Scattering Analysis of Periodic Composite Metallic and Dielectric Structures with Synthetic Basis Functions

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Abstract ─ Synthetic basis functions method (SBFM) is used in this paper to analyze scattering properties of periodic arrays composed of composite metallic and dielectric structures based on EFIE-PMCHW equation. Compared to traditional method of moment (MoM) based on volume integral equations (VIE) or surface integral equations (SIE), SBFM uses fewer synthetic basis functions to approximate scattering properties of a target which decreases the number of unknowns as well as memory cost significantly. Auxiliary sources are introduced to imitate the mutual coupling effects between different blocks. By solving targets' responses to these auxiliary sources, scattering solution space will be determined. Then, singular value decomposition (SVD) is adopted to extract synthetic basis functions' coefficients matrix from scattering solution space. For periodic structures, synthetic basis functions of each block are exactly the same which means previously computed coefficients matrix can be recycled; therefore, SBFM is of great advantages in analyzing large scale periodic mixed problems.

Index Terms ─ Periodic structures, PMCHW formulation, scattering properties, singular value decomposition.

I. INTRODUCTION

Surface integral equation solvers based on MoM are often adopted in the field of scattering analysis. However, standard MoM implemented with low order basis functions (e.g., RWG [1]) is hardly applied to electrically large targets for the rapid increase of computational complexity and memory cost. To solve this problem, fast multi-pole method (FMM) [2,3] and its extension (multilevel fast multi-pole method, MLFMM [4,5]) made great progresses. These algorithms are based on the sub-domain basis functions, aiming to accelerate the computational efficiency of evaluating interactions among blocks. Another approach dealing with electrically large problems is reducing the number of unknowns (e.g., characteristic basis function method, CBFM [6] and synthetic basis function method, SBFM [7-9]). These approaches firstly divide the target into several blocks according to its physical and geometrical features. Then, high order functions, which are usually linear combinations of low order functions, are adopted to describe the target's electromagnetic properties.

SBFM was first put forward by Matekovits in 2001 [7]. Not until its systematical representation was published in 2007 [9], this approach caught much more attention. Based on sub-domain decomposition, SBFM introduces the concept of degree of freedom (DOF) in its solution space to control the generation of each block's synthetic basis functions. More importantly, synthetic basis functions can be recycled in periodic system which can improve the computational efficiency significantly. There had been many applications of SBFM both in radiating and scattering analysis since 2007 [10-14].

Although SBFM has been put forward for more than ten years, researches on this approach are mainly laid on two kinds of applications: perfectly electric conducting (PEC) and dielectric structures. For example, paper [9] utilized EFIE and SBFM to analyze large complex conducting structures, while paper [14] applied SBFM to the scattering analysis of inhomogeneous dielectric bodies based on generalized surface integral equations (GSIE) [13,14]. However, in this paper, emphasis is laid on the third application: mixed problems. SBFM is used to analyze scattering properties of large scale periodic structures composed of different kinds of mediums based on EFIE-PMCHW formulation. Compared to the GSIE used in paper [14], PMCHW formulation is adopted in the process of addressing dielectric problems in this paper, which lowers the computational accuracy to some extent but decreases the number of unknowns drastically, and thus, improves the computational efficiency. What's more, the paper also explores principles on how to get synthetic basis functions' solution space for mixed problems based on equivalence theorem. Compared to traditional analysis of mixed problems based on MoM and VIE or SIE, this approach not only reduces the number of unknowns as well as computational time

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sharply, but also can make a compromise between complexity and accuracy.

II. THEORY OF SBFM

SBFM is an improved algorithm on the basis of MoM which uses synthetic basis functions to replace traditional low order basis functions.

$$f\left(\mathbf{X}\right)=g. \tag{1}$$

Assume Equation (1) as a linear integral formula and $\boldsymbol{X}$ as the unknowns defined on the surface of a target. In traditional MoM, RWG functions are usually used to discretize $\boldsymbol{X}$ and to make inner product which leads to the following matrix equations:

$$\boldsymbol{X}=\sum_{n=1}^{N}x_n\boldsymbol{f}_n(\boldsymbol{r}'), \tag{2}$$

$$\begin{aligned}\left[\left\langle \boldsymbol{f}_m(\boldsymbol{r}), f\left(\boldsymbol{f}_n(\boldsymbol{r}')\right)\right\rangle\right]_{N\times N}\left[x_n\right]_{N\times 1}\\ =\left[\left\langle \boldsymbol{f}_m(\boldsymbol{r}), g\right\rangle\right]_{N\times 1},\end{aligned} \tag{3}$$

where $\boldsymbol{f}_n(\boldsymbol{r})$ stands for the RWG functions and $<\boldsymbol{A},\boldsymbol{B}>$ represents the inner product of $\boldsymbol{A}$ and $\boldsymbol{B}$.

However, in SBFM, synthetic functions are employed in place of RWG functions which can sharply decrease the number of unknowns. There are three main steps in SBFM [9,15].

A. Domain decomposition

To reduce the number of unknowns, the first step is breaking the target into several geometrically small and simple sub-blocks. Compared to dealing with the whole target, it will be much easier to analyze these sub-blocks. Notably, each sub-block should share the same geometrical shape for the sake of making each block's synthetic basis functions the same. For periodic structures, a single element of the structure is usually defined as a sub-block.

Suppose a target is divided into N_{SB} blocks, as is shown in Fig. 1, and label each block's surface as S block for the sake of simplicity. Considering the fact that there may be public edges between different S blocks, L block needs to be introduced on behalf of these public edges. Denote the number of L blocks is N_{LB}.

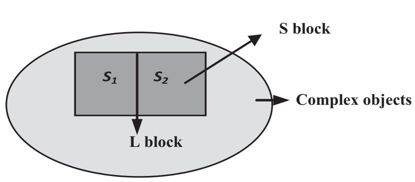


Fig. 1. Sketch map of domain decomposition.

B. Construct synthetic basis functions

Traditionally, low order functions (e.g., RWG) are used to discretize the unknowns defined on blocks. However, this kind of discretization usually needs quantities of basis functions which will greatly increase computational complexity and memory cost. Thus, in SBFM, we try to approximate the unknowns with less high order functions. High order functions, also called synthetic basis functions, vary with the geometrical features of blocks.

For S blocks, synthetic basis functions are set as linear combinations of a series of RWG functions:

$$\boldsymbol{F}_m^b\left(\boldsymbol{r}\right)=\sum_{k=1}^{N_b}P_{k,m}^b\boldsymbol{f}_k\left(\boldsymbol{r}\right), m=1,\cdots,M_b; b=1,\cdots,N_{SB}, \tag{4}$$

where N_b is the number of RWG functions defined on block b and M_b refers to the number of synthetic basis functions defined on the block. As for N_{SB}, it represents the number of S-blocks.

Obviously, N_b is determined by the triangulation carried on block b while M_b is determined by SBFM's truncation error which will be discussed in Section IV. To reach the purpose of reducing the number of unknowns, we want M_b<<N_b.

For L blocks, define synthetic basis functions as RWG functions since RWG functions already meet the current continuity conditions on public edges:

$$\boldsymbol{F}_m^b\left(\boldsymbol{r}\right)=\boldsymbol{f}_m\left(\boldsymbol{r}\right), m=1,\cdots,N_b; b=1,\cdots,N_{LB}, \tag{5}$$

where N_b represents the number of RWG basis functions defined on block b.

C. Establish synthetic matrix equation

Having got synthetic basis functions, Equations (2) and (3) can be re-written as:

$$\begin{cases}\boldsymbol{X}=\sum\limits_{b=1}^{N_{SB}+N_{LB}}\sum\limits_{n=1}^{M_b}x_n\boldsymbol{F}_n(\boldsymbol{r}')=\sum\limits_{u=1}^{N_{SBF}}y_u\boldsymbol{F}_u(\boldsymbol{r}')\\ N_{SBF}=\sum\limits_{b=1}^{N_{SB}+N_{LB}}M_b\end{cases}, \tag{6}$$

$$\begin{aligned}\left[\left\langle \boldsymbol{F}_m(\boldsymbol{r}), f\left(\boldsymbol{F}_n(\boldsymbol{r}')\right)\right\rangle\right]_{N_{SBF}\times N_{SBF}}\left[y_u\right]_{N_{SBF}\times 1}\\ =\left[\left\langle \boldsymbol{F}_m(\boldsymbol{r}), g\right\rangle\right]_{N_{SBF}\times 1},\end{aligned} \tag{7}$$

where N_{SBF} is the total number of unknowns in Equation (7), which depend on the number of synthetic basis functions defined on each block. While in traditional MoM, the total number of unknowns N_{MoM} is calculated in the following equation:

$$N_{MoM}=\sum_{b=1}^{N_{SB}+N_{LB}}N_b. \tag{8}$$

For each block, the number of synthetic basis functions is less than that of RWG functions: M_b<<N_b. Thus, the total number of unknowns of MoM and SBFM meet the condition: N_{SBF} <<N_{MoM}, which means memory cost of SBFM will be much smaller than that of MoM.

Further, in MoM, computational complexity and memory cost are $O(N^3)$ and $O(N^2)$ respectively, where

N represents the number of unknowns. Since the main difference between MoM and SBFM is the selection of basal functions, the method of evaluating SBFM's computational complexity and memory cost is the same to that of MoM. Therefore, compared to MoM, SBFM has great advantages both in computational complexity and memory cost for $N_{SBF} << N_{MoM}$.

Compared to Equation (3), Equation (7) replaces all the RWG functions with the synthetic basis functions got in B and is called synthetic matrix equation.

Taking block b_1 and b_2 into consideration, elements in Equation (7) can be calculated from that in Equation (3):

$$\left[\left\langle \boldsymbol{F}_m(\boldsymbol{r}), f\left(\boldsymbol{F}_n(\boldsymbol{r}')\right)\right\rangle\right]_{b_1 b_2} = \left[\boldsymbol{F}^{b_1}\right]^T \left[\left\langle \boldsymbol{f}_m(\boldsymbol{r}), f\left(\boldsymbol{f}_n(\boldsymbol{r}')\right)\right\rangle\right]_{b_1 b_2} \boldsymbol{F}^{b_2}, \tag{9}$$

$$\left[\left\langle \boldsymbol{F}_m(\boldsymbol{r}), g\right\rangle\right]_{b_1 b_2} = \left[\boldsymbol{F}^{b_1}\right]^T \left[\left\langle \boldsymbol{f}_m(\boldsymbol{r}), g\right\rangle\right]_{b_1 b_2}, \tag{10}$$

where $\boldsymbol{F}^b$ represents the matrix of synthetic basis functions' coefficients defined on block b:

$$\begin{cases} \boldsymbol{F}^b = \left[\boldsymbol{F}_1^b, \boldsymbol{F}_2^b, \cdots, \boldsymbol{F}_{M_b}^b\right] \\ \boldsymbol{F}_m^b = \left[P_{1,m}^b, P_{2,m}^b, \cdots, P_{N_b,m}^b\right]^T \end{cases}. \tag{11}$$

Obviously, synthetic basis functions' solution space influences accuracy and complexity. Thus, how to determine the solution space is of vital importance and this will be discussed in Section IV.

III. SURFACE INTEGRAL EQUATION

For scattering problems, SIE (e.g., EFIE and MFIE) are often used when it comes to PEC structures, with RWG functions being its basis functions. As for dielectric structures, VIE and volume basis functions (e.g., SWG [16]) are usually adopted. However, volume meshing usually means a large amount of unknowns which is unbearable for a personal computer. Compared to VIE, SIE can also be used to analyze homogeneous dielectric structures with its results being less accurate but much more efficient. In this paper, EFIE and PMCHW formulation are used to analyze PEC and homogenous dielectric structures respectively. Finally, mixed problems are also explored in the method of EFIE+PMCHW.

For PEC targets, EFIE is usually expressed as [17]:

$$\hat{n} \times Z^{(i)} L^{(i)}(\boldsymbol{J}_{si}) = \hat{n} \times \boldsymbol{E}_{inc}^{(i)}, \tag{12}$$

where L is the electric integral operator and defined as:

$$L(\boldsymbol{X}) = jk\int_v \left[\boldsymbol{X}(\boldsymbol{r}')G(\boldsymbol{r}-\boldsymbol{r}') + \frac{1}{k^2}\nabla'\cdot \boldsymbol{X}(\boldsymbol{r}')\nabla G(\boldsymbol{r}-\boldsymbol{r}')\right]dv'. \tag{13}$$

For homogenous dielectric targets, PMCHW formulation is compactly written as [18]:

$$\begin{cases} \hat{n} \times \left\{Z^{(i)}L^{(i)}(\boldsymbol{J}) - K^{(i)}(\boldsymbol{M}) + Z^{(j)}L^{(j)}(\boldsymbol{J}) - K^{(j)}(\boldsymbol{M})\right\} = \hat{n} \times \boldsymbol{E}_{inc}^{(i)} \\ \hat{n} \times \left\{K^{(i)}(\boldsymbol{J}) + Y^{(i)}L^{(i)}(\boldsymbol{M}) + K^{(j)}(\boldsymbol{J}) + Y^{(j)}L^{(j)}(\boldsymbol{M})\right\} = \hat{n} \times \boldsymbol{H}_{inc}^{(i)} \end{cases}, \tag{14}$$

where K is the magnetic integral operator and defined as:

$$K(\boldsymbol{X}) = \int_v \left[\boldsymbol{X}(\boldsymbol{r}') \times \nabla G(\boldsymbol{r}-\boldsymbol{r}')\right]dv'. \tag{15}$$

For composite metallic and dielectric targets, SIE is described as [18]:

$$\begin{cases} \hat{n}_{ij} \times \left\{ \begin{array}{l} \sum\limits_{k=0,k\neq i}^{n} \left[Z^{(i)}L^{(i)}(\boldsymbol{J}_{sik}) - K^{(i)}(\boldsymbol{M}_{sik})\right] + \\ \sum\limits_{k=0,k\neq j}^{n} \left[Z^{(j)}L^{(j)}(\boldsymbol{J}_{sjk}) - K^{(j)}(\boldsymbol{M}_{sjk})\right] \end{array} \right\} \Bigg|_{S_{ij}} = \hat{n}_{ij} \times (\boldsymbol{E}_{inc}^{(i)} - \boldsymbol{E}_{inc}^{(j)})\Big|_{S_{ij}} \\ \hat{n}_{ij} \times \left\{ \begin{array}{l} \sum\limits_{k=0,k\neq i}^{n} \left[K^{(i)}(\boldsymbol{J}_{sik}) + Y^{(i)}L^{(i)}(\boldsymbol{M}_{sik})\right] + \\ \sum\limits_{k=0,k\neq j}^{n} \left[K^{(j)}(\boldsymbol{J}_{sjk}) + Y^{(j)}L^{(j)}(\boldsymbol{M}_{sjk})\right] \end{array} \right\} \Bigg|_{S_{ij}} = \hat{n}_{ij} \times (\boldsymbol{H}_{inc}^{(i)} - \boldsymbol{H}_{inc}^{(j)})\Big|_{S_{ij}} \end{cases}, \tag{16}$$

where S_{ij} refers to the interface of medium i and j, $\boldsymbol{J}$ and $\boldsymbol{M}$ represent electric and magnetic currents on the interface respectively.

Specifically, magnetic currents $\boldsymbol{M}$ and incident magnetic field $\boldsymbol{H}_{inc}$ tend to be zero when medium j is PEC bodies. Then, Equation (16) can be re-written as:

$$\hat{n}_{ij} \times \left\{ \sum_{k=0,k\neq i,j}^{n} \left[Z^{(i)}L^{(i)}(\boldsymbol{J}_{sik}) - K^{(i)}(\boldsymbol{M}_{sik})\right] + Z^{(i)}L^{(i)}(\boldsymbol{J}_{sij}) \right\}\Bigg|_{S_{ij}} = \hat{n}_{ij} \times \boldsymbol{E}_{inc}^{(i)}\Big|_{S_{ij}}. \tag{17}$$

Equations (16) and (17) is called EFIE-PMCHW equation for composite metallic and dielectric targets.

To solve these SIEs listed above, $\boldsymbol{J}$ and $\boldsymbol{M}$ need to be discretized by basis functions firstly:

$$\boldsymbol{J}(\boldsymbol{r}') = \sum_{n=1}^{N} \alpha_n \boldsymbol{f}_n(\boldsymbol{r}'), \boldsymbol{M}(\boldsymbol{r}') = \sum_{n=1}^{N} \beta_n \boldsymbol{f}_n(\boldsymbol{r}'). \tag{18}$$

Then, according to MoM and Galerkin principle, these SIEs can be transformed into the following linear matrix equations.

- PEC Targets

$$ZI = V, \tag{19}$$

where Z is the impedance matrix and V is the exciting matrix:

$$\begin{cases} Z_{mn} = \left\langle \boldsymbol{f}_m(\boldsymbol{r}), Z_0 L(\boldsymbol{f}_n(\boldsymbol{r}'))\right\rangle \\ V_m = \left\langle \boldsymbol{f}_m(\boldsymbol{r}), \boldsymbol{E}_{inc}(\boldsymbol{r})\right\rangle \end{cases}, \tag{20}$$

where Z_0 is the wave impedance of free space.

- Dielectric Targets

$$\begin{bmatrix} z^{EJ} & z^{EM} \\ z^{HJ} & z^{HM} \end{bmatrix} \begin{bmatrix} I^J \\ I^M \end{bmatrix} = \begin{bmatrix} V^E \\ V^H \end{bmatrix}. \tag{21}$$

The impedance matrix and exciting matrix will be obtained through Equations (22) and (23):

$$\begin{cases} z_{mn}^{EJ} = \left\langle \boldsymbol{f}_m(\boldsymbol{r}), \left[Z^{(i)} L^{(i)}(\boldsymbol{f}_n(\boldsymbol{r}')) + Z^{(j)} L^{(j)}(\boldsymbol{f}_n(\boldsymbol{r}')) \right] \right\rangle \\ z_{mn}^{EM} = \left\langle \boldsymbol{f}_m(\boldsymbol{r}), -\left[K^{(i)}(\boldsymbol{f}_n(\boldsymbol{r}')) + K^{(j)}(\boldsymbol{f}_n(\boldsymbol{r}')) \right] \right\rangle \\ z_{mn}^{HJ} = \left\langle \boldsymbol{f}_m(\boldsymbol{r}), \left[K^{(i)}(\boldsymbol{f}_n(\boldsymbol{r}')) + K^{(j)}(\boldsymbol{f}_n(\boldsymbol{r}')) \right] \right\rangle \\ z_{mn}^{HM} = \left\langle \boldsymbol{f}_m(\boldsymbol{r}), \left[Y^{(i)} L^{(i)}(\boldsymbol{f}_n(\boldsymbol{r}')) + Y^{(j)} L^{(j)}(\boldsymbol{f}_n(\boldsymbol{r}')) \right] \right\rangle \end{cases}, \tag{22}$$

$$\begin{cases} V_m^E = \left\langle \boldsymbol{f}_m(\boldsymbol{r}), \boldsymbol{E}_{inc}(\boldsymbol{r}) \right\rangle \\ V_m^H = \left\langle \boldsymbol{f}_m(\boldsymbol{r}), \boldsymbol{H}_{inc}(\boldsymbol{r}) \right\rangle \end{cases}. \tag{23}$$

- Composite Metallic and Dielectric Targets

$$\begin{bmatrix} Z^{CC} & Z^{CD} & Z^{CM} \\ Z^{DC} & Z^{DD} & Z^{DM} \\ Z^{MC} & Z^{MD} & Z^{MM} \end{bmatrix} \begin{bmatrix} I^C \\ I^D \\ I^M \end{bmatrix} = \begin{bmatrix} V^C \\ V^D \\ V^M \end{bmatrix}, \tag{24}$$

where V^C is the exciting matrix of PEC bodies and V^D and V^M represent the exciting matrixes of dielectric bodies:

$$\begin{cases} V_m^C = \left\langle \boldsymbol{f}_m(\boldsymbol{r}), \boldsymbol{E}_{inc}(\boldsymbol{r}) \right\rangle, \ m = 1, ..., N^C \\ V_m^D = \left\langle \boldsymbol{f}_m(\boldsymbol{r}), \boldsymbol{E}_{inc}(\boldsymbol{r}) \right\rangle, \ m = 1, ..., N^D \\ V_m^M = \left\langle \boldsymbol{f}_m(\boldsymbol{r}), \boldsymbol{H}_{inc}(\boldsymbol{r}) \right\rangle, \ m = 1, ..., N^D \end{cases}. \tag{25}$$

As for impedance matrix, it is calculated similar to that in PEC and dielectric targets. For the sake of simplicity, formulas of each element are not listed here but can be found in paper [18]:

$$\begin{cases} \boldsymbol{J}(\boldsymbol{r}') = \sum_{b=1}^{N_B} \sum_{m=1}^{M_b^J} j_m^b \boldsymbol{F}_m^b(\boldsymbol{r}') \\ \boldsymbol{M}(\boldsymbol{r}') = \sum_{b=1}^{N_B} \sum_{m=1}^{M_b^M} m_m^b \boldsymbol{F}_m^b(\boldsymbol{r}') \end{cases}. \tag{26}$$

In MoM, electric and magnetic currents are discretized by RWG functions, as is shown in Equation (18), while in SBFM, currents are discretized by synthetic basis functions, as is shown in Equation (26).

Then, all the matrix Equations (19), (21) and (24) can be transformed into synthetic matrix equations in the following method:

$$\begin{cases} \left[Z_{SBF} \right] \left[j \right] = \left[V_{SBF} \right] \\ \left[Z_{SBF} \right]_{b_1 b_2} = \left[\boldsymbol{F}^{b_1} \right]^T \left[Z \right]_{b_1 b_2} \boldsymbol{F}^{b_2} \\ \left[V_{SBF} \right]_{b_1} = \left[\boldsymbol{F}^{b_1} \right]^T \left[V \right]_{b_1} \end{cases}, \tag{27}$$

where $[Z]_{b1b2}$ is the impedance matrix of block b_1 and b_2 calculated by RWG functions. Since Equation (27) tackles the target block by block, the scale of equation is much smaller than that in traditional MoM.

Solving Equation (27), currents coefficients matrix corresponding to synthetic basis functions [j] will be available. Then, according to Equation (26), the original currents coefficients matrix corresponding to RWG functions [I] can be transformed from [j] and synthetic basis functions' coefficients matrix [P];

$$I_k^b = \sum_{m=1}^{M_b} P_{k,m}^b j_m^b, \quad b = 1, 2, \cdots, N_B. \tag{28}$$

Having got the coefficients of initial RWG functions, it is easy to do some further processing (radiating field, scattering properties, etc.).

Overall, Equation (28) shows that the key of SBFM is determining the number of synthetic basis functions M_b and their coefficients matrix [P]. This will be discussed in Section IV.

IV. GENERATION OF SYNTHETIC BASIS FUNCTIONS

Paper [19] claims that DOF of a certain scatterer's scattering field is restricted to [D_∞,D_1], where D_1 donates the total number of RWG functions defined on the surface of the scatterer. Thus, it is possible to approximate the scattering fields' solution space with fewer synthetic functions [14]. To find proper synthetic functions, there are two main steps: setting auxiliary sources and SVD.

Taking an isolated block into consideration, exciting voltage not only comes from the incident plane wave but also comes from mutual coupling effects of other blocks. Paper [14] points out mutual coupling effects can be imitated by a series of auxiliary sources based on equivalence theorem. Figure 2 demonstrates that a block is surrounded by an enclosed stereoscopic space, and a series of auxiliary sources (RWG functions) are defined on the surface of the space. By evaluating the block's responses to these auxiliary sources, solution space will be available. Synthetic basis functions' coefficients are these independent columns of the solution space. To extract independent elements, SVD will be adopted.

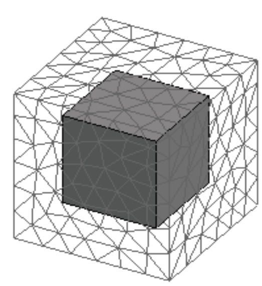


Fig. 2. An isolated block is surrounded by auxiliary sources.

- PEC Targets

Paper [9] shows the method of determining PEC's solution space. Referring to this method, re-write it as:

$$\begin{cases} \left[I^1 \right] = \left[Z^1 \right]^{-1} \left(V^1 - V^{1e} \right) \\ V^{1e} = \left\langle \boldsymbol{f}_m(\boldsymbol{r}), Z_0 L(\boldsymbol{f}_\alpha(\boldsymbol{s})) \right\rangle \end{cases}, \tag{29}$$

where V^{1e} represents exciting voltage caused by mutual coupling effects.

It should be noticed that Equation (29) is slightly different with that in paper [9], but corresponds to Equation (12).

Similarly, for dielectric and composite structures, solution space can also be obtained through PMCHW formulation. However, in comparison to PEC bodies, both auxiliary electric and magnetic sources are needed for dielectric bodies.

- Dielectric Targets

$$\begin{bmatrix} z^{EJ} & z^{EM} \\ z^{HJ} & z^{HM} \end{bmatrix}^1 \begin{bmatrix} I^J \\ I^M \end{bmatrix}^1 = \begin{bmatrix} V^E - V^{E1e} \\ V^H - V^{H1e} \end{bmatrix}, \tag{30}$$

where V^{E1e} and V^{H1e} represent exciting voltage caused by auxiliary electric and magnetic sources:

$$\begin{cases} V^{E1e} = \\ \begin{bmatrix} \left\langle \boldsymbol{f}_m(\boldsymbol{r}), \left[Z^{(i)} L^{(i)}(\boldsymbol{f}_\alpha(\boldsymbol{s})) + Z^{(j)} L^{(j)}(\boldsymbol{f}_\alpha(\boldsymbol{s})) \right] \right\rangle \\ + \left\langle \boldsymbol{f}_m(\boldsymbol{r}), -\left[K^{(i)}(\boldsymbol{f}_\alpha(\boldsymbol{s})) + K^{(j)}(\boldsymbol{f}_\alpha(\boldsymbol{s})) \right] \right\rangle \end{bmatrix} \\ V^{H1e} = \\ \begin{bmatrix} \left\langle \boldsymbol{f}_m(\boldsymbol{r}), \left[K^{(i)}(\boldsymbol{f}_\alpha(\boldsymbol{s})) + K^{(j)}(\boldsymbol{f}_\alpha(\boldsymbol{s})) \right] \right\rangle + \\ \left\langle \boldsymbol{f}_m(\boldsymbol{r}), \left[Y^{(i)} L^{(i)}(\boldsymbol{f}_\alpha(\boldsymbol{s})) + Y^{(j)} L^{(j)}(\boldsymbol{f}_\alpha(\boldsymbol{s})) \right] \right\rangle \end{bmatrix} \end{cases}. \tag{31}$$

- Composite Metallic and Dielectric Targets

$$\begin{bmatrix} Z^{CC} & Z^{CD} & Z^{CM} \\ Z^{DC} & Z^{DD} & Z^{DM} \\ Z^{MC} & Z^{MD} & Z^{MM} \end{bmatrix}^1 \begin{bmatrix} I^C \\ I^D \\ I^M \end{bmatrix}^1 = \begin{bmatrix} V^C - V^{C1e} \\ V^D - V^{D1e} \\ V^M - V^{M1e} \end{bmatrix}, \tag{32}$$

where V^{C1e} is exciting voltage caused by auxiliary electric sources around PEC bodies, V^{D1e} and V^{H1e} represent exciting voltage caused by auxiliary electric and magnetic sources around dielectric bodies:

$$\begin{cases} V^{C1e} = \left[Z^{CC} \right]^{1e} + \left[Z^{CD} \right]^{1e} + \left[Z^{CM} \right]^{1e} \\ V^{D1e} = \left[Z^{DC} \right]^{1e} + \left[Z^{DD} \right]^{1e} + \left[Z^{DM} \right]^{1e} \\ V^{M1e} = \left[Z^{MC} \right]^{1e} + \left[Z^{MD} \right]^{1e} + \left[Z^{MM} \right]^{1e} \end{cases}. \tag{33}$$

Once the solution space is got, coefficients of synthetic basis functions defined in Equation (4) will be obtained by SVD. Denote the solution space as R, then:

$$R = U \rho V^H, \rho = diag(\rho_1, \rho_2, ..., \rho_N), \tag{34}$$

where ρ_i (i=1,2,…,N) represent singular values of R and $\rho_1 > \rho_2 >, \ldots, > \rho_N$.

Coefficients $[P]$ of synthetic basis functions are elements of column vectors of U. Set truncation error as ρ_{SBF} and take the top M_1 columns of U as effective independent coefficients, where $\rho_{M1}/\rho_1 < \rho_{SBF}$. Thus, the number of synthetic basis functions is M_1 and their coefficients:

$$\left[P \right] = \left\{ \left[U_1 \right], \left[U_2 \right], ..., \left[U_{M_1} \right] \right\}. \tag{35}$$

For periodic structures, coefficient space of each block is exactly the same since all the blocks share the same geometrical features. Thus, computational efficiency will be greatly improved.

V. NUMERICAL RESULTS AND VALIDATION

As a part of scattering analysis, bistatic RCS is calculated by SBFM in this section and three examples are given to validate the accuracy of SBFM. Besides, results of commercial software Feko and MoM are also given here for comparison. Before presenting the examples, it is noteworthy that all the arrays listed in examples are placed in the plane *xoy*, with the incident wave coming from +*z* and polarizing +*x*. Observing plane is set as *xoz* and *yoz*. What's more, to fully validate the accuracy of SBFM, frequency of incident wave and truncation error are set differently.

Example 1 shows a 5×5 array composed of PEC paraboloids, as is shown in Fig. 3. Row and column gap between elements are 2λ and focal distance and radius of the paraboloid are both set as 0.5λ. Frequency of incident wave is 3 GHz. Surface of the target is discretized into 5875 triangles and 8375 edges by setting the maximum size of meshing as 0.1λ. In SBFM, 525 synthetic basis functions are used to analyze the target with truncation error ρ_{SBF}=0.1. Results of the example are shown in Fig. 4 and Table 1.

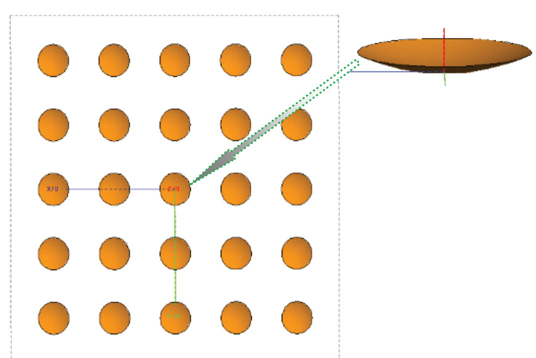


Fig. 3. 5×5 PEC array of paraboloids.

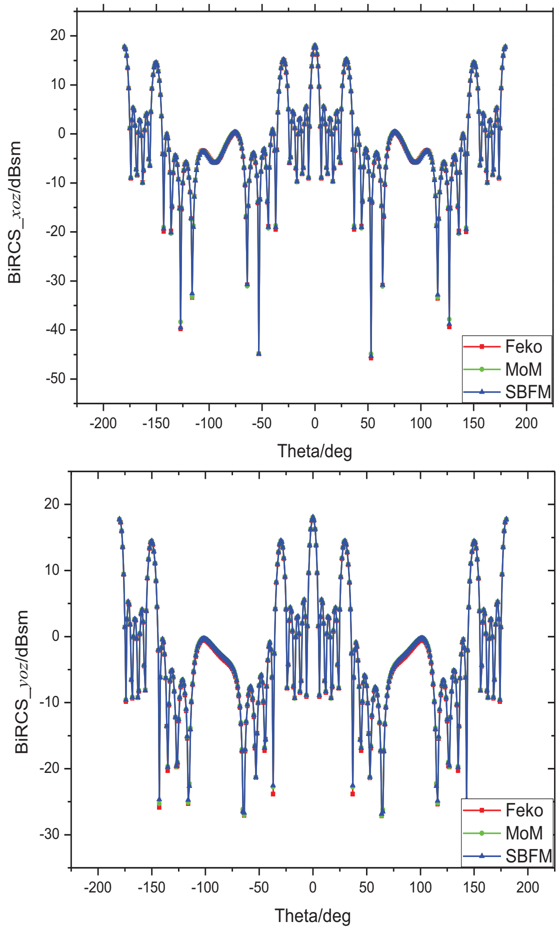


Fig. 4. Bi-RCS of 5×5 PEC array.

Table 1: Properties of MoM and SBFM in example 1

	Number of Unknowns	RAM Cost of Impedance Matrix	Time of Solving Matrix Equation	Total Elapsed Time
SBFM	525	7.69 MB	0.06s	168.53s
MoM	8375	1.79 GB	260.22s	431.86s

Example 2 is a 3×3 array consisting of 9 dielectric rectangular grooves with relative permittivity ε_r=3.6, as is shown in Fig. 5. Size of the outer rectangle is $0.4\lambda\times0.4\lambda\times0.3\lambda$ while size of the inner slot is $0.25\lambda\times0.25\lambda\times0.2\lambda$. Row and column gap are set as 0.9λ. Frequency of incident wave is 1 GHz. There are 3582 triangles and 5373 edges defined on the surface. Since both electric and magnetic currents are there on the surface, 10746 RWG functions in total are needed in MoM. By setting truncation error ρ_{SBF}=0.2, only 9 synthetic basis function for each block and 81 for total are used in SBFM. Results of this example will be displayed in Fig. 6 and Table 2.

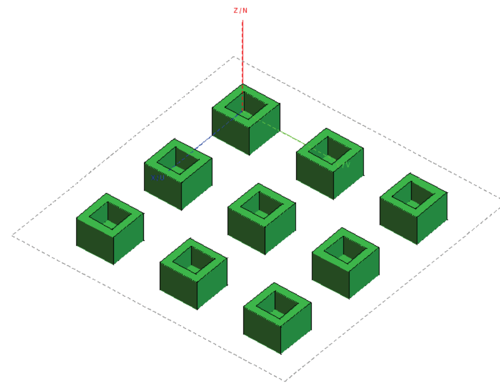


Fig. 5. 3×3 dielectric array of grooves.

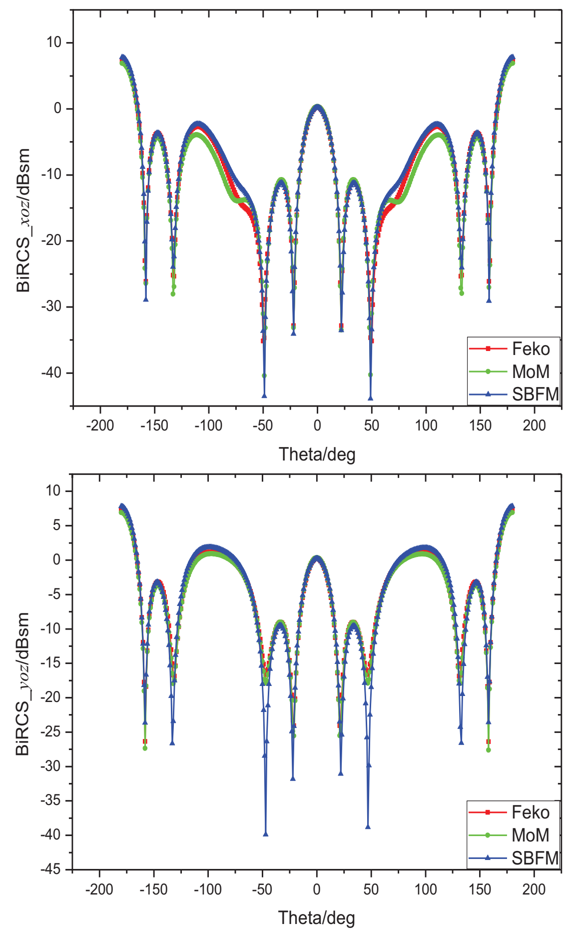


Fig. 6. Bi-RCS of 3×3 dielectric array.

Table 2: Properties of MoM and SBFM in example 2

	Number of Unknowns	RAM Cost of Impedance Matrix	Time of Solving Matrix Equation	Total Elapsed Time
SBFM	81	0.19 MB	0.001 s	571.99s
MoM	10746	3.19 GB	596.61 s	1240.96s

Example 3 is a 3×3 composite array of monopole antennas. 9 blocks are included in the array with its row and column gap being 0.9λ, as is shown in Fig. 7. For each block, a PEC monopole (0.5λ in length) is placed on a dielectric rectangular base (0.4λ×0.4λ×0.3λ). Relative permittivity of the base is 2.6 and incident plane wave's frequency is 1 GHz. Since both PEC and dielectric structures are there in the system, the process of meshing needs to be carried on independently. For PEC structures, the surface is discretized into 720 triangles and 1080 edges while for dielectric structures, 2574 triangles and 3861 edges are obtained. Thus, 8802 RWG functions are needed totally in MoM. When it comes to SBFM, three kinds of auxiliary sources need to be set independently in this system: electric sources around PEC structures, electric and magnetic sources around dielectric structures. By solving the solution space and SVD, 44 synthetic basis functions are adopted for each block, where truncation error ρ_{SBF}=0.15. Results of this example are demonstrated in Fig. 8 and Table 3.

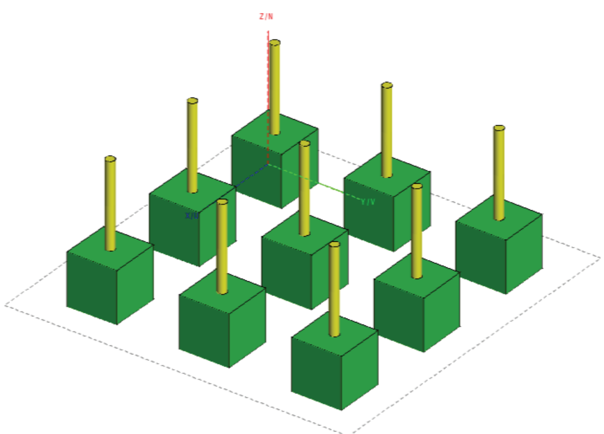
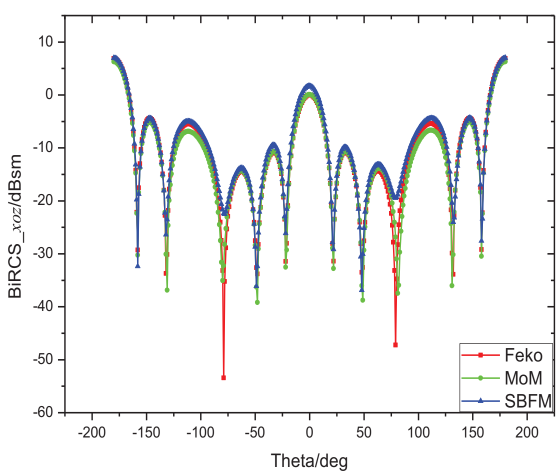


Fig. 7. 3×3 composite array of monopole antennas.



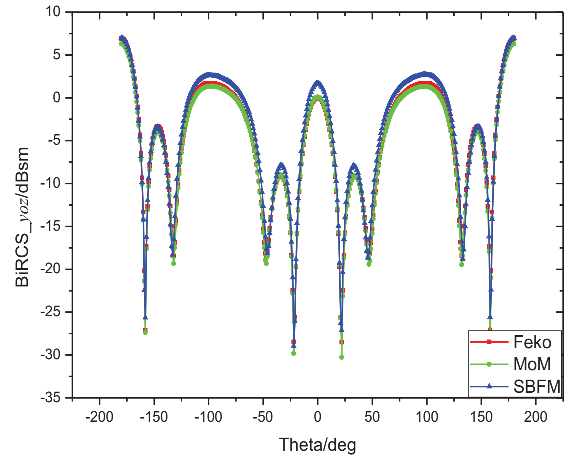


Fig. 8. Bi-RCS of 3×3 composite array.

Table 3: Properties of MoM and SBFM in example 3

	Number of Unknowns	RAM Cost of Impedance Matrix	Time of Solving Matrix Equation	Total Elapsed Time
SBFM	396	4.41 MB	0.03 s	398.63s
MoM	8802	2.14 GB	308.63 s	575.68s

Among these Bi-RCS pictures, θ=0° and θ=180° represent backward and forward scattering respectively. These examples exhibit that results of SBFM yield well to that of Feko and MoM except for slight differences in lateral scattering. Table 1-3 give computational properties of MoM and SBFM. In MoM, the number of unknowns is depend on the triangulation carried on the surface of the target, while in SBFM, it depends on the value of truncation error and auxiliary sources. Generally, higher truncation error means less synthetic basis functions and thus less memory cost. As for computing time, elapsed time of filling impedance matrix and solving matrix equation are the two main parts in MoM while in SBFM, except for the two parts listed above, elapsed time of SVD is the third part. Since the scale of matrix equation in SBFM is much smaller than that in MoM, the solving process will be much faster.

Obviously, SBFM has great advantages over MoM in memory cost as well as computing time especially for periodic targets. More importantly, it is noteworthy that the scale of periodic array which SBFM can address is not restricted to what listed in the paper. To get the results of MoM for comparison, examples in the paper are relatively small scale problems since computational complexity and memory cost of larger scale problems are unbearable for a personal computer. Thus, compared to traditional MoM, SBFM makes it possible to analyze large scale problems in personal computer.

VI. CONCLUSION

SBFM is used in this paper to analyze scattering properties of periodic structures based on EFIE-PMCHW formulation. Three kinds of applications are introduced here: PEC, dielectric and composite metallic and dielectric structures. Results verify the accuracy and efficiency of the algorithm. Compared to traditional MoM, SBFM not only reduces quantities of unknowns and memory but also enables us to make a balance between accuracy and efficiency through truncation error. Besides, SBFM can also be combined with other fast approaches such as MLFMA and adaptive integral method (AIM) to further improve its properties. Thus, SBFM makes it possible to analyze large scale targets on a personal computer.

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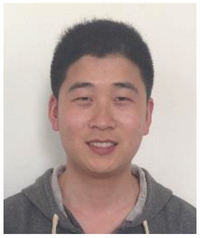

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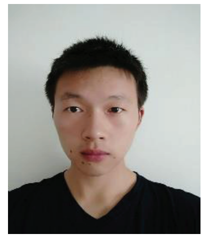

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