Oblique Incidence Plane Wave Scattering from Large Finite Arrays Using EV-AEF Method

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Abstract **─** An efficient method for calculating scattering patterns of large finite arrays illuminated by an obliquely incident plane wave is presented in this paper. Based on the element-varying active element factor (AEF) technique, this method takes the element mutual coupling and edge effects into account, and it results in accurate solutions. From the periodicity and distribution properties of induced currents, various elements are combined according to a certain rule. The determination of element combination is related to the plane wave incident angle. Thus, the scattering pattern of a large finite array can be calculated with superposition of combined-elements in a relatively small array. Numerical examples of one-dimensional (1-D) and two-dimensional (2-D) arrays verify the accuracy and efficiency of the proposed method.

Index Terms **─** Active element factor, large finite arrays, oblique plane wave, scattering pattern.

I. INTRODUCTION

Direct numerical methods are often used to calculate the scattering field from a small array or an infinite periodic array. Numerical methods, such as the method of moment (MoM) [1], finite element method (FEM) [2], and finite-difference time-domain (FDTD) method [3], lead to an accurate and effective solution for a small array calculation. For an infinite periodic array, only an element is required to be simulated with the Floquet's theorem or periodic Green function [4-5]. The numerical methods, however, become inefficient or even infeasible for rather large finite arrays when considering the mutual coupling effects in the whole array environment. In this case, approximate methods are needed to reduce memory requirements and computing time for array calculation.

An active element pattern (AEP) technique has been proposed to solve radiation problems considering mutual coupling effects between array elements [1]. Based on the AEP, an average active element factor (AEF) with the reduced window array (RWA) approximation is introduced to analyze the plane wave scattering problem of finite arrays on an infinite ground plane [7]. This method changes a large array scattering problem into a superposition of various simplified subarray problems. In [8], an element-varying (EV) AEF method is introduced to analyze normal plane wave scattering patterns of finite arrays on a finite ground plane. From the induced current distribution on an array with odd or even elements, the array is divided into different parts and neglects the weak mutual coupling affected by far elements. In [9] , an improved induced element pattern method (IIEPM) transforms a large finite array problem into two small ones when the normal incidence plane wave is adopted.

In this paper, instead of the case of normal incidence in [8], the scattering of an obliquely incident plane wave from a large finite array is studied with an improved EV-AEF method. Beginning with the induced current distribution in the case of oblique incidence, the whole array elements are divided into relatively small edge combined-elements and interior combined-elements for one-dimensional (1-D) problems. Different from [7], in which exterior equivalent sources are calculated as the incident field to model the edge effects of a finite array, only AEPs of edge combined-elements need to be extracted to

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essentially account for the edge effects in our method. The scattered field from a large finite array with an oblique incident plane wave can be easily calculated by a superposition of the two types of combined-elements in this small array. In addition, the relationship between the current distribution periodicity and incidence angle is analyzed when the oblique plane wave is adopted. To verify the accuracy and efficiency of the proposed method, results of some numerical examples are provided. With the proposed method, therefore, some important features of the radar cross section (RCS) signatures of 1-D and 2-D finite arrays can be examined efficiently.

II. THEORIES

A. Induced current distribution with an oblique plane wave

A 1-D array model illuminated by an oblique plane wave is shown in Fig. 1. In [8] , the induced current density on a 1-D array is symmetrical when the array is illuminated by a normal plane wave. In fact, the induced current density takes on the asymmetric distribution at oblique incidence. Figure 2 plots the induced current density, with FEKO simulation, on four finite arrays of seven, eight, nine, and ten elements, respectively, with the obliquely incident angle of $\theta_0 = 30^0$ and $\varphi_0 = 0^0$.

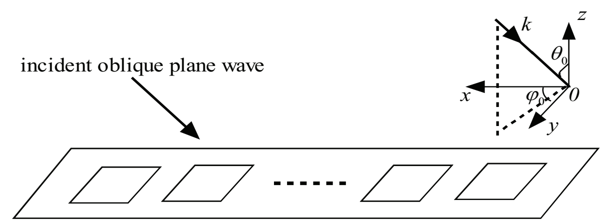
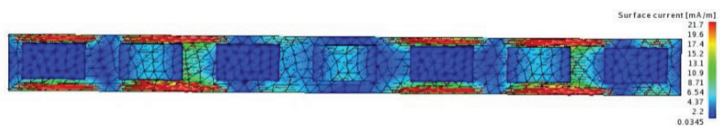
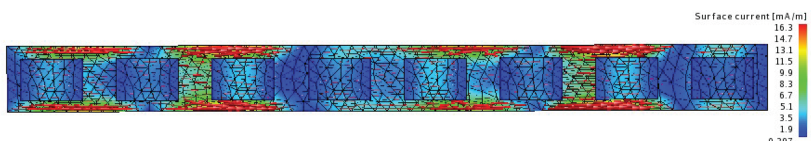


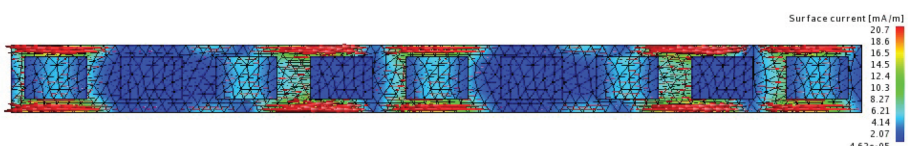
Fig. 1. Obliquely incident plane wave on a 1-D finite array.



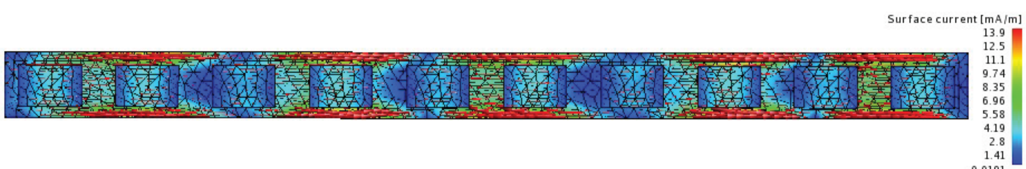
(a) 7-element array



(b) 8-element array



(c) 9-element array



(d) 10-element array

Fig. 2. Current density on 1-D arrays with incident plane wave of $\theta_0 = 30^0$ and $\varphi_0 = 0^0$ (arrows densities denote current densities).

B. Improved EV-AEF method

Based on the EV-AEF method [8] , the far electric field scattered by an array can be expressed as:

$$\boldsymbol{E}^{\mathrm{s}}_{\mathrm{total}} = \sum_{n=1}^{N} \boldsymbol{E}^{\mathrm{e}}_{n}\left(\theta,\varphi\right) e^{\mathrm{j}k\hat{\boldsymbol{r}}\cdot\boldsymbol{r}_n}\ , \tag{1}$$

where $\boldsymbol{E}^{\mathrm{e}}_{n}(\theta,\varphi)$ is the AEF field from the nth element.

For an oblique plane wave, the concept of combined-element is introduced into the EV-AEF method. A combined-element in Fig. 3 consists of several adjoining elements, and its size is determined by the current distribution periodicity.

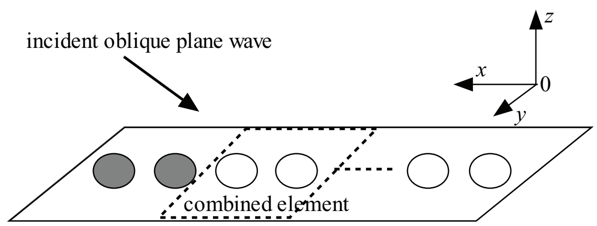


Fig. 3. Combined-element in a 1-D finite array.

Similar to the normal incidence case [8] , only a few neighboring combined-elements are involved in calculating the EV-AEF of a combined-element with an obliquely incident wave. Thus, (1) can be rewritten as:

$$\boldsymbol{E}^{s}_{\text{total}}\left(\theta,\varphi\right)=\boldsymbol{E}_{\text{E}}\left(\theta,\varphi\right)+\boldsymbol{E}_{\text{I}}\left(\theta,\varphi\right), \tag{2}$$

$$\boldsymbol{E}_{\text{E}}\left(\theta,\varphi\right)=\sum_{e=1}^{N_e}\boldsymbol{E}_{e}\left(\theta,\varphi\right)e^{\text{j}k\hat{r}\cdot r_e}, \tag{3}$$

$$\boldsymbol{E}_{\text{I}}\left(\theta,\varphi\right)=\sum_{i=1}^{N_i}\boldsymbol{E}_{i}\left(\theta,\varphi\right)e^{\text{j}k\hat{r}\cdot r_i}, \tag{4}$$

where $\boldsymbol{E}_{\text{E}}(\theta,\varphi)$ and $\boldsymbol{E}_{\text{I}}(\theta,\varphi)$ are the EV-AEF superposition of all edge combined-elements and of all interior combined-elements, respectively. N_e and N_i are the amounts of all edge combined-elements and of all interior combined-elements, respectively.

It is noted that, for oblique incident plane wave, only if the resolved component of electric field is along the 1-D array distribution, the periodicity of induced current distribution is observed and our proposed method can be applied to the scattering modeling.

C. Relationship between incident angle and combined-element properties

For the 1-D array illuminated by an oblique plane wave with $\theta_0 = 30^0$ and $\varphi_0 = 0^0$ in Fig. 1, a four-element periodicity of current distribution can be observed in Fig. 2. In the following, how to determine the element number in a current distribution periodicity will be discussed.

When the array is illuminated by an oblique plane wave with $\theta_0 = 30^0$ and $\varphi_0 = 0^0$, as shown in Fig. 4, the phase difference between two adjacent elements is $\Delta\xi = d\sin\theta_0$. When the element spacing $d = \lambda/2$ (λ is the wavelength of plane wave), the phase difference $\Delta\xi = \lambda/4$. Therefore, the phase difference between the 1st element and (N+1)th element is $N\Delta\xi = N\lambda/4$. If N is a multiple of 4, $N\Delta\xi$ will be a multiple of λ. That means that the induced current densities on the 1st element and (N+1)th element have periodic feature.

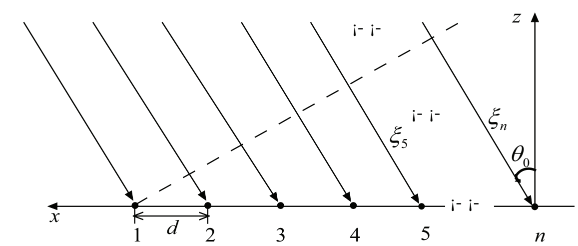


Fig. 4. Phase difference of a 1-D array with an oblique plane wave.

Under the oblique incidence condition, half of elements in a periodicity are chosen to form a combined-element. Thus, when $\theta_0 = 30^0$ and $\varphi_0 = 0^0$, a combined-element consists of two original elements. Figure 5 shows that an 8-element array is divided into edge combined-elements and interior combined-elements. The edge combined-elements fall into two classes: the left one and the right one. And the interior combined-elements also fall into two classes: the odd one and the even one.

For an array at oblique incidence, because the strong mutual coupling only takes place in neighboring combined-elements, the influence of combined-elements far away can be ignored for the EV-AEF calculation of a certain combined-element. Once the EV-AEF of each combined-element in this 8-element model is extracted, the scattering pattern of an arbitrary 4n-element array can be obtained through superposition theory, as shown in Fig. 5.

Similarly, a 9-element array shown in Fig. 6 is used to calculate the scattering pattern of an arbitrary (4n+1)-element array. In the same way, (4n+2)- and (4n+3)-element arrays can also be modeled by using rather small arrays.

Furthermore, when an array is illuminated by oblique plane wave with other angles, for example $\theta_0 = 45^0$ and $\varphi_0 = 0^0$, the phase difference between the 1st element and (N+1)th element is $N\Delta\xi = \sqrt{2}N\lambda/4$. When $N = 14$, this phase difference approximately equals to 5λ , a multiple of the wavelength. Thus, a periodicity of the array involves 14 elements.

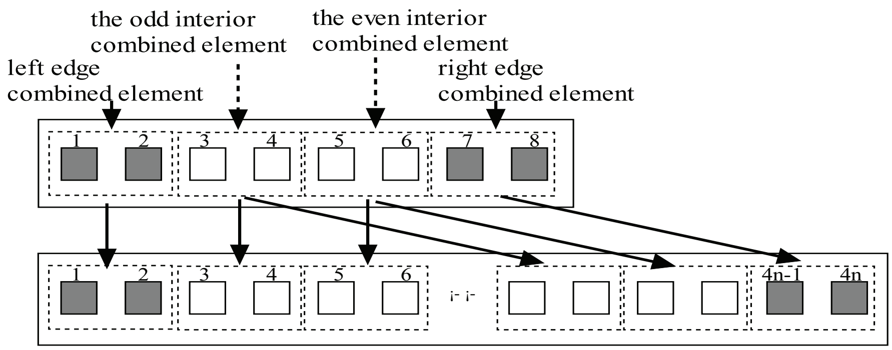


Fig. 5. Illustration of using an 8-element array to calculate a 4*n*-element array.

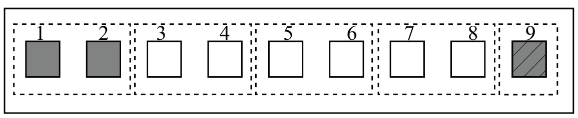


Fig. 6. Combined-elements in a 9-element array with $\theta_0 = 30^0$ and $\varphi_0 = 0^0$.

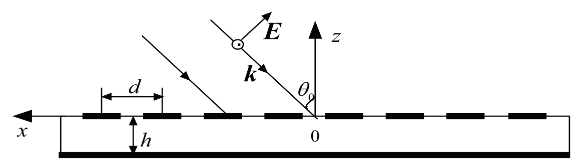
In general, for a certain incident angle θ_0, if the phase difference between the 1st element and (N+1)th one, $Nd\sin\theta_0$, is a multiple of wavelength λ, the number of elements in a periodicity of the induced current density can be obtained. Since half of elements in a periodicity make up a combined-element, it is required that N has to be an even number. For N=2, 4, 6, … , $Nd\sin\theta_0/\lambda$ is calculated in sequence. Once a calculated result is an integer or very close to an integer, the corresponding N is just the number of elements in a periodicity we want.

III. EXAMPLES AND DISCUSSIONS

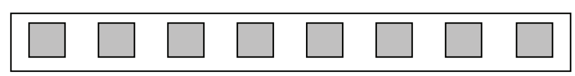
A. Far field calculation for 1-D finite arrays

A 1-D 8-element array illuminated by an oblique plane wave with $\theta_0 = 30^0$ and $\varphi_0 = 0^0$ is plotted in Fig. 7. The patch element of array is 26mm × 37mm. The structure parameters are h = 1.58mm and d = 0.5 λ_0, and the relative permittivity of the substrate is chosen as ε_r = 2.

Based on the EV-AEF result of each combined-element in the 8-element subarray, the scattered fields of 24- and 100-element arrays are shown in Figs. 8 (a) and (b), respectively. The results show that the scattered fields with the improved EV-AEF method closely match those with the whole-array simulation in FEKO.

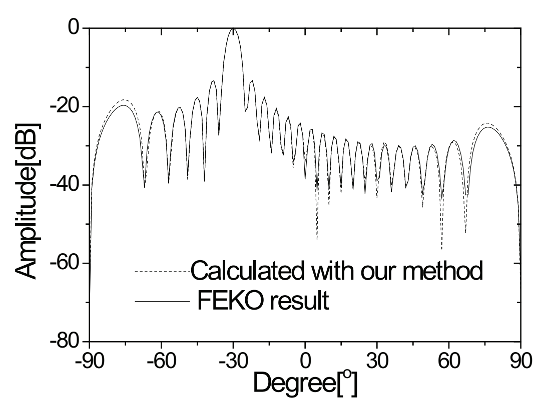


(a) Side view

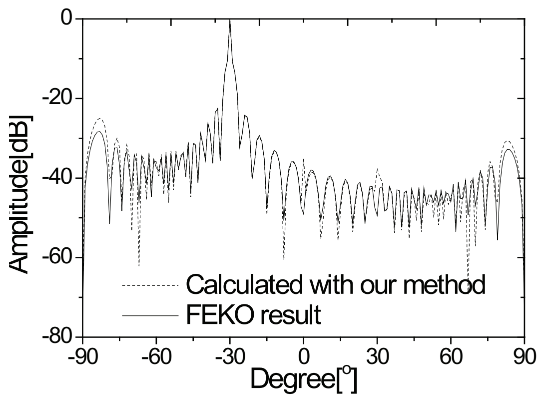


(b) Top view

Fig. 7. 8-element array illuminated by an oblique plane wave.



(a) 24-element array



(b) 100-element array

Fig. 8. Scattered fields with the EV-AEF method and FEKO's simulation.

For a ($4n$+1)-element array with an oblique wave of $\theta_0 = 30^0$ and $\varphi_0 = 0^0$, a 9-element subarray is required to calculate the scattered field. Figure 9 plots the scattered fields of a 101-element array whose parameters are the same as that in Fig. 7. The results of scattered field with the improved EV-AEF method and the whole-array FEKO simulation are in good agreement.

Table 1 presents the comparison of computing time between the proposed method and FEKO's simulation. The proposed method shows a significant improvement in computational efficiency. All calculations are performed on a computer with a Dual-Core 2.93 GHz CPU and 2.0 GB RAM.

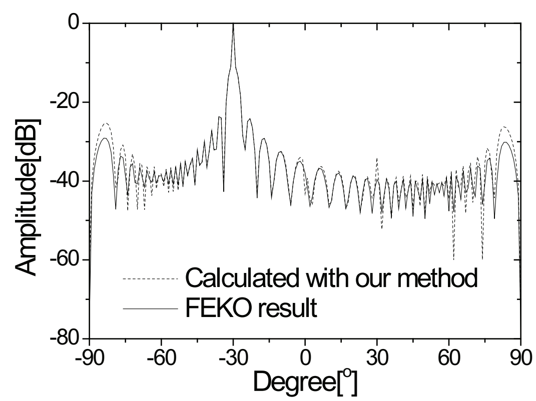


Fig. 9. Scattered fields with the EV-AEF method and FEKO's simulation.

Table 1: Comparison of computing time between the EV-AEF and FEKO's simulation

n-Element Array	n = 24		n = 100		n = 101	
Method	EV-AEF	FEKO	EV-AEF	FEKO	EV-AEF	FEKO
Subarray simulation (s)	4.97	------	4.97	------	7.13	------
Pattern superposition (s)	0.15	------	0.19	------	0.19	------
Total time (s)	5.12	50.02	5.16	5720.94	7.32	7082.52

B. Far feld calculation for 2-D finite array

In order to further prove the validity of the proposed method for computing 2-D finite arrays, an (8×8)-element array illuminated by an oblique plane wave with $\theta_0 = 30^0$ and $\varphi_0 = 0^0$, as shown in Fig. 10, is analyzed. The material and the element parameters are the same as the 1-D 8-element array in Fig. 6. The induced current distribution on the array surface is related to the polarization direction of obliquely incident wave. Along one resolved component of electric fields, x-direction in Fig. 10, the current distribution of a 2-D array is with the same regularity of that of a 1-D array. Along the non-polarization direction, the current distribution is dependent of the array environment and does not have periodic feature. The 2-D array elements can be divided into the corner combined-elements, edge combined-elements and interior combined-elements, respectively, as shown in Fig. 10.

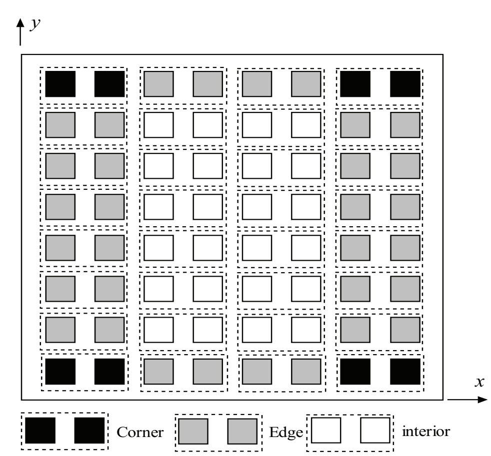


Fig. 10. Combined-elements in a 2-D (8 × 8)-element array.

An (8×3)-element subarray is required to calculate the type of $(4n\times n)$-element arrays. Figure 11 illustrates the scattered fields for the (8×8)-element array using the EV-AEF method and whole-array simulation in FEKO.

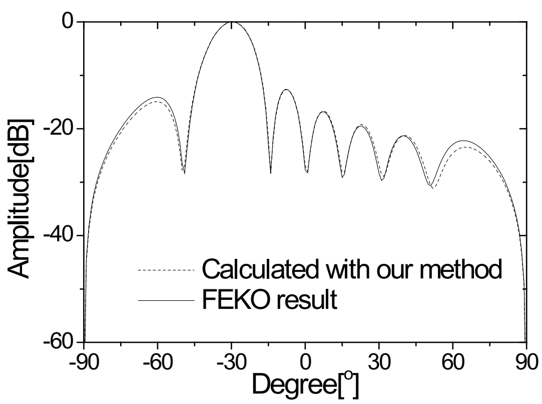


Fig. 11. Scattered fields for 2-D (8×8)-element array.

Table 2 shows that the proposed method for the 2-D (8×8)-element array gets a significant improvement in computational efficiency compared to whole-array simulation in FEKO.

Table 2: Comparison of computing time between our method and whole-array simulation

Approach	EV-AEF	FEKO
Subarray simulation (s)	102.16	-------
Pattern superposition (s)	0.17	-------
Total time (s)	102.33	5054.31

IV. CONCLUSION

In this paper, an improved EV-AEF method for calculating scattering patterns from a large finite array at oblique incidence is proposed. The relationship between incident angle of plane wave and combined-element properties is discussed. In order to validate the proposed method, numerical examples of 1-D and 2-D arrays are calculated. The results show that the improved EV-AEF method leads to the much faster solution than the whole-array simulation with good accuracy.

In addition, this work can also extend to non-regular or conformal arrays. Taking a thinned array which is thinned from a full array with the regular lattice of N positions spaced by a uniform distance for example, its element states (positions with element or without element) are unknown and there are a variety of possibilities. For a certain element in the thinned array, the distributions of its neighboring elements are various. Therefore, extracting the AEPs of all element combinations is indispensable.

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