Modal Analysis of Frequency Selective Surface Containing Ring Loops

Mehran Shamaei Samani, Reza Sarraf Shirazi, and Gholamreza Moradi

Department of Electrical Engineering
Amirkabir University of Technology, Tehran, Iran
Mehranshamaey@aut.ac.ir, Sarraf@aut.ac.ir, Ghmoradi@aut.ac.ir

Abstract **─** Modal analysis of frequency selective surface (FSS) containing ring loops arranged periodically along any two skewed coordinates, will be calculated. In proposed modal analysis, a compatible set of orthonormal mode functions used for ring loops in addition to Floquet modes which are used in every infinite two dimensional arrays. This set of modes provides faster convergence in solving integral equation in accordance with this boundary condition problem. Calculations are compared with results of CST (Finite Integration Technique) and HFSS (Finite Element Method) full-wave simulators and they are in good agreement with each other.

Index Terms **─** Frequency Selective Surface (FSS), ring loop, Substrate Integrated Waveguide (SIW), TE and TM polarizations.

I. INTRODUCTION

Frequency selective surface (FSS) as spatial filter [1-2], should have low insertion loss in passband and sharp roll-off rejection in out-of-band. However, these two characteristics are sufficient in filter designing, in spatial form of filters, stability of these characteristics about polarization and angle of incident is more important [3]. One of the significant parameters in this stability is symmetrical geometry. As circular loop is the most symmetrical object in all candidate elements in FSS designing, it is preferred to implement a stable FSS [4].

For the analytical calculation of electromagnetics problems such as boundary value conditions, modal analysis is the inseparable part of computations [5-6], so in the analysis of this paper, modal method is utilized to extract reflection coefficient.

Boundary value problem of open ended circular waveguide phased array, were solved generally by Amitay and Galindo [5]. They used the Ritz-Galerkin method to solve the integral equation.

For the case of ring loops or apertures, if the method similar to Amitay and Galindo is utilized, the convergence is so slowly [7]. To speed up the convergence, in this paper, the fields of ring loop are expanded to a set of orthonormal mode functions which are compatible with geometry of ring loops and satisfy loop boundary conditions [7-8].

In this paper, thin perfectly conducting loops distributed periodically along any two skewed (nonorthogonal) coordinates is exposed with a plane wave of arbitrary polarization incident from any oblique angle like whatever seen in Fig. 1. The distance between loop elements should be less than $\lambda/2$. In the following parts of this paper, all the details of this method will be described and results are compared with simulations of CST and HFSS.

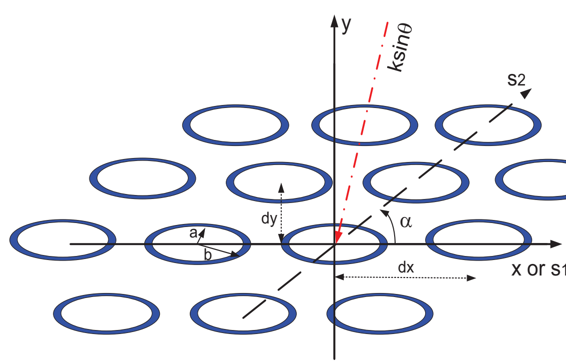

Fig. 1. Geometry of FSS with ring loops.

II. THEORITICAL ANALYSIS

According to Fig. 1, array of ring loops which is distributed periodically along skewed coordinates s_1 and s_2, is illuminated by a plane wave with propagation vector, $\overline{k}$. The angle between $\overline{k}$ and the normal to the surface is θ and between s_1 and projection of $\overline{k}$ is φ.

Near the array of FSS, the electromagnetic fields must satisfy the periodicity condition of Floquet theorem. By employing this theorem, the scalar wave equation (the time dependence is eliminated) is in the form of [7]:

$$\psi_{pq} = e^{-j(U_{pq}x+V_{pq}y+W_{pq}z)}, \tag{1}$$

where

Submitted On: October 26, 2014
Accepted On: June 18, 2015

$$U_{pq} = k\sin\theta\cos\varphi + \frac{2\pi p}{dx}, \tag{2}$$

$$V_{pq} = k\sin\theta\sin\varphi + \frac{2\pi q}{dy} - \frac{2\pi p}{dx\tan\alpha}, \tag{3}$$

$$W_{pq} = \begin{cases} \sqrt{k^2 - T_{pq}^2} & for \quad k^2 > T_{pq}^2 \\ -j\sqrt{T_{pq}^2 - k^2} & for \quad k^2 < T_{pq}^2 \end{cases}, \tag{4}$$

which

$$T_{pq}^2 = U_{pq}^2 + V_{pq}^2,$$

p and q are Floquet indices. W_{pq}, the modal propagation constants, is positive for propagating modes and negative imaginary for evanescent modes. The space between elements and the direction of k, determine the number of propagating modes. The electric field components transverse to the z axis (TE and TM mode functions), then can be expressed in terms of ψ as:

$$\bar{\Phi}_{pqTE} = \frac{1}{\sqrt{d_x d_y}}(\frac{V_{pq}}{T_{pq}}\hat{x} - \frac{U_{pq}}{T_{pq}}\hat{y})\psi_{pq} \quad TE \quad \text{modes}, \tag{5}$$

$$\bar{\Phi}_{pqTM} = \frac{1}{\sqrt{d_x d_y}}(\frac{U_{pq}}{T_{pq}}\hat{x} + \frac{V_{pq}}{T_{pq}}\hat{y})\psi_{pq} \quad TM \quad \text{modes}. \tag{6}$$

These fields are related to each other by modal impedance, as:

$$\eta_{pqTE} = \frac{k\eta_0}{W_{pq}}, \tag{7}$$

$$\eta_{pqTM} = \frac{W_{pq}\eta_0}{k}. \tag{8}$$

A plane wave with unit intensity electric field and in direction of (θ, φ), can be expressed as the sum of two E and H polarized plane waves. These plane waves, correspond to TE and TM Floquet modes with $p,q=0$. So we have for incident electric and magnetic fields:

$$\bar{E}^i = \sum_{r=1}^{2} A_{00r}\bar{\Phi}_{00r}, \tag{9}$$

$$\bar{H}^i = \sum_{r=1}^{2} \frac{A_{00r}}{\eta_{00r}}(\hat{z}\times\bar{\Phi}_{00r}), \tag{10}$$

In the above expressions, r designates TE and TM modes. A_{00r} is the magnitude of component. With R_{pqr} as the reflection coefficient, the scattered fields can be expressed as:

$$\bar{E}^s = \sum_p\sum_q\sum_{r=1}^{2} R_{pqr}\bar{\Phi}_{pqr}, \tag{11}$$

$$\bar{H}^s = -\sum_p\sum_q\sum_{r=1}^{2} \frac{R_{pqr}}{\eta_{pqr}}(\hat{z}\times\bar{\Phi}_{pqr}). \tag{12}$$

The unknown reflection coefficient, R_{pqr}, according to orthonormality of modes is:

$$R_{pqr} = \eta_{pqr}\iint_{Loop} \hat{z}\times\bar{H}^s\cdot\bar{\Phi}_{pqr}^* da, \tag{13}$$

$\bar{\Phi}_{pqr}^*$ is the complex conjugate of $\bar{\Phi}_{pqr}$. The boundary conditions then force that:

$$\bar{E}^i + \bar{E}^s = 0 \quad over \quad each \quad loop, \tag{14}$$

$$2\hat{z}\times(\bar{H}^i + \bar{H}^s) = \bar{K} \quad over \quad each \quad loop. \tag{15}$$

Substitution (9), (10) and (13) into (14) yields the integral equation:

$$\begin{aligned} &\sum_{r=1}^{2} A_{00r}\bar{\Phi}_{00r} = \\ &-\sum_p\sum_q\sum_{r=1}^{2}\eta_{pqr}\bar{\Phi}_{pqr}\iint_{Loop}\hat{z}\times\bar{H}^s\cdot\bar{\Phi}_{pqr}^* da. \end{aligned} \tag{16}$$

Here we replace the induced current, $-\hat{z}\times\bar{H}^s$ with another set of orthonormal mode functions which are compatible to geometry of loops and satisfy boundary conditions, as:

$$-\hat{z}\times\bar{H}^s = \sum_m\sum_n\sum_{l=1}^{2}(B_{mnl}\bar{\psi}_{mnl}) + B_{TEM}\bar{\psi}_{TEM}. \tag{17}$$

$l=1$ and $l=2$ designate TE and TM mode respectively, then $\bar{\psi}_{mnl}$ and $\bar{\psi}_{TEM}$ for a ring loop can be expressed as [9]:

$$\begin{aligned} \bar{\psi}_{mn}^{TE} &= \hat{\rho}\frac{n}{\rho}Z_n(k_{cmn}'\rho)\sin n\varphi \\ &+\hat{\varphi}k_{cmn}'Z_n'(k_{cmn}'\rho)\cos n\varphi, \end{aligned} \tag{18}$$

$$\begin{aligned} \bar{\psi}_{mn}^{TM} &= \hat{\rho}k_{cmn}\bar{Z}_n'(k_{cmn}\rho)\sin n\varphi \\ &+\hat{\varphi}\frac{n}{\rho}\bar{Z}_n(k_{cmn}\rho)\cos n\varphi, \end{aligned} \tag{19}$$

$$\bar{\psi}_{TEM} = \frac{1}{\rho\ln b/a}\left[\hat{x}\cos\varphi + \hat{y}\sin\varphi\right], \tag{20}$$

which [10]:

$$Z_n = \frac{\sqrt{\pi\varepsilon_n}}{2\sqrt{(\frac{J_n(k_{cmn}'b)}{J_n(k_{cmn}'a)})^2(1-(\frac{n}{a})^2) - (1-(\frac{n}{b})^2)}}$$
$$\times(Y_n'(k_{cmn}'a)J_n(k_{cmn}'\rho) - J_n'(k_{cmn}'a)Y_n(k_{cmn}'\rho)),$$

$$\bar{Z}_n = \frac{\sqrt{\pi\varepsilon_n}}{2\sqrt{(\frac{J_n(k_{cmn}b)}{J_n(k_{cmn}a)}) - 1}}$$
$$\times(Y_n(k_{cmn}a)J_n(k_{cmn}\rho) - J_n(k_{cmn}a)Y_n(k_{cmn}\rho)),$$

k_{cmn} is the mth root of:

$$Y_n'(k_c')J_n'(k_c' b/a) - J_n'(k_c')Y_n'(k_c' b/a) = 0.$$

And k_{cmn}' is the mth root of:

$$Y_n(k_c)J_n(k_c b/a) - J_n(k_c)Y_n(k_c b/a) = 0.$$

If both sides of (16) multiplied by the complex conjugate of $\bar{\psi}_{mnl}$ and $\bar{\psi}_{TEM}$, after that the products are integrated over the ring loop, the results are [7]:

$$\sum_{r=1}^{2} A_{00r} C_{00r}^{*TEM} = -\sum_{p}\sum_{q}\sum_{r=1}^{2} \eta_{pqr} C_{pqr}^{*TEM} \iint_{Loop} -\hat{z}\times\bar{H}^{s}\cdot\bar{\Phi}_{pqr}^{*} da, \tag{21}$$

$$\sum_{r=1}^{2} A_{00r} C_{00r}^{*MNL} = -\sum_{p}\sum_{q}\sum_{r=1}^{2} \eta_{pqr} C_{pqr}^{*MNL} \iint_{Loop} -\hat{z}\times\bar{H}^{s}\cdot\bar{\Phi}_{pqr}^{*} da, \tag{22}$$

where

$$C_{pqr}^{*MNL} = \iint_{Loop} \bar{\psi}_{MNL}\cdot\bar{\Phi}_{pqr}^{*} da, \tag{23}$$

$$C_{pqr}^{*TEM} = \iint_{Loop} \bar{\psi}_{TEM}\cdot\bar{\Phi}_{pqr}^{*} da. \tag{24}$$

The integral equation can be rewritten as:

$$\left[Z_{MNL}^{mnl}\right]\left[B_{mnl}\right] = \left[D_{mnl}\right]. \tag{25}$$

Which Z is a square matrix in which the row index is designated by M,N,L and the column index is designated by m,n,l. As there is no $mn=00$ mode, the TEM mode is called 00 mode (Don't care l) in matrix implementation. The matrix elements are given by:

$$\left[Z_{MNL}^{mnl}\right] = \sum_{p}\sum_{q}\sum_{r=1}^{2} \eta_{pqr} C_{pqr}^{*MNL} C_{pqr}^{mnl}, \tag{26}$$

and

$$\left[D_{mnl}\right] = \sum_{r=1}^{2} A_{00r} C_{00r}^{*mnl}. \tag{27}$$

C_{pqr}^{mnl} is the coupling coefficient between modes in the both sides of interface and it can be seen as scalar products of these modes. Keep in mind that:

r=l=1 related to TE modes.

r=l=2 related to TM modes.

mn=00 related to TEM mode.

After a lot of mathematics calculations the close form of these integrals extract as bellow:

$$C_{pq1}^{mn1} = \left\langle \psi_{mn}^{TE}\cdot\varphi_{pq}^{TE}\right\rangle = \frac{k_{cmn}^{'}}{2\sqrt{dxdy}}\left\{(\frac{V}{T})(X_1+X_2)+(\frac{-U}{T})(Y_1+Y_2)\right\},$$

$$C_{pq2}^{mn1} = \left\langle \psi_{mn}^{TE}\cdot\varphi_{pq}^{TM}\right\rangle = \frac{k_{cmn}^{'}}{2\sqrt{dxdy}}\left\{(\frac{U}{T})(X_1+X_2)+(\frac{V}{T})(Y_1+Y_2)\right\},$$

$$C_{pq1}^{mn2} = \left\langle \psi_{mn}^{TM}\cdot\varphi_{pq}^{TE}\right\rangle = \frac{k_{cmn}}{2\sqrt{dxdy}}\left\{(\frac{V}{T})(\bar{X}_1+\bar{X}_2)+(\frac{-U}{T})(\bar{Y}_1+\bar{Y}_2)\right\},$$

$$C_{pq2}^{mn2} = \left\langle \psi_{mn}^{TM}\cdot\varphi_{pq}^{TM}\right\rangle = \frac{k_{cmn}}{2\sqrt{dxdy}}\left\{(\frac{U}{T})(\bar{X}_1+\bar{X}_2)+(\frac{V}{T})(\bar{Y}_1+\bar{Y}_2)\right\},$$

$$C_{pq1}^{TEM} = \left\langle \psi^{TEM}\cdot\varphi_{pq}^{TE}\right\rangle = \frac{-2j\pi}{\ln(b/a)}\left(\frac{V}{T}\cos\varphi_T-\frac{U}{T}\sin\varphi_T\right)\left\{J_1(Ta)-J_1(Tb)\right\},$$

$$C_{pq2}^{TEM} = \left\langle \psi^{TEM}\cdot\varphi_{pq}^{TM}\right\rangle = \frac{-2j\pi}{\ln(b/a)}\left(\frac{U}{T}\cos\varphi_T+\frac{V}{T}\sin\varphi_T\right)\left\{J_1(Ta)-J_1(Tb)\right\},$$

$$X_1 = \begin{cases} \varepsilon_{\frac{n+1}{2}}(-1)^{n+1/2} S_1 \dfrac{bB_{n+1,b}-aB_{n+1,a}}{T^2-k_{cmn}^{'\,2}} & (n \quad odd) \\ -2j(-1)^{n/2} S_1 \dfrac{bB_{n+1,b}-aB_{n+1,a}}{T^2-k_{cmn}^{'\,2}} & (n \quad even) \end{cases},$$

$$X_2 = \begin{cases} \varepsilon_{\frac{n-1}{2}}(-1)^{n-1/2} S_2 \dfrac{bB_{n-1,b}-aB_{n-1,a}}{T^2-k_{cmn}^{'\,2}} & (n \quad odd) \\ -2j(-1)^{n-2/2} S_2 \dfrac{bB_{n-1,b}-aB_{n-1,a}}{T^2-k_{cmn}^{'\,2}} & (n \quad even) \end{cases},$$

$$Y_1 = \begin{cases} \varepsilon_{\frac{n+1}{2}}(-1)^{n+1/2} C_1 \dfrac{bB_{n+1,b}-aB_{n+1,a}}{T^2-k_{cmn}^{'\,2}} & (n \quad odd) \\ 2j(-1)^{n/2} C_1 \dfrac{bB_{n+1,b}-aB_{n+1,a}}{T^2-k_{cmn}^{'\,2}} & (n \quad even) \end{cases},$$

$$Y_2 = \begin{cases} -\varepsilon_{\frac{n-1}{2}}(-1)^{n-1/2} C_2 \dfrac{bB_{n-1,b}-aB_{n-1,a}}{T^2-k_{cmn}^{'\,2}} & (n \quad odd) \\ -2j(-1)^{n-2/2} C_2 \dfrac{bB_{n-1,b}-aB_{n-1,a}}{T^2-k_{cmn}^{'\,2}} & (n \quad even) \end{cases},$$

$$\bar{X}_1 = \begin{cases} -\varepsilon_{\frac{n+1}{2}}(-1)^{n+1/2} S_1 \dfrac{b\bar{B}_{n+1,b}-a\bar{B}_{n+1,a}}{T^2-k_{cmn}^{2}} & (n \quad odd) \\ 2j(-1)^{n/2} S_1 \dfrac{b\bar{B}_{n+1,b}-a\bar{B}_{n+1,a}}{T^2-k_{cmn}^{2}} & (n \quad even) \end{cases},$$

$$\bar{X}_2 = \begin{cases} \varepsilon_{\frac{n-1}{2}}(-1)^{n-1/2} S_2 \dfrac{b\bar{B}_{n-1,b}-a\bar{B}_{n-1,a}}{T^2-k_{cmn}^{2}} & (n \quad odd) \\ -2j(-1)^{n-2/2} S_2 \dfrac{b\bar{B}_{n-1,b}-a\bar{B}_{n-1,a}}{T^2-k_{cmn}^{2}} & (n \quad even) \end{cases},$$

$$\bar{Y}_1 = \begin{cases} -\varepsilon_{\frac{n+1}{2}}(-1)^{n+1/2} C_1 \dfrac{b\bar{B}_{n+1,b}-a\bar{B}_{n+1,a}}{T^2-k_{cmn}^{2}} & (n \quad odd) \\ -2j(-1)^{n/2} C_1 \dfrac{b\bar{B}_{n+1,b}-a\bar{B}_{n+1,a}}{T^2-k_{cmn}^{2}} & (n \quad even) \end{cases},$$

$$\bar{Y}_1 = \begin{cases} -\varepsilon_{\frac{n-1}{2}}(-1)^{n-1/2} C_2 \dfrac{b\bar{B}_{n-1,b}-a\bar{B}_{n-1,a}}{T^2-k_{cmn}^{2}} & (n \quad odd) \\ -2j(-1)^{n-2/2} C_2 \dfrac{b\bar{B}_{n-1,b}-a\bar{B}_{n-1,a}}{T^2-k_{cmn}^{2}} & (n \quad even) \end{cases},$$

$$B_{n\pm1,r_i} = TJ_{\substack{n+2 \\ n}}(Tr_i)Z_{n\pm1}(k_{cmn}^{'}r_i) - k_{cmn}^{'}J_{n\pm1}(Tr_i)Z_{\substack{n+2 \\ n}}(k_{cmn}^{'}r_i) \quad (r_i=a,b),$$

$$\bar{B}_{n\pm1,r_i} = TJ_{\substack{n+2\\n}}(Tr_i)\bar{Z}_{n\pm1}(k_{cmn}r_i)$$
$$-k_{cmn}J_{n\pm1}(Tr_i)\bar{Z}_{\substack{n+2\\n}}(k_{cmn}r_i) \quad (r_i = a,b),$$
$$S_1 = \pi\sin\left((n+1)\varphi_T\right),$$
$$S_2 = \pi\sin\left((n-1)\varphi_T\right),$$
$$C_1 = \pi\cos\left((n+1)\varphi_T\right),$$
$$C_2 = \pi\cos\left((n-1)\varphi_T\right),$$
$$\varphi_T = \tan^{-1}(\frac{V}{T}).$$

To calculate the reflection coefficient R_{pqr}, first, the unknown coefficient B_{mnl} and B_{TEM} should be evaluated from (25) and then these values substituted in (17). The number of modes for each region should be chosen carefully which only the significant modes are assigned [7].

III. RESULTS

According to the formulation achieved in the previous section, and for the case of geometry of Fig. 1, and for all Floquet modes which T_{pq} (transverse wave number) in them are less than 10 times the k (wave number) and for 12 lowest modes for circular loop, the reflection coefficient, is calculated. Increasing the number of modes cause to obtain more accurate response in the expense of times (Because the output of integral equation is a multiterminal network and there are a lot of nested loop in it, therefore it grows rapidly). For a new geometrical parameters and frequency band, sometimes it is time consuming procedure to find the optimized number of modes (often up to 9 and less than 14). But by determining the optimized number, for a wide range of parameters and frequency bands in the vicinity of that case, it can be ok.

The geometrical parameters of a sample FSS array are described in Table 1.

Table 1: Geometrical parameters for FSS array

Parameter	Value	Parameter	Value
a	5 (mm)	b	6 (mm)
dx	15 (mm)	dy	15 (mm)
α	90°		

The results are plotted in Fig. 2, for normal incidence (TE or TM plane wave) in compare with CST and HFSS simulators. In Fig. 3, the reflection coefficient for some θ and TE polarization is plotted. Also in Fig. 4, it is repeated for TM polarization. As it is clear in both polarizations, by increasing θ the resonant frequency reduced somewhat.

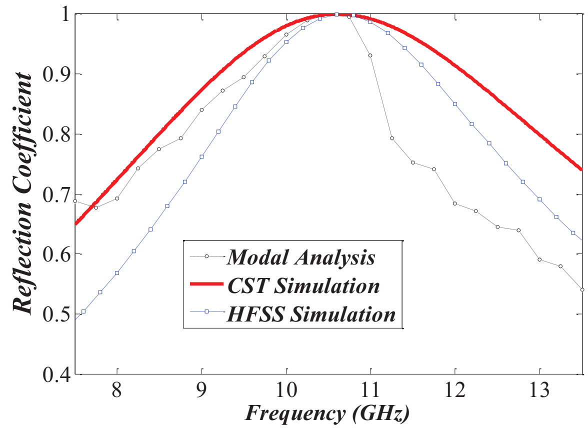

Fig. 2. Comparison of reflection coefficient between modal analysis, CST and HFSS simulation.

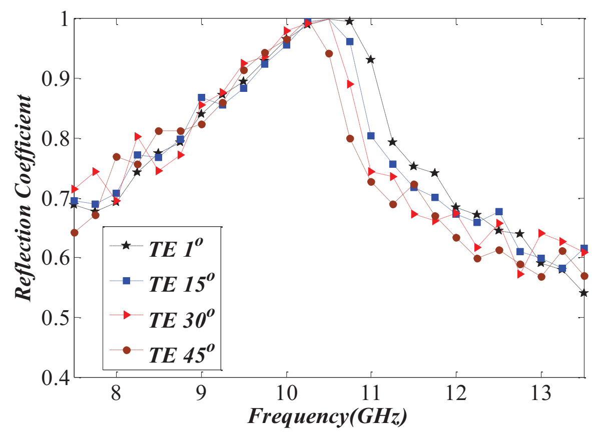

Fig. 3. Reflection coefficient for some angles of incidence and TE polarization.

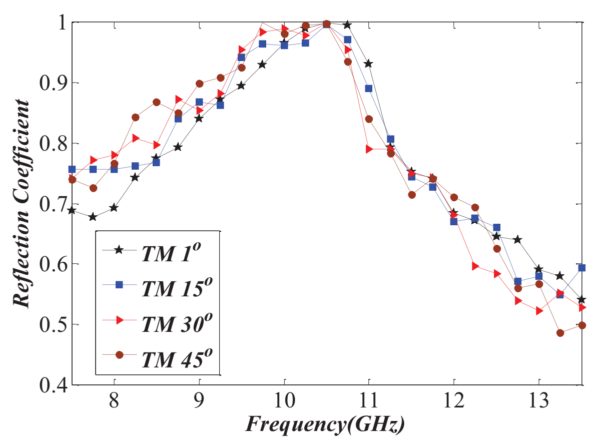

Fig. 4. Reflection coefficient for some angles of incidence and TM polarization.

IV. CONCLUSION

Modal analysis of a FSS containing ring loops is formulated and calculated. For this geometry, a set of orthonormal mode functions compatible with circular loop were defined to replace the induced current. This replacement and setting an optimized number of modes can cause to faster convergence than a simple integral equation with just Floquet modes.

The results were compared with CST and HFSS simulators and relatively good agreement was achieved. Also, as the ring element has the best geometrical symmetry, the frequency response for proposed FSS has stable characteristics about incident polarization and angle up to 45°.

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Mehran Shamaei was born in 1984 and received the B.S. and M.S. degrees from the Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran, in 2008 and 2010, respectively, all in Electrical Engineering. He is currently working toward the Ph.D. degree at same university. His research interests include frequency selective surfaces and reflect array antennas.

Reza Sarraf Shirazi was born in Yazd, Iran in 1958. He received the Ph.D. degrees in Electrical Engineering from Amirkabir University of Technology, Tehran, Iran in 2006. His main research interests are numerical Electromagnetics and radio wave propagation. He is currently an Assistant Professor with the Electrical Engineering Department, Amirkabir University of Technology. He has published several papers in the refereed journals and international conferences.

Gholamrreza Moradi was born in Shahriar, Iran in 1966. He received the Ph.D. degree in Electrical Engineering from Amirkabir University of Technology, Tehran, Iran in 2002. His main research interests are numerical Electromagnetics, antennas, active microwave and mm-wave circuits and systems. He is currently an Associate Professor with the Electrical Engineering Department, Amirkabir University of Technology. He has published several papers in the refereed journals and international conferences.