

# Harmonics Amplitude Measurement in UHF Band by Multi Harmonic Multiplication

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**Abstract** — Analysis of a system to measure the amplitude of harmonics of a signal in UHF band is presented. First, analytical considerations are proposed, in which multi harmonic signal is used as local oscillator signal. The results are used to obtain optimal implementation by which conversion loss of mixer becomes minimal. The proposed method is implemented by single diode mixer. In mixer, harmonics of the desired signal to be measured are fed into RF port and a comb signal is synthesized to be fed into LO port to down convert the desired signal. Bottle neck is the amplitude of each harmonic in comb signal fed in LO port of mixer. The proposed design is validated and optimized by ADS (a full wave electromagnetic simulator). Optimization of conversion loss of the mixer boots the system bandwidth.

**Index Terms** - Conversion loss, harmonics amplitude, and single diode mixer.

## I. INTRODUCTION

Nowadays, spectrum analyzers are widely used to determine frequency contents of signals. These instruments are available in wide range of frequency and dynamic range. The overall method is to down convert the signal and determine its frequency content by digital signal processing. The signal which we intend to determine its frequency content is fed into a mixer and frequency of local oscillators is swept to ensure that all signal frequency content is down converted to frequency range that digital processing block works [1]. The behavioral modeling of RF and microwave amplifiers has attracted a great deal of

attention [2, 3]. One method for accurate characterization of nonlinear RF and microwave devices involves use of time domain measurements [4, 5]. Various methods differ in DUT stimulating, methods of analyzing transmitted and reflected waves and also in models they provide. Behavioral modeling includes measurement of voltage and current waveforms in nonlinear regime, which are full of harmonics. LSNAs and NVNAs are two powerful devices for these measurements [6-8]. In recent years, X-parameters are defined for nonlinear modeling and design; these parameters could be measured by NVNAs [9, 10]. Time domain method proposed in [5] includes multi harmonic signal for driving the mixer. Like all other methods, the system in [5] utilizes a down convertor. We analyze mixer operation to obtain minimum conversion loss of mixer for multi harmonic signals. The problem we consider is that we have a single tone signal with its harmonics and we want to measure the amplitude of harmonics. It is assumed that frequency of fundamental tone is known. The following method, which is described in part II, is faster than measurement by spectrum analyzer, since it down converts harmonics simultaneously, not one by one, as the method used in spectrum analyzer (it sweeps frequency of local oscillator to down converts RF signal). It is also supposed that an attenuator is used to make small signal in RF port of the mixer. First we describe the method and then mixer consideration will be proposed.

## II. METHODOLOGY

The signal whose frequency contents are to be determined is expressed as,

$$RF(t) \triangleq \sum_{n=1}^k A_n \cos(n\omega_{RF}t + \alpha_n). \quad (1)$$

The LO signal used to down convert the RF signal expressed in equation (1) is given by,

$$LO(t) \triangleq \sum_{m=1}^k B_m \cos(m\omega_{LO}t + \beta_m) \quad (2)$$

in which,

$$\omega_{LO} - \omega_{RF} \triangleq \omega_{IF}. \quad (3)$$

As mentioned before,  $\omega_{RF}$  is known and  $\omega_{LO}$  is chosen such that,

$$\frac{\omega_{IF}}{\omega_{LO}} \ll 1, \quad \frac{\omega_{IF}}{\omega_{RF}} \ll 1 \quad (4)$$

in which  $f_{RF}$  is in UHF band. We choose  $f_{IF} = 10$  MHz, so in UHF band we have

$$\frac{\omega_{IF}}{\omega_{RF}} \leq \frac{10}{300} \ll 1. \quad (5)$$

Multiplying equations (1) and (2) gives

$$\begin{aligned} RF(t) \times LO(t) &= \left( \sum_{n=1}^k A_n \cos(n\omega_{RF}t + \alpha_n) \right) \times \\ &\left( \sum_{m=1}^k B_m \cos(m\omega_{LO}t + \beta_m) \right) \\ &= \sum_{n=1}^k \sum_{m=1}^k A_n B_m \cos(n\omega_{RF}t + \alpha_n) \cos(m\omega_{LO}t + \beta_m) \\ &= \sum_{n=1}^k \sum_{m=1}^k \frac{A_n B_m}{2} \left( \cos(n\omega_{RF}t + \alpha_n + m\omega_{LO}t + \beta_m) + \right. \\ &\quad \left. \cos(n\omega_{RF}t + \alpha_n - m\omega_{LO}t - \beta_m) \right). \end{aligned} \quad (6)$$

Considering equation (4), the low frequency content of equation (6) is,

$$IF(t) = \sum_{m=1}^k \frac{A_m B_m}{2} \cos(m\omega_{IF}t + \alpha_m - \beta_m). \quad (7)$$

The maximum frequency content of equation (7) is  $kf_{IF}$ . Since we synthesize LO signal,  $B_m$  is known. If we detect equation (7),  $A_m$ , the amplitude to be measured, could be determined since  $B_m$  is known.

### III. MIXER CONSIDERATION

As observed in Fig.1, in order to multiply equation (1) by equation (2), these signals should be fed to RF and LO ports of a mixer.

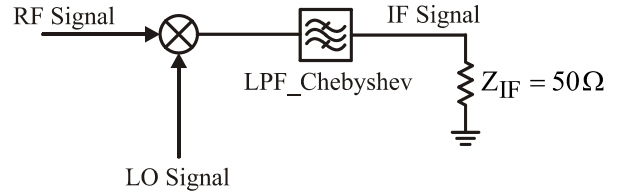


Fig. 1. Diagram of RF, LO, and IF ports.

In usual applications, a single tone LO signal is used in down converter mixers. But we want to multiply two multi harmonic signals. If single tone LO signal is fed to an LO port, the nonlinear element (i.e., here diode) produces harmonics of this tone and frequencies, which are linear combinations of  $f_{LO}$  and  $f_{RF}$ . These frequencies include desired IF frequencies, but conversion losses of harmonics of RF signal will be so large that result in the decrease of the level of IF signal to noise level. Since it is supposed that RF signal is small signal made by an attenuator, large conversion loss can decrease the level of IF signal to noise level. Let us consider on-off model for diode. So its conductance waveform is

$$g(t) = \begin{cases} g_{on} & \text{LO signal} > 0 \\ 0 & \text{LO signal} \leq 0 \end{cases} \quad (8)$$

The diode current is  $i(t) = g(t) RF(t)$ . If LO signal includes only  $f_{LO}$ , the fundamental tone of LO signal, then  $g(t)$  does not include even harmonics of  $f_{LO}$  [11]. Therefore, even harmonics of RF signal will not be down converted and conversion losses for even harmonics of RF signal will be infinite. Then it should be investigated whether or not LO signal includes harmonics of  $f_{LO}$ . We choose LO signal to have first and second harmonics and the calculate conversion losses of RF harmonics. In this case, LO signal is given by

$$LO(t) = a \cos(\omega_{LO}t) + b \cos(2\omega_{LO}t), \quad (9)$$

in which  $a$  and  $b$  are amplitudes with positive values. According to equation (8),  $g(t)$  is

$$g(t) = \begin{cases} g_{on} & a \cos(\omega_{LO}t) + b \cos(2\omega_{LO}t) > 0 \\ 0 & a \cos(\omega_{LO}t) + b \cos(2\omega_{LO}t) \leq 0. \end{cases} \quad (10)$$

The Fourier series representation of  $g(t)$  derived by finding the roots of  $LO(t)$  is given in equation (11). Conversion losses variations for the first to sixth harmonics of RF signal, as a function of  $c$ , are depicted in Fig. 2.

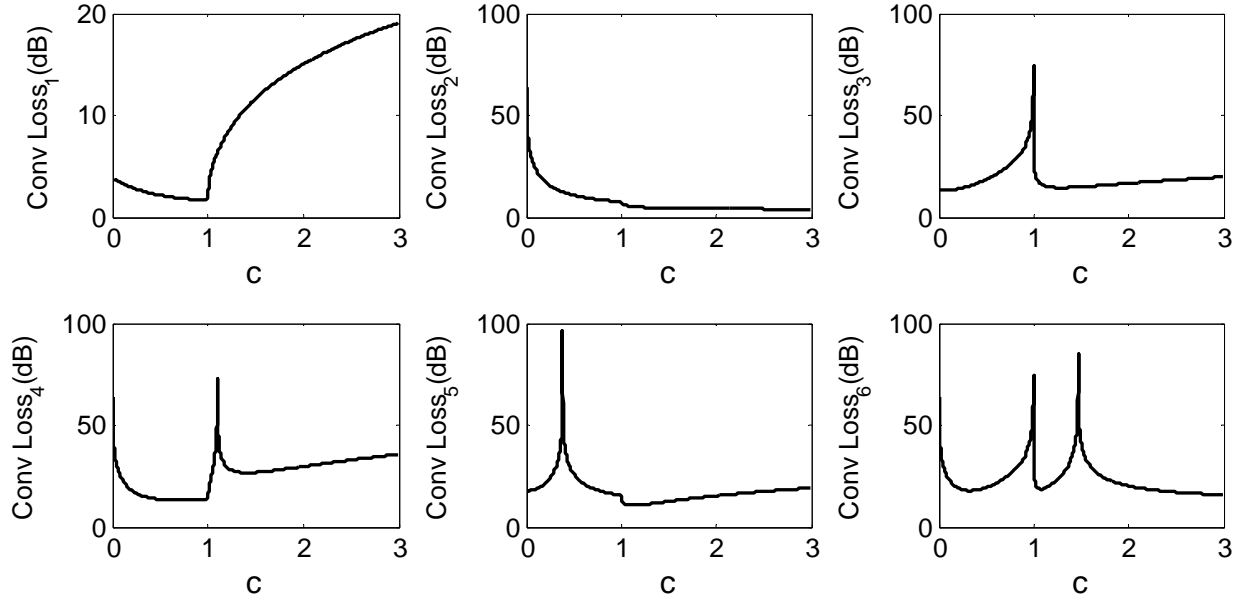


Fig. 2. Conversion losses of the first to sixth harmonics of the RF signal.  $Conv Loss_m$  is the conversion loss of the  $m^{\text{th}}$  harmonic.

$$\begin{aligned} \frac{b}{a} \leq 1 &\rightarrow g(t) = g_{on} \frac{\cos^{-1}(B)}{\pi} + \\ &g_{on} \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin(n \cos^{-1}(B)) \cos(n\omega_{LO}t) \\ \frac{b}{a} > 1 &\rightarrow g(t) = g_{on} \frac{\cos^{-1}(B) + \cos^{-1}(-A)}{\pi} + \\ &g_{on} \sum_{n=1}^{\infty} \frac{2}{n\pi} (\sin(n \cos^{-1}(B)) - \sin(n \cos^{-1}(A))) \cos(n\omega_{LO}t) \end{aligned}$$

where  $A = \frac{-1 - \sqrt{1+8c^2}}{4c}$   $B = \frac{-1 + \sqrt{1+8c^2}}{4c}$  ,  $c = \frac{b}{a}$  .

(11)

As shown in Fig. 2, the conversion losses of the third and higher harmonics are very large. Conversion losses of the first and second harmonics are depicted in Fig. 3. For  $c \geq 1$ , these conversion losses vary in opposite directions.

In [11] for the single tone LO signal and on-off model for diode, it is shown that the conversion loss is 3.92 dB. Now for the two harmonics, we see that the conversion losses change rapidly by varying  $c$ , which is relative amplitude of harmonics of LO signal. For  $c = 1.071$  conversion losses of first and second

harmonics are 5.72 dB, which is 1.8 dB more than the single tone case.

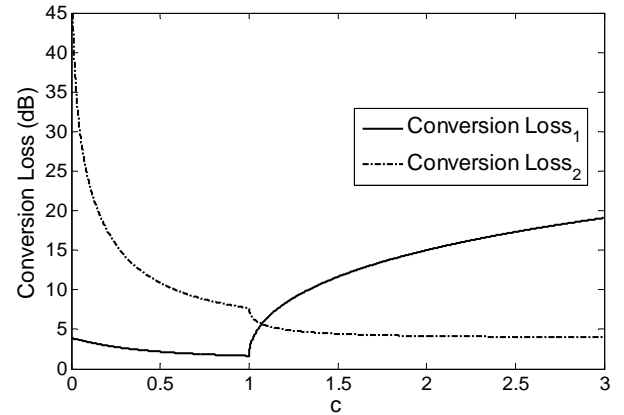


Fig. 3. The conversion losses of the first and second harmonics of the RF signal.

The LO signal includes first and third harmonics as shown,

$$LO(t) = a \cos(\omega_{LO}t) + b \cos(3\omega_{LO}t) . \quad (12)$$

The function  $g(t)$  does not include even harmonics of  $f_{LO}$ , so even harmonics of RF signal will not be down converted. Therefore, conversion losses for even harmonics of RF signal will be infinite. Minimum conversion

losses for odd harmonics are 8.42 dB when  $b/a = 0.663$ . So conversion losses of the first and third harmonics that constitute LO signal are too smaller than other harmonics.

The LO signal which includes first and fourth harmonics is stated as,

$$LO(t) = a \cos(\omega_{LO}t) + b \cos(4\omega_{LO}t). \quad (13)$$

The minimum conversion losses of first to sixth harmonics of RF signal (by numerical calculation) are 6.72, 15.41, 21.15, 7.88, 18.69, and 15.17 dB, respectively, when  $b/a$  is 0.98. Conversion losses of first and fourth harmonics, that constitute LO signal, are too smaller than those of the other harmonics.

The LO signal which contains first three harmonics is given by,

$$LO(t) = a \cos(\omega_{LO}t) + b \cos(2\omega_{LO}t) + c \cos(3\omega_{LO}t) \quad (14)$$

in which  $a$ ,  $b$ , and  $c$  are amplitudes (positive numbers). For this LO signal, the minimum conversion losses of first three harmonics are 6.83, 7, and 7.18 dB, respectively, when  $b/a$  is 1.4 and  $c/a$  is 1.1. The Fourier series of  $g(t)$  is a complicated function of  $a$ ,  $b$ , and  $c$ . So numerical calculation is carried out and it is concluded that conversion losses of fourth and higher harmonics are large. Conversion losses of fourth to sixth harmonics will be minimized subject to  $b/a=1.95$  and  $c/a=1.15$ . By these values, conversion losses of first to sixth harmonics are 12.9, 5.88, 10.33, 11.78, 14.11, and 9.38 dB, respectively.

The LO signal which contains first to fourth harmonics can be described as,

$$LO(t) = a \cos(\omega_{LO}t) + b \cos(2\omega_{LO}t) + c \cos(3\omega_{LO}t) + d \cos(4\omega_{LO}t). \quad (15)$$

For this LO signal, minimum conversion losses of first to sixth harmonics are 11.09, 7.05, 8.49, 11.8, 9.54, and 12.43 dB, respectively when  $b/a=1.12$ ,  $c/a=1.14$  and  $d/a=0.1$ .

Considering these results for conversion losses, it is concluded that to keep conversion losses of harmonics in RF signal minimum, LO signal should contain those harmonics too, and by more harmonics consideration, the minimum conversion losses will be larger.

#### IV. MIXER IMPLEMENTATION

In this section, a single diode mixer is used, which has the least complication to consider a multi harmonic signal as an LO signal. In single diode mixer, a multi section Wilkinson coupler in UHF band to mix RF and LO signal is used. HSMS 8101, which is a surface mount microwave Schottky mixer diode is used as Schottky diode to mix LO and RF signals. Resistor in IF section is set to 50 ohm ( $Z_{IF} = 50$  ohm), which is the resistance of the digital processing block.

A common way to produce multi harmonic signal to make desired LO signal is to use comb generator, typically including step recovery diode (SRD). But amplitudes of harmonics in output signal of comb generators generally decrease or remain approximately constant by the increase of harmonic index; we have no control on the amplitude of the output harmonics. The power of different harmonics of an LO signal should be tuned to make the conversion losses of different harmonics of the RF signal minimum. Frequency multipliers with attenuators could be used to produce comb signal and control the power of harmonics.

A conversion Loss of the designed single diode mixer with a single tone RF signal and a single tone LO signal is 8.4 dB when the power of the LO signal is 8 dBm, it should be noted that 3 dB of this conversion loss is due to the coupler that dissipates half power of RF signal. If we fed the first two harmonics of LO signal in LO port and the first two harmonics of RF signal in RF port, then conversion loss<sub>1</sub> and conversion loss<sub>2</sub> will be so larger than 8.4 dB. Varying the power of LO harmonics changes the conversion losses of different harmonics. If conversion loss<sub>1</sub> decreases, conversion loss<sub>2</sub> increases and vice versa. For two harmonics of RF signal, the conversion loss<sub>1,2} ≈ 12 dB can be reached and for the three harmonics of the RF signal we can reach conversion loss<sub>1,2,3} ≈ 14.75 dB. The variations of conversion losses by an RF frequency for two and three harmonics are shown in Figs. 4 and 5. Simulations have been carried out by multi tone harmonic balance method in Advanced Design System (ADS) and genetic algorithm was used to optimize conversion losses. It should be noted that the 3dB conversion loss is due to the coupler (balanced mixer does not have this loss), and a 1 dB of conversion loss is due to the ohmic resistance of the diode, here we</sub></sub>

have  $R_s = 6\Omega$ . In IF port, amplitudes of IF harmonics are detected. Since we have the conversion loss of the mixer for different harmonics, we can determine the amplitude of RF harmonics. The results of part III and IV for the conversion losses are compared in Table 1.

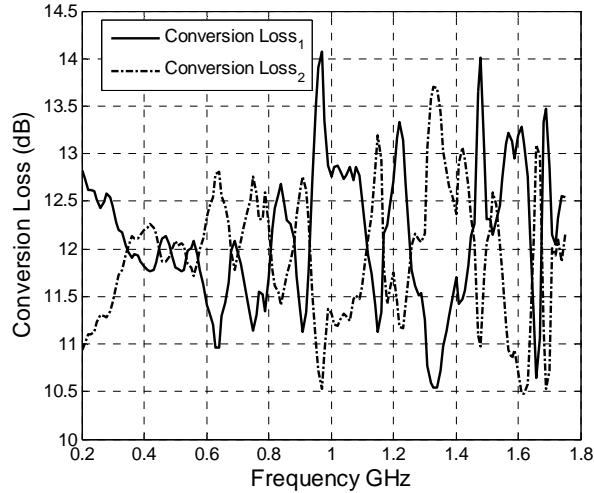


Fig. 4. Conversion losses of two harmonics of an RF signal. The power of the LO signal harmonics are 8 and 10.5 dBm, respectively.

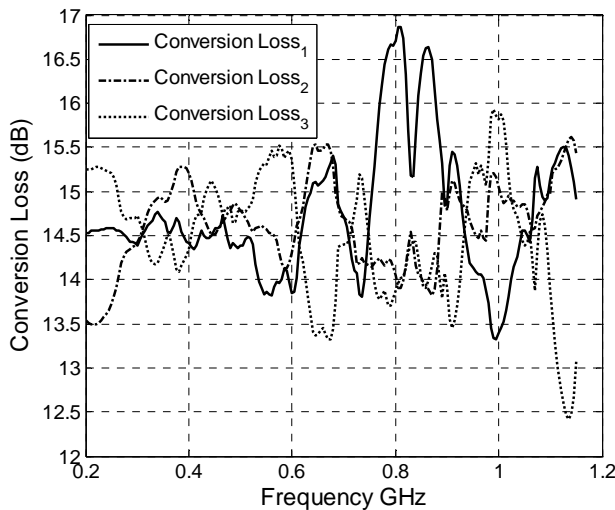


Fig. 5. Conversion losses of three harmonics of an RF signal. The power of the LO signal harmonics are -2.15, 4.6, and 4.6 dBm, respectively.

Table 1: Comparison of conversion losses.

Number of harmonics considered	2	3
Minimum conversion loss by ideal diode (dB)	5.72	~ 7
Minimum conversion loss by single diode mixer (dB)	~ 12	~ 14.75

## V. CONCLUSION

A new approach to measure the harmonic's amplitude was introduced. Mixer considerations and limitations have been examined for ideal and real diode. Conversion losses of different harmonics were made minimum.

It was concluded that by more harmonics consideration, conversion loss will be larger. Additionally, using common comb generator causes large conversion losses since we have no control on amplitude and phase of different harmonics in comb signal. We optimized the power of the LO harmonics to obtain minimum conversion losses and as a result the dynamic range increased.

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