

# A Spherical Harmonic Expansion Method for Accelerating the Interface Between the NEC-REF and NEC-BSC Codes

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**Abstract** — A spherical harmonic expansion is proposed to provide a more efficient interface between a reflector antenna simulation code and an Uniform Geometrical Theory of Diffraction (UTD) ray tracing code. The principal bottleneck is a large number of dipole sources. As a consequence, a large number of dipole sources lead to a large number of ray traces, which increases the computational times in the UTD code. We propose to use a spherical harmonic expansion of the reflector fields as an efficient interface. This method saves significant computational times in the UTD code provided that the number of dipole sources per cell is large.

**Index Terms** — Dipoles, spherical harmonics, Uniform Theory of Diffraction.

## I. INTRODUCTION

The issue of co-site electromagnetic interference must be carefully handled when integrating SAT-COM reflector antennas into Navy topsides. The analysis of optimizing antenna placement within a topside environment is performed here with a hybrid analysis between The Ohio State University Reflector Antenna (NEC-REF) [1] and the Numerical Electromagnetic Code-Basic Scattering Code (NEC-BSC) [2]. NEC-REF simulates the performance of the reflector antenna. Its solution results in an equivalent current representation of the reflector antenna. These equivalent currents are modeled as dipoles in NEC-BSC. The amplitude and phase of the dipoles are computed

to produce the reflector antenna fields in NEC-BSC in the forward direction. NEC-BSC then computes the field as the superposition from the total fields of each of the NEC-REF dipoles based on the topside environment. For an accurate backscattering, a metal reflector plate must be placed in the NEC-BSC computational environment. Figure 1 depicts a general scenario of interest for the SAT-COM reflector antenna with rays being shown in the topside environment.

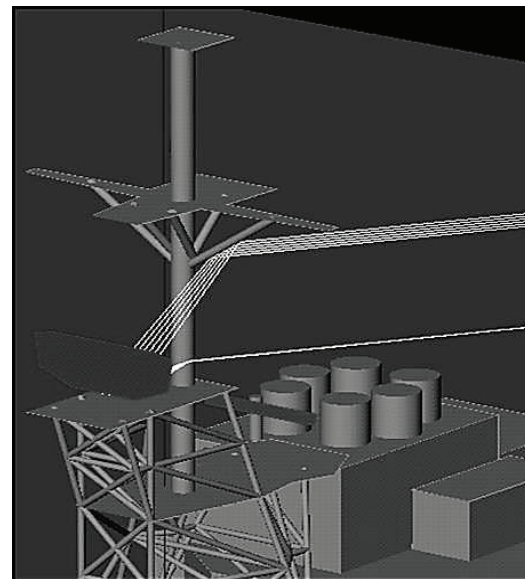


Fig. 1. Rays being depicted from an SAT-COM reflector antenna.

Due to recent antenna requirements, computational times for a NEC-REF/NEC-BSC

analysis can be on the order of several days to run a single design iteration. Typically, a designer will want to run significantly more iterations to obtain the best result. Therefore, this increase in run times can become laborious, limiting the quality of design. For example, the analysis of the SAT-COM reflector antenna onboard a ship (Fig. 1) required several different designs scenarios. When an exact replica of the SAT-COM antenna was modeled in NEC-REF, the large number of dipoles model froze the NEC-BSC code. The model required many dipole sources, which exceeded the memory allocation. The model was eventually analyzed but only after a reduced model was used in a reduced NEC-BSC computational environment (e.g., ground and only part of the superstructure and number of dipoles). The analysis still required several days.

The principal bottleneck in the current analysis is the NEC-REF equivalent current solution, which results in a large number of dipole sources. Each dipole is a ray origin in the UTD code from which multiple rays must be traced. A more efficient method must be utilized to reduce or group the number of dipoles/ray origins while still maintaining accuracy in the analysis. The more ray centers, the longer the computational times, but the proper number still needs to be maintained to provide an accurate pattern and avoid grating lobes by having the centers too far apart.

The Spherical Harmonic Interface Procedure (SHIP) [3,4] transfers spherical harmonic coefficients computed from NEC-REF dipole model to the UTD, NEC-BSC. SHIP creates a spherical harmonic expansion of the NEC-REF dipole model. By incorporating a multi-cell SHIP approach, the spherical harmonic expansion of the NEC-REF dipole model can also handle sources within the near field of the topside environment.

The paper is organized as follows. Section II discusses the overall methodology of the SHIP procedure. Details are discussed for expanding the dipole model from NEC-REF into multiple cell origins and spherical harmonic expansions. Finally, section III provides numerical results using the SHIP interface to NEC-BSC to analyze a SAT-COM antenna installed on two Navy ships. The computational times and the accuracy

of the radiation patterns computed by the environmental computational code will be used to validate and show the improved efficiency for design.

## II. SHIP METHODOLOGY

An efficient interface between NEC-REF and NEC-BSC is possible using the Spherical Harmonic Interface Procedure (SHIP). This method expands the free space fields generated by the reflector antenna using a spherical harmonic expansion [5,6]. The spherical harmonics coefficients are then used as an interface into the UTD code, NEC-BSC, rather than the NEC-REF dipoles model. The benefits of this procedure are the spherical harmonic expansion of the antenna fields has a single origin, which is at the cell center based upon the antenna currents. NEC-BSC utilizes these cell origins to trace the UTD rays through the environment. Figure 2 depicts the ray tracing from a single cell with the origin at the center. An optimal solution is imperative to balance the number of cells and their location with the resulting computation time to achieve sufficient accuracy.

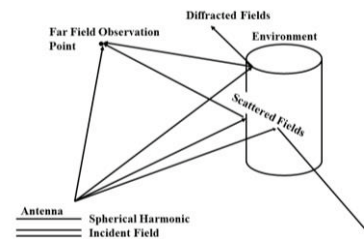


Fig. 2. UTD rays from a single cell.

SHIP decomposes the NEC-REF dipole model of the reflector antenna into a single or multiple cells. The multi-cell SHIP increases the computational time since NEC-BSC must trace rays from a number of cell origins. However, in the case of a complex environment, shadowing considerations can cause the resulting reduction in accuracy of the overall pattern, especially if grating lobes results due to large distances between origins. A greater number of ray traces will provide a higher level of accuracy when these ray origins are in the near zone of the geometry of the environment. This is especially true when the origins are placed near the

reflector plate of the SAT-COM antenna.

### A. Spherical harmonic expansion

SHIP procedure begins with an initial cell of radius  $r_1$  encompassing the entire dipole model of the SAT-COM reflector antenna. The initial cell is then segmented into smaller  $P$  cells of radius  $r_p=(r_1/P^{1/3})$ . The electromagnetic fields for a single cell are the sum of the spherical wave expansions of all the dipoles contained inside that cell. This expansion is stored in the spherical harmonic coefficients; thus, enabling NEC-BSC to compute the total electromagnetic fields of the dipoles from a cell. Using the spherical wave expansion, the electromagnetic fields of a given cell  $P$  is given by:

$$\begin{aligned} E_\theta^P &= -j\omega [A_\theta^P + \eta F_\theta^P] \\ H_\phi^P &= \frac{j\omega}{\eta} [A_\theta^P + \eta F_\theta^P] \end{aligned} \quad (1)$$

and

$$\begin{aligned} E_\phi^P &= -j\omega [A_\phi^P + \eta F_\phi^P] \\ H_\theta^P &= \frac{j\omega}{\eta} [A_\phi^P - \eta F_\phi^P] \end{aligned} \quad (2)$$

The magnetic vector potential in (1) and (2) are given by:

$$A^P = \frac{-j\mu}{2} \frac{e^{-jkr}}{r} \sum_{n=0}^{\infty} \sum_{m=0}^n a_{nm}^P(\theta', \varphi') Y_{nm}^e(\theta, \varphi) + b_{nm}^P(\theta', \varphi') Y_{nm}^o(\theta, \varphi), \quad (3)$$

where

$$\begin{aligned} \bar{a}_{nm} &= j^{n+1} \int_V \bar{J}_V(kr') j_n(kr') Y_{nm}^e(\theta', \varphi') dV' \\ \bar{b}_{nm} &= j^{n+1} \int_V \bar{J}_V(kr') j_n(kr') Y_{nm}^o(\theta', \varphi') dV' \end{aligned} \quad (4)$$

are the even,  $a_{nm}$ , and odd,  $b_{nm}$ , spherical harmonic coefficients for the magnetic vector potential and  $Y_{nm}^e$  and  $Y_{nm}^o$  are the even and odd orthonormal spherical harmonics. The electric vector potential can be written similarly as:

$$F^P = \frac{-j\varepsilon}{2} \frac{e^{-jkr}}{r} \sum_{n=0}^{\infty} \sum_{m=0}^n c_{nm}^P(\theta', \varphi') Y_{nm}^e(\theta, \varphi) + d_{nm}^P(\theta', \varphi') Y_{nm}^o(\theta, \varphi), \quad (5)$$

with

$$\begin{aligned} \bar{c}_{nm} &= j^{n+1} \int_V \bar{M}_V(kr') j_n(kr') Y_{nm}^e(\theta', \varphi') dV' \\ \bar{d}_{nm} &= j^{n+1} \int_V \bar{J}_V(kr') j_n(kr') Y_{nm}^o(\theta', \varphi') dV' \end{aligned} \quad (6)$$

as the even,  $c_{nm}$ , and odd,  $d_{nm}$ , spherical harmonic coefficients for the electric vector

potential.

### B. Computation of the spherical harmonic coefficients

The multi-cell and single cell SHIP uses the dipoles model of the SAT-COM reflector antennas to compute the  $a_{nm}$ ,  $b_{nm}$ ,  $c_{nm}$  and  $d_{nm}$  coefficients for each cell  $P$ . The current description of the dipoles in the case of the magnetic vector current distribution,  $\bar{J}_V$ , is:

$$\bar{J}_V = \begin{cases} \bar{u} I_0 e^{i l_{ph}} & -l/2 \leq l' \leq l/2 \\ 0 & l' < -l/2 \\ 0 & l' > l/2 \end{cases}, \quad (7)$$

where  $\bar{u}$  is the directional unit vector for an infinitesimal dipole of length  $l$ ,  $I_0$  is the amplitude of the current distribution, and  $l_{ph}$  is the phase of the current distribution. The value of  $\bar{u}$  is determined from the NEC-REF code. For a dipole of constant current distribution, the even spherical harmonic coefficient becomes:

$$\bar{a}_{nm}^A = j^{n+1} \bar{u} I_0 e^{i l_{ph}} \int_l j_n(kr') Y_{nm}^e(\theta', \varphi') dl'. \quad (8)$$

A similar analysis can be used for the electric vector current distribution.

### C. Computing of environmental fields given the spherical harmonic representation

The final step in the SHIP is the use of the spherical harmonic coefficients as an input into the UTD-based such as NEC-BSC. In general, the spherical harmonic coefficients contain all phase information. This phase information includes the phase of the dipole model itself as well as the phase due to an offset from the dipole model's origin. Transferring this information to NEC-BSC should be done in such a manner that the ray origins of the spherical harmonic representation for the individual cells be properly located close to the geometric center or the center weighted by dipole model amplitudes. NEC-BSC can then translate the antenna from its origin to the desired destination.

### D. Truncation of spherical harmonic expansion and computational times

The truncation of the infinite series in Equation (3) and (5) must be handled properly to obtain reasonable accuracy for the

electromagnetic fields. The truncation limit [7,8,9],  $N_{max}$ , is:

$$N_{max,1} = kr_1 + 3\ln(\pi + kr_1), \quad (9)$$

where  $r_1$  is the maximum dimension of the cell enclosing the cell dipoles and the cell origin. As the electrical size of the antenna increases or the antenna is offset from the cell origin,  $N_{max,1}$  increases as well as the number of spherical harmonic coefficients. The number of spherical harmonics coefficients is:

$$N_{H1} = N_{max,1}^2 \approx (kr_1)^2 \quad \text{if } kr > 1. \quad (10)$$

The NEC-BSC computational time is dependent upon the number of harmonics, cells, and complexity of the ray traces. For either complex geometries or large number of cells, the ray tracing computational times will dominant the computational time of the spherical harmonic wave expansion. The computational times for a single cell is [3],

$$T_1 = CN_{H1}^\beta = C(kr_1)^{2\beta}, \quad (11)$$

where  $\beta \approx 1.4$  and  $C$  is a constant dependent on the complexity of the ray trace and hardware.

For multi-cell SHIP, the number of harmonics in each of the  $P$  cells is given as:

$$\begin{aligned} N_{HP} &= N_{max,P}^2 = (kr_p)^2 = \left(k \frac{r_1}{P^{1/3}}\right)^2 \\ &= \frac{N_{H1}}{P^{2/3}} \quad \text{if } kr_p \gg 1. \end{aligned} \quad (12)$$

Since the UTD code must trace rays from each of the  $P$  origins, the total computational time for the multi-cell SHIP is:

$$\begin{aligned} T_P &= P \left( CN_{HP}^\beta \right) = PC \left( \frac{N_{H1}}{P^{2/3}} \right)^\beta \\ &= P^{1-2\alpha\beta} T_1 = F_P T_1, \end{aligned} \quad (13)$$

where

$$F_P = P^{1-2\alpha\beta} = P^{1-2(1/3)(1.45)} = P^{+0.033}, \quad (14)$$

is the factor to convert the single cell to the  $P$  cell computational time.

### E. Multi-level cell approach for reflector antenna

The multi-cell SHIP approach uses an edge modification technique to handle correctly the reflector plate for the SAT-COM reflector antenna. The reflector plate and feed source is incorporated into the analysis by placing them as

an environmental element in NEC-BSC. NEC-BSC must compute the reflector, diffraction, and scattering of the dipoles model and SHIP due to these elements. In the case of the reflector plate with the NEC-REF dipole model, this environmental element is placed very close to the dipole sources in NEC-BSC. The large number of resulting ray traces from this model provides the higher accuracy levels; avoiding a decrease of accuracy due to shadowing. In the case of the reflector plate with the SHIP model, a multi-level SHIP procedure was developed to maintain the desired accuracy and ensures the desired increase in performance.

The multi-level SHIP procedure uses an edge modification to maintain the desired accuracy due to the reflector plate. The approach uses two different cells grouping as seen in Fig. 3. The first cell grouping uses larger cell sizes to model the NEC-REF dipoles, which are located near the center of the reflector plate. The first group of cells maintains SHIP's computational enhancement. The second cell grouping uses smaller cell size to model the NEC-REF dipole, which are located near the edge of the reflector plate. The second cell grouping produces a large number of ray traces, which increases the accuracy in the side lobe regions. The first and second cell groupings maintain the overall accuracy in the broadside region.

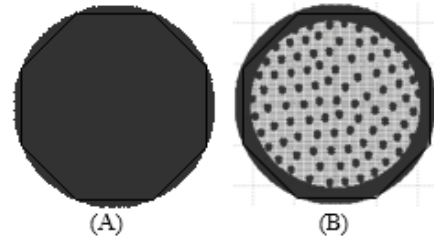


Fig. 3. (A) Infinitesimal dipole representation of a reflector dish currents, and (B) spherical harmonics representation of reflector dish currents.

## III. NUMERICAL RESULTS

This section illustrates the SHIP procedure to model reflector antennas within Navy topsides. The SAT-COM reflector antenna connects the US Navy to NATO satellites. The antenna is modeled as a source feed in front of a



reflector. The NEC-REF dipole model of the SAT-COM reflector antenna is between 40,000-100,000 dipoles depending upon the desired level of accuracy. To get a realistic model of the reflector, a reflector plate must also be included near the dipoles in NEC-BSC. This reflector plate is modeled as a flat PEC plate in the NEC-BSC code. For the examples below, the SAT-COM reflector antenna will be integrated onto two examples of Navy topside environments, a generic ship and the LHA-1 amphibious assault craft. A comparison is made between the reduced computational time for the SHIP procedures versus the standard hybrid approach using NEC-REF and NEC-BSC. The UTD codes computational times will be analyzed based on a far zone pattern for all 360 angles.

The first example is the analysis of the SAT-COM reflector antenna on the generic ship. The NEC-BSC representation of the GENERIC SHIP with placement of the reflector antenna is shown in Fig. 4. The antenna is the small red plate near the front mast. In this example, the reflector PEC plate has been removed, so all NEC-REF or SHIP sources are in the far-field to NEC-BSC.

In Figs. 5 and 6, the far-field (magnitude and phase) results for SHIP accurately compares to the NEC-REF/NEC-BSC analysis approach. The results are for left-handed polarization. The SHIP analysis used 392 cells to accurately represent the same field behavior as NEC-REF using  $\sim 94,000$  dipoles. The difference in computational times in NEC-BSC between SHIP and NEC-REF dipoles for the same accuracy is 15 minutes ( $P=392$ ) versus 1,672 minutes (NEC-REF $\sim 94,000$ ).



Fig. 4. NEC-BSC representation of a generic ship and location of SAT-COM reflector antenna.

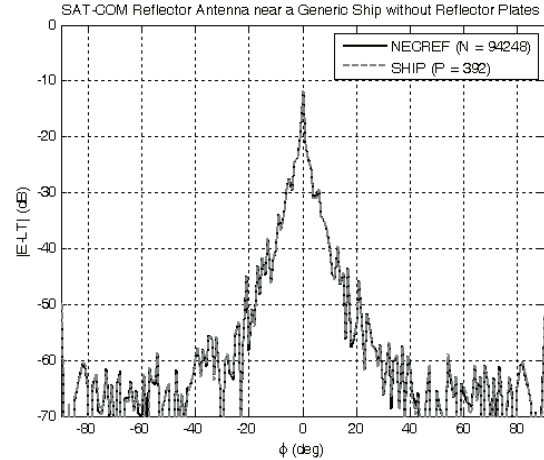


Fig. 5.  $|E-LT|$  (dB) of the SAT-COM reflector antenna near a generic ship without a reflector.

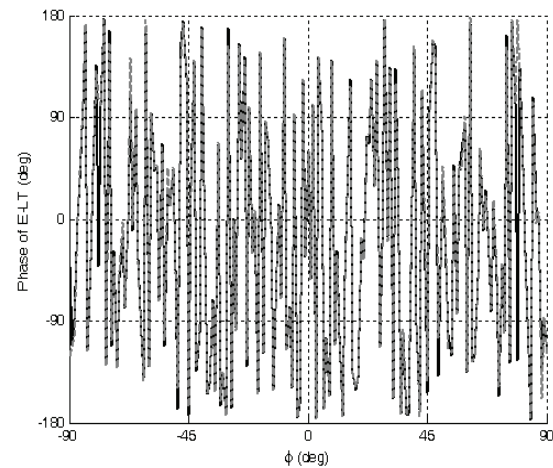


Fig. 6. Phase of E-LT (deg) of the SAT-COM reflector antenna near generic ship without reflector plates.

The second example models the SAT-COM reflector antenna near the Navy's LHA-1 amphibious assault ship. The reflector antenna of interest is shown in red in Fig. 7, surrounded by the rest of the topside environment simulated in NEC-BSC. Again, in this example, there is no reflector plate near the NEC-REF sources. All dipole sources are in the far-field to any NEC-BSC geometries.

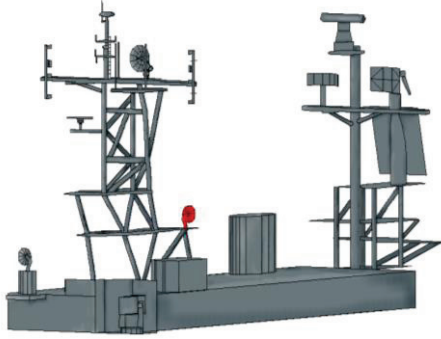


Fig. 7. NEC-BSC representation of the topside on the LHA-1 and location of the SAT-COM reflector antenna.

In Figs. 8 and 9, the far-field (magnitude and phase) has been computed using the NEC-REF/NEC-BSC analysis approach and SHIP. Again, the SHIP analysis required only 392 multiple cells to accurately represent the same field behavior as NEC-REF using ~94,248 dipoles. The difference in computational times in NEC-BSC between SHIP and NEC-REF dipoles for the similar accuracy is 2.8 hours ( $P=392$ ) versus 415 hours (NEC-REF~94248).

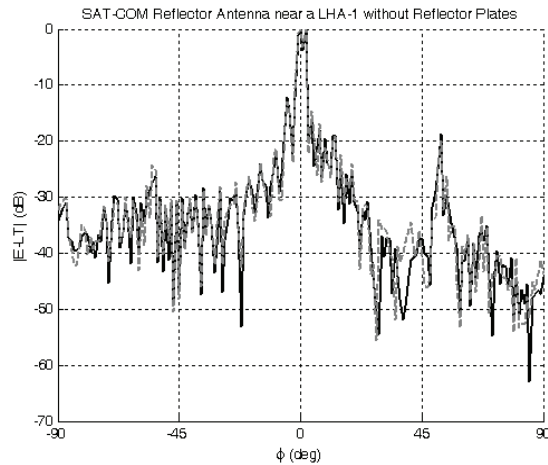


Fig. 8.  $|E-LT|$  (dB) of the SAT-COM reflector antenna near a LHA-1 without reflector plates.

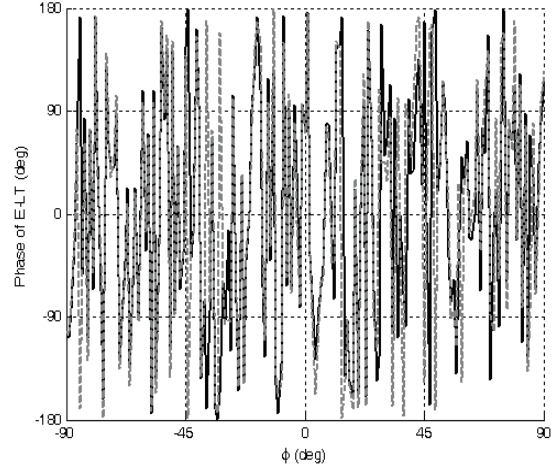


Fig. 9. Phase of E-LT (deg) of the reflector antenna near a LHA-1 without reflector plates.

In the last example, the reflector plates are used in the NEC-BSC model on a generic ship. Since the flat PEC plate is in the near field to the NEC-REF dipole model, the edge modification routine must be taken into account. This will result in a greater number of cells but with higher accuracy in the side lobes region. In Figs. 10 and 11, the far-field (magnitude and phase) results for SHIP accurately compares to the NEC-REF/NEC-BSC analysis approach, and are for left-handed polarization. The SHIP analysis used 392 dipoles to represent (~5%) the NEC-REF dipole model of ~94,000 dipoles. The multi-level SHIP analysis used 16,208 multiple cells to accurately represent (~4.2%) the same field behavior as NEC-REF model using ~94,000 dipoles. The increase in accuracy between the multi-level SHIP and SHIP is prevalent in the side lobe regions between  $-90^\circ$  to  $-40^\circ$  and  $40^\circ$  to  $90^\circ$  degrees. The accuracy of the multi-level SHIP is greater than SHIP in this region. At  $-40^\circ$  to  $40^\circ$  degrees, SHIP and multi-level SHIP give similar accuracy with respect to NEC-REF dipole model. The difference in computational times between the SHIP and

NEC-REF dipoles model for their respective accuracy was 33 minutes ( $P=392$ ), 447 minutes ( $P=16,208$ ), versus 1,773 minutes (NEC-REF~94,000).

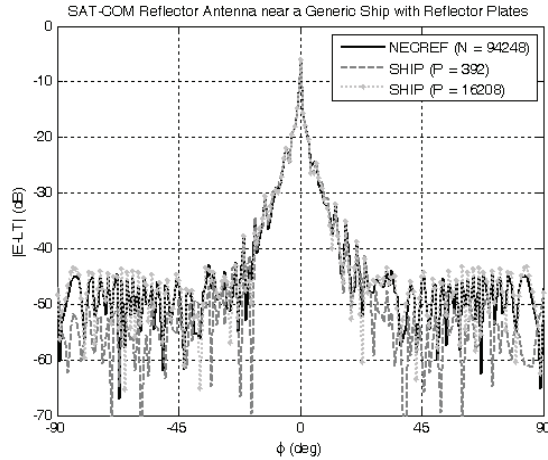


Fig. 10.  $|E-LT|$  (dB) for the SAT-COM reflector antenna (with plate) near a generic ship.

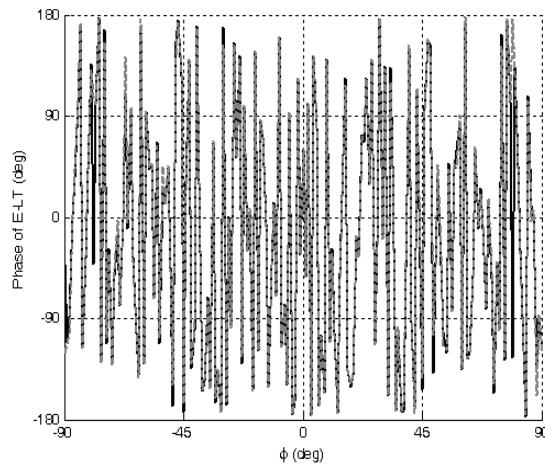


Fig. 11. Phase of E-LT (deg) for the reflector antenna with reflector plates near a generic ship.

#### IV. CONCLUSION

The traditional design of integrating SAT-COM reflector antenna into Navy topside environment begins with the use of a hybrid analysis between NEC-REF and the UTD code, NEC-BSC. Excessive computational times are often required by this analysis. Instead, a spherical wave expansion of the SAT-COM reflector antenna fields was used as an efficient interface between the NEC-REF and NEC-BSC.

Table 1 summarized the results presented here. Computational times and number of unknowns between the hybrid analysis and the SHIP method are indicated. This method saves significant computational time in the UTD code provided that the number of dipoles per cubic wavelength is large.

Table 1: Summary of results

	NEC-REF (NF)	NEC-REF (time)	SHIP (cells)	SHIP (time)	Factor Decrease
Reflector antenna on generic ship w/o plate	94,248	1,672 mins.	392	15 mins.	107 x
Reflector antenna on LHA-1 w/o plate	94,248	415 hrs.	392	2.8 hrs.	166 x
Reflector antenna on generic ship w/plate	94,248	1,773 mins.	16,208	447 mins.	3 x

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