number of iterations, construction time and iteration time (in seconds) for the X43

	Construction Time (in seconds)	Iteration Time (in seconds)	Number of Iterations
MLFMM	*	2183	4324
SAI preconditioned two-level spectral method	478	309	355
MUMPS preconditioned two-level spectral method	547	118	116

The second example is an analysis of monostatic RCS from a satellite. The length of the cube in the middle is 2 m, the length of the solar panels beside the cube is 8 m and the interval between the cube and the solar panels is 1.87 m. The incident plane wave direction is fixed at $\theta^{inc}=0^\circ-180^\circ,\phi^{inc}=90^\circ$, the frequency is 2.0 GHz and the scattering angle is fixed at $\theta_s=0^\circ-180^\circ,\phi_s=0^\circ$. The number of unknowns is 861,204.

Figure 3 shows the convergence histories of the GMRES method with or without preconditioning for solving the linear system associated with the first right-hand side. It can be found that the two-level spectral preconditioned method decreases the number of iterations by a factor of 5.73 when compared with the MUMPS preconditioned method. Larger improvements can also be found when compared with the GMRES method without preconditioning in terms of iterations. As shown in Fig. 4, the number of iterations with both MUMPS and two-level spectral preconditioning are displayed for solving systems with respect to different incident angles. The result of monostatic RCS calculation is shown in Fig. 5. As shown in Table 3, the construction time and the iteration time are listed with different preconditioners. The construction time of the MUMPS preconditioning method is for constructing M_1, while the construction time of the MUMPS preconditioned two-level spectral method is for constructing both M_1 and M_2. It demonstrates the effectiveness of the proposed method. The parallel efficiency for the proposed new two-level spectral preconditioner is tested in the Table 2. The construction time in Table 2 is for constructing both M_1 and M_2.

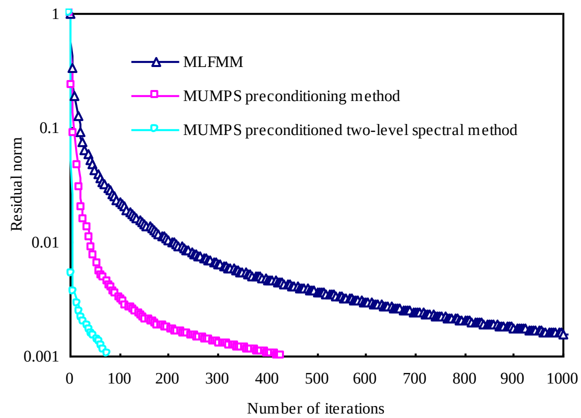


Fig. 3. Convergence histories of GMRES method with or without preconditioning for solving the linear system associated with first right-hand side.

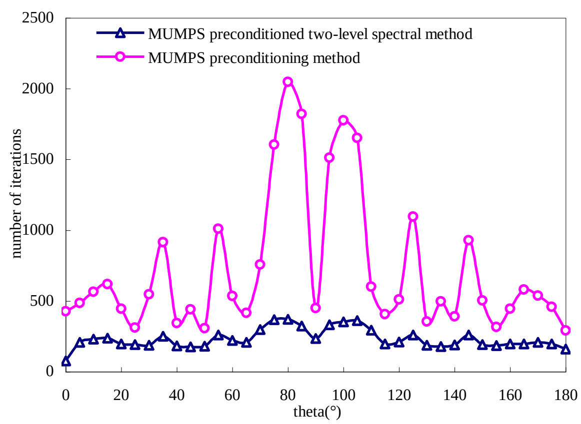


Fig. 4. Number of iterations with both MUMPS and two-level preconditioning for solving systems with respect to different incident angles.

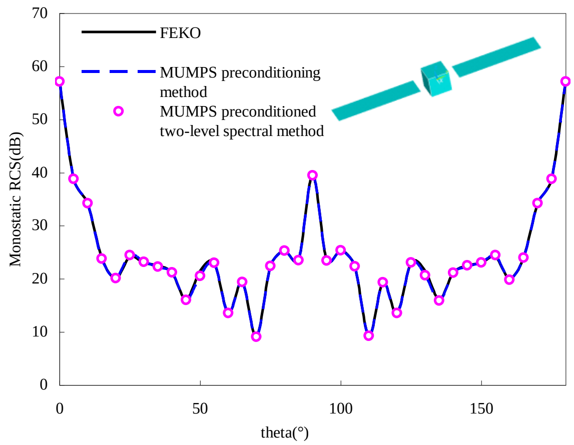


Fig. 5. Monostatic RCS of the satellite at 2 GHz.

Table 2: The parallel efficiency of the proposed new two-level spectral preconditioner for the satellite

	For 8 Cores	For 16 Cores	For 32 Cores
Construction time (in seconds)	1149	575	351
Total time (in seconds)	33,632	25,686	21,166

Table 3: Construction time and iteration time (in seconds) for the satellite

	Construction Time (in seconds)	Iteration Time (in seconds)
MUMPS preconditioning method	396	59,762
MUMPS preconditioned two-level spectral method	1149	32,822

At last, the proposed method is used to analyze scattering from a war craft. The war craft is an open structure with the size of 1.91 m$\times$2.73 m$\times$0.6 m. There are 5,741,073 unknowns after discretization, and the frequency is 8.0 GHz. The incident plane wave direction is fixed at $\theta^{inc} = 0^{\circ} - 180^{\circ}, \phi^{inc} = 0^{\circ}$, and the scattering angle is fixed at $\theta_s = 0^{\circ} - 180^{\circ}, \phi_s = 0^{\circ}$.

Figure 6 shows the convergence histories of the GMRES method with or without preconditioning for solving the linear system associated with the first right-hand side. As shown in Fig. 7, the number of iterations with both MUMPS and two-level spectral preconditioning is displayed for solving systems with respect to different incident angles. It can be observed that the use of the spectral preconditioner in a two-level manner improves the convergence of the MUMPS method by a factor of 2.8 on average. The result of monostatic RCS calculation is shown in Fig. 8. As shown in Table 4, the construction time and the iteration time are listed with different preconditioners. The construction time of the MUMPS preconditioning method is for constructing M_1, while the construction time of the MUMPS preconditioned two-level spectral method is for constructing both M_1 and M_2. It demonstrates the effectiveness of the proposed method.

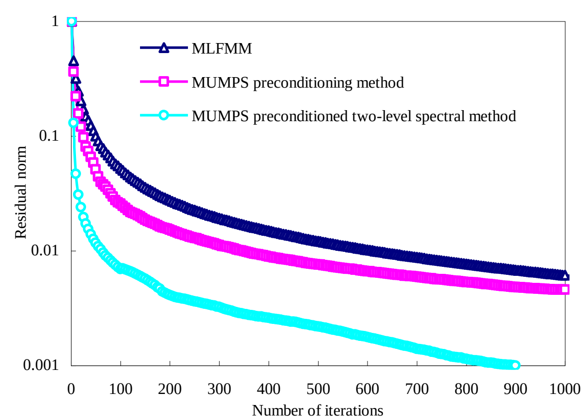


Fig. 6. Convergence histories of GMRES method with or without preconditioning for solving the linear system associated with first right-hand side.

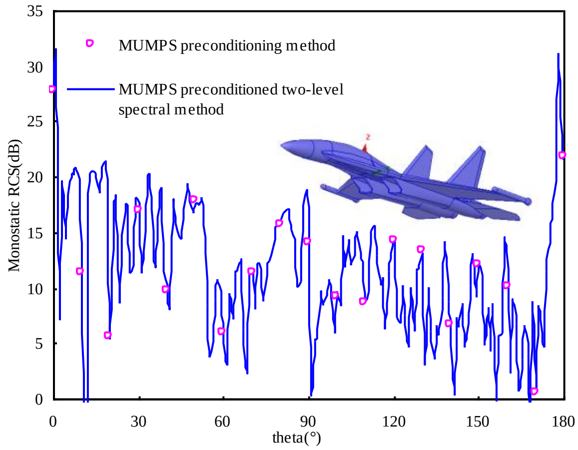


Fig. 7. Monostatic RCS of the war craft at 8 GHz.

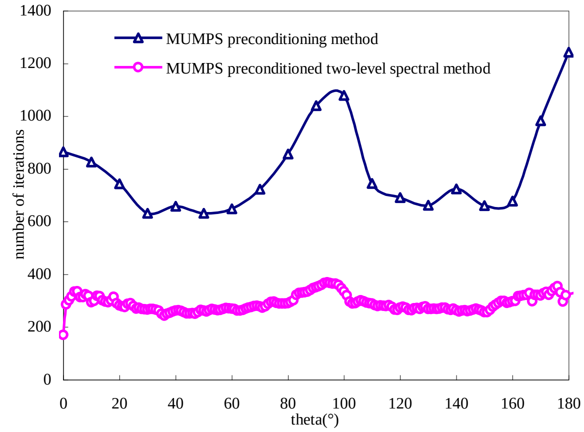


Fig. 8 Number of iterations with both MUMPS and two-level preconditioning for solving systems with respect to different incident angles.

Table 4: Construction time and iteration time (in seconds) for the war craft

	Construction Time (in seconds)	Iteration Time (in seconds for 19 angles)
MUMPS preconditioning method	634	60,547
MUMPS preconditioned two-level spectral method	1342	16,704

IV. CONCLUSION

In this paper, a parallel two-level spectral preconditioner utilizing MUMPS is proposed for solving systems with multiple right-hand sides in monostatic RCS calculation. The MUMPS preconditioner is used to damp the high frequencies of the error, and the low frequencies of the error are eliminated by a spectral preconditioner in a two-level manner. The first right-hand side system is solved by the use of the GMRES-DR algorithm, and the approximate smallest eigenvector information is obtained for constructing the spectral preconditioner for subsequent systems. Numerical results are presented to demonstrate the efficiency of the proposed method. And the comparisons are made among different preconditioners. It can be found that the proposed preconditioner can not only get better convergence, but can also reduce the overall simulation time when compared with other preconditioners.

ACKNOWLEDGMENT

We would like to thank the support of Natural Science Foundation of 6143100, Jiangsu Natural Science Foundation of BK2012034, Natural Science Foundation of 61271076, 61171041, 61371037, Ph.D. Programs Foundation of Ministry of Education of China of 20123219110018; the Fundamental Research Funds for the central Universities of No. 30920140111003, No. 30920140121004.

REFERENCES

[1] R. F. Harrington, "Field computation by moment methods malabar," *FL: R. E. Krieger*, 1968.

[2] J. J. H. Wang, "Generalized moment methods in electromagnetics," *New York: Wiley*, 1991.

[3] Y. Zhang, D. Huang, and J. Chen, "Combination of asymptotic phase basis functions and matrix interpolation method for fast analysis of monostatic RCS," *Applied Computational Electromagnetics Society (ACES) Journal*, vol. 28, no. 1, pp. 49-56, January 2013.

[4] S. M. Rao, D. R. Wilton, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Transactions on Antennas and Propagation*, vol. 30, no. 3, pp. 409-418, 1982.

[5] W. C. Chew, J. M. Jin, E. Midielssen, and J. M. Song, "Fast and efficient algorithms in computational electromagnetics," *Boston, MA: Artech House*, 2001.

[6] R. S. Chen, Z. H. Fan, Y. Y. An, M. M. Zhu, and K. W. Leung, "Modified adaptive cross

approximation algorithm for analysis of electromagnetic problems," *Applied Computational Electromagnetics Society (ACES) Journal*, vol. 26, no. 2, pp. 160-169, February 2011.

[7] R. S. Chen, X. W. Ping, and E. K. N. Yung, "Application of diagonally perturbed incomplete factorization preconditioned CG algorithms for edge FEM analysis of helmholtz equations," *IEEE Trans. Antennas Propag.*, vol. 54, no. 5, pp. 1064-1068, 2006.

[8] K. Sertel and J. L. Volakis, "Incomplete LU preconditioner for FMM implementation," *Microwave and Optical Technology Letters*, vol. 26, no. 7, pp. 265-267, 2000.

[9] E. Chow and Y. Saad, "Experimental study of ILU preconditioners for indefinite matrices," *J. Comput. App. Math.*, vol. 86, pp. 387-414, 1997.

[10] M. Benzi and M. Tuma, "A sparse approximate inverse preconditioner for nonsymmetric linear systems," *SIAM Journal on Scientific Computing*, vol. 19, pp. 968-994, 1998.

[11] X. Q. Hu, M. Chen, D. Z. Ding, and R. S. Chen, "A modified complex shifted preconditioner combined with sparse approximate inversion preconditioner for electromagnetic scattering," *Microwave and Optical Technology Letters*, vol. 53, no. 1, pp. 55-58, January 2011.

[12] M. Chen, R. S. Chen, D. Z. Ding, and Z. H. Fan, "Accelerating the multilevel fast multipole method with parallel preconditioner for large-scale scattering problems," *Applied Computational Electromagnetic Society (ACES) Journal*, vol. 26, no. 10, pp. 815-822, October 2011.

[13] N. Carpentieri, Y. F. Jing, T. Z. Huang, W. C. Pi, and X. Q. Sheng, "Combining the CORS and BiCORSTAB iterative methods with MLFMA and SAI preconditioning for solving large linear systems in electromagnetics," *Applied Computational Electromagnetics Society (ACES) Journal*, vol. 27, no. 2, pp. 102-111, February 2012.

[14] Z. W. Liu, J. Q. Chen, and R. S. Chen, "An adaptive preconditioning technique using fuzzy controller for efficient solution of electric field integral equations," *Applied Computational Electromagnetics Society (ACES) Journal*, vol. 26, no. 5, pp. 411-417, May 2011.

[15] P. L. Rui and R. S. Chen, "Application of a two-step preconditioning strategy to the finite element analysis for electromagnetic problems," *Microw. Opt. Technol. Lett.*, vol. 48, no. 8, pp. 1623-1627, 2006.

[16] P. L. Rui, R. S. Chen, Z. H. Fan, and D. Z. Ding, "Multi-step spectral preconditioner for fast monostatic radar cross-section calculation," *Electronics Letters*, vol. 43, no. 7, March 29, 2007.

[17] D. Z. Ding, R. S. Chen, Z. H. Fan, and P. L. Rui, "A novel hierarchical two-level spectral preconditioning technique for electromagnetic wave scattering," *IEEE Transactions on Antennas and Propagation*, vol. 56, no. 4, April 2008.

[18] X. Hu, R. Chen, D. Ding, Z. Fan, and Y. Xu, "Two-step preconditioner of multilevel simple sparse method for electromagnetic scattering problems," *Applied Computational Electromagnetics Society (ACES) Journal*, vol. 27, no. 1, pp. 14-21, January 2012.

[19] "Multifrontal massively parallel solver (MUMPS 4.10.0) users' guide [online]," *available: http://mumps.enseeiht.fr/.*

[20] P. R. Amestoy, I. S. Duff, J. Koster, and J. Y. L'Excellent, "A fully asynchronous multifrontal solver using distributed dynamic scheduling," *SIAM Journal of Matrix Analysis and Applications*, vol. 23, no. 1, pp. 15-41, 2001.

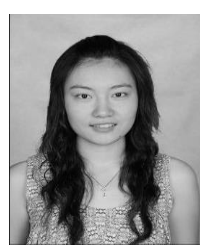
[21] P. R. Amestoy, A. Guermouche, J. Y. L'Excellent, and S. Pralet, "Hybrid scheduling for the parallel solution of linear systems," *Parallel Computing*, vol. 32 (2), pp. 136-156, 2006.

[22] R. B. Lehoucq, D. C. Sorensen, and C. Yang, "ARPACK user's guide: solution of large-scale problem with implicitly restarted arnoldi methods," *SIAM, Philadelphia*, 1998.

[23] J. Erhel, K. Burrage, and B. Pohl, "Restarted GMRES preconditioned by deflation," *Journal of Computational and Applied Mathematics*, vol. 69, pp. 303-318, 1996.

[24] R. B. Morgan, "GMRES with deflated restarting," *SIAM J. Sci. Comput.*, vol. 24, pp. 20-37, 2002.

[25] H. A. van der Vorst and C. Vuik, "The superlinear convergence behavior of GMRES," *Journal of Computational and Applied Mathematics*, vol. 48, pp. 327-341, 1993.



Zi He received her B.Sc. degree in Electronic Information Engineering from the School of Electrical Engineering and Optical Technique, Nanjing University of Science and Technology, Nanjing, China, in 2011. She is currently working towards her Ph.D. degree in Electromagnetic Fields and Microwave Technology at the School of Electrical Engineering and Optical Technique, Nanjing University of Science and Technology. Her research interests include antenna, RF-integrated circuits, and computational electromagnetics.