A Vector Parabolic Equation Method Combined with MLFMM for Scattering from a Cavity

Z. He, Z. H. Fan, D. Z. Ding, and R. S. Chen

Department of Communication Engineering
Nanjing University of Science and Technology, Nanjing, 210094, China
eerschen@njust.edu.cn

Abstract **─** The three-dimensional vector parabolic equation (PE) method was used for the fast analysis of the electromagnetic scattering with reasonable accuracy. The algorithm was marched from plane to plane along some preferred directions so that the computational load could be reduced significantly. However, large errors will be introduced for cavities with the multiple scattering. In this paper, the vector parabolic equation method combined with the multilevel fast multipole method (MLFMM) is presented for the analysis of the electromagnetic scattering from a cavity. Numerical results demonstrate that the proposed technique can efficiently give reasonably accurate results for the cavities when compared with the conventional PE method.

Index Terms **─** Cavity, electromagnetic scattering, MLFMM, vector parabolic equation method.

I. INTRODUCTION

The parabolic equation (PE) method provides an approximate solution for the wave equation with the energy propagating along the paraxial direction [1]. The calculation of the PE is taken to march gradually from one plane to another plane along the paraxial direction so that the 3D problem can be converted into a series of 2D problems to be solved by the PE method. Therefore, the electromagnetic scattering from electrically large objects can be analyzed efficiently. However, the PE is applicable only to the object which does not have large changes in direction, because large errors will be introduced for both nonconvex objects and cavities with the multiple scattering. The electromagnetic scattering from cavities has been paid more and more attention due to its wide range of applications in both the industry and the military. Many numerical methods, including the method of moment (MoM) [2-3], the finite difference method (FDM) [4], and the finite element method (FEM) [5-6] have been used to analyze the electromagnetic scattering from cavities. The error of each solution is large unless fine discretization is used for the scatterers. But a fine mesh will bring a large system of equations which may be computationally prohibitive. Therefore, fast and accurate calculation of the electromagnetic scattering from cavities becomes significant problem to be solved.

The MoM accelerated by the multilevel fast multipole method (MLFMM) can be used to analyze the electromagnetic scattering from cavities accurately [2]. However, a large number of computations are required for the very electrically large three-dimensional PEC objects. In this paper, the PE method is combined with MLFMM for the analysis of electromagnetic scattering from electrically large PEC objects with cavities. Firstly, the scattered electric fields in the aperture can be obtained by calculating the current density on the inner sides of the cavity with the MLFMM. Then, the PE method can be applied to analyze the scattered electric fields of other parts with the corresponding boundary conditions on the PEC objects. By this way, both the accuracy and the efficiency for electromagnetic scattering from electrically large PEC cavities can be assured.

This paper is organized as follows. Section 2 gives a brief introduction to the traditional vector parabolic equation method. At the same time, the vector parabolic equation method combined with the multilevel fast multipole method is explained in detail. A slant cavity embedded in a block is analyzed as the numerical example to show the

Submitted On: October 11, 2014
Accepted On: April 16, 2015

validity and the efficiency of the proposed method in Section 3. Section 4 gives some conclusions and comments.

II. THEORY AND FORMULATIONS

A. Vector parabolic equation method for electromagnetic scattering problems

The three-dimensional vector wave equation can be used as an efficient tool to analyze the electromagnetic scattering problems. The scattered field components $E_{\mathrm{x}}^{S}, E_{y}^{S}, E_{z}^{S}$ satisfy scalar wave equation in the Cartesian coordinate as follows:

$$\frac{\partial^2 E_\xi^s}{\partial x^2}+\frac{\partial^2 E_\xi^s}{\partial y^2}+\frac{\partial^2 E_\xi^s}{\partial z^2}+k^2 E_\xi^s=0 \qquad \xi=x,y,z, \tag{1}$$

where k is the wave number.

The reduced scattered fields u_x^s, u_y^s, u_z^s are defined as:

$$u_\xi^s(x,y,z)=e^{-jkx}E_\xi^s(x,y,z) \qquad \xi=x,y,z. \tag{2}$$

Substitute equation (1) with equation (2), the vector parabolic equations can be gotten in air domain:

$$\frac{\partial u_\xi^s}{\partial x}(x,y,z)=\frac{j}{2k}\left(\frac{\partial^2 u_\zeta^s}{\partial y^2}(x,y,z)+\frac{\partial^2 u_\xi^s}{\partial z^2}(x,y,z)\right) \qquad \xi=x,y,z. \tag{3}$$

It should be noted that the first order Taylor expansions of the square root and the exponential are used in this paper. And it can be seen from the equation (3) that the solutions at ($x+\Delta x$) plane can be calculated from those at x plane. As shown in Fig. 1, the computation can start in the plane before the object and stop in the plane beyond the object.

When the FD scheme of the Crank-Nicolson type is applied to the equation (3), the computational format for the vector parabolic equations can be written as follows:

$$\begin{aligned}
&\frac{\Delta x}{2jk(\Delta y)^2}u_\xi^s(x+\Delta x,y+\Delta y,z)\\
&+\frac{\Delta x}{2jk(\Delta z)^2}u_\xi^s(x+\Delta x,y,z+\Delta z)\\
&+\left(1-\frac{\Delta x}{jk(\Delta y)^2}-\frac{\Delta x}{jk(\Delta z)^2}\right)u_\xi^s(x+\Delta x,y,z)\\
&+\frac{\Delta x}{2jk(\Delta y)^2}u_\xi^s(x+\Delta x,y-\Delta y,z)\\
&+\frac{\Delta x}{2jk(\Delta z)^2}u_\xi^s(x+\Delta x,y,z-\Delta z)=u_\xi^s(x,y,z)\\
&\xi=x,y,z.
\end{aligned} \tag{4}$$

The incident field is taken into account by the boundary conditions on the scatterer in each transverse plane. Perfect matching layers (PML) are placed around the object to truncate an infinite space to a finite computation domain. For PML domain, the following coordinate transformation is introduced [7-9]:

$$\begin{aligned}
\hat{y}&=y-i\int_0^y\sigma(\xi)d\xi,\\
\hat{z}&=z-i\int_0^z\sigma(\xi)d\xi.
\end{aligned} \tag{5}$$

In equation (5), $\sigma(\xi)=\frac{3}{2\delta}\times\frac{1}{\eta}\times\log(\frac{1}{10^{-3}})\times(\frac{\xi}{\delta})^2$, δ is the thickness of the PML and η is the wave impedance. As in air domain, the FD scheme of the Crank-Nicolson type is used. Therefore, the equation (6) can be replaced with equation (4) for the computation in the PML domain:

$$\begin{aligned}
&\frac{\Delta x e_i e_{i-1/2}}{2ik\Delta y^2}u_\xi^s(x+\Delta x,y-\Delta y,z)+\\
&\frac{\Delta x e_j e_{j-1/2}}{2ik\Delta z^2}u_\xi^s(x+\Delta x,y,z-\Delta z)+\\
&\left(1-\frac{\Delta x e_j\left(e_{j+1/2}+e_{j-1/2}\right)}{2ik\Delta z^2}-\right.\\
&\left.\frac{\Delta x e_i\left(e_{i+1/2}+e_{i-1/2}\right)}{2ik\Delta y^2}\right)u_\xi^s(x+\Delta x,y,z)+\\
&\frac{\Delta x e_i e_{i+1/2}}{2ik\Delta y^2}u_\xi^s(x+\Delta x,y+\Delta y,z)+\\
&\frac{\Delta x e_j e_{j+1/2}}{2ik\Delta z^2}u_\xi^s(x+\Delta x,y,z+\Delta z)=u_\xi^s(x,y,z)\\
&\xi=x,y,z,
\end{aligned} \tag{6}$$

where

$$e_i=\frac{1}{1-i\sigma(y_i)}, \tag{7}$$

$$\sigma(y_i)=\frac{1}{\Delta z}\int_{y_{i-1/2}}^{y_{i+1/2}}\sigma(\xi)d\xi, \tag{8}$$

$$e_j=\frac{1}{1-i\sigma(z_j)}, \tag{9}$$

$$\sigma(z_j)=\frac{1}{\Delta z}\int_{z_{j-1/2}}^{z_{j+1/2}}\sigma(\xi)d\xi. \tag{10}$$

The narrow-angle approximation is only accurate when energy does not have large changes along the paraxial direction. As a result, the traditional parabolic equation method cannot be used to analyze the scattering from cavities.

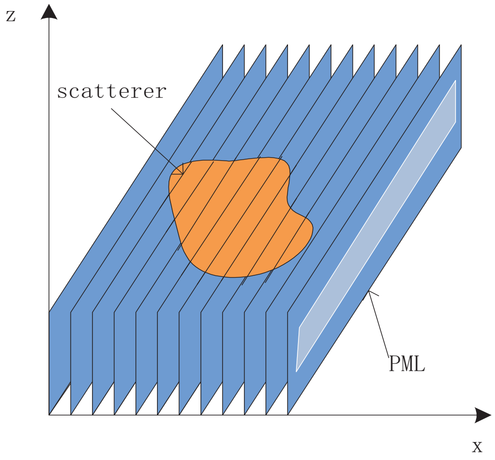


Fig. 1. Transverse planes for PE marching.

B. Vector parabolic equation method combined with the MLFMM for scattering from a cavity

As shown in Fig. 2, a PEC object with cavity in free space is illuminated by a plane wave e^{ikx} propagating along $-x$ direction with wave number k.

The scattering by 3-D arbitrary conducting objects can be formulated by the electrical-field integral equation (EFIE). As shown in Fig. 2, the current density on the inner sides of the cavity can be calculated with the equation (11) by the MLFMM

$$-ik\mathbf{n}(\mathbf{r})\times\int_{S_{inner}} G(\mathbf{r},\mathbf{r}')\mathbf{J}(\mathbf{r}')dS' + \frac{1}{ik}\mathbf{n}(\mathbf{r})\times\nabla\int_{S_{inner}} G(\mathbf{r},\mathbf{r}')\nabla'\Box\mathbf{J}(\mathbf{r}')dS' = -\mathbf{n}(\mathbf{r})\times\mathbf{E}^{\mathbf{i}}(\mathbf{r}) \qquad \forall\mathbf{r}\in \mathrm{S}_{inner}, \tag{11}$$

where S_{inner} denotes the surface of the inner sides in the cavity, μ_0 and ε_0 are the free-space permittivity and permeability, $k=\omega\sqrt{\mu_0\varepsilon_0}$ is the wave number, $\mathbf{J}$ is the current density in the inner sides of the cavity, $\mathbf{E}^{\mathbf{i}}(\mathbf{r})$ is the incident plane wave, $\mathbf{n}$ denotes the surface outward pointing unit normal and $G(\mathbf{r},\mathbf{r}')$ is the Green's function in free space [2].

For the aperture, the scattered electric fields on S_2 can be obtained by the integration of the electric current on the inner sides S_{inner} of the cavity, as written in equation (12).

$$\mathbf{E}^{\mathbf{s}}(\mathbf{r}) = -\frac{i\omega\mu_0}{4\pi}\int_{S_{inner}}\left(I-\frac{1}{k^2}\nabla\nabla'\right)G(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}(\mathbf{r}')dS' \qquad \forall\mathbf{r}\in\mathrm{S}_2. \tag{12}$$

To efficiently analyze the electromagnetic scattering from PEC objects with cavities, the vector PE method is introduced to calculate the scattered electric fields from the surface S_1 and the aperture surface S_2 with equation (3).

The FD scheme of the Crank-Nicolson type is used to the equation (3), as shown in equation (4). Therefore, the $u_\xi^s(x+\Delta x,y,z)$ at ($x+\Delta x$) plane can be calculated from $u_\xi^s(x,y,z)$ at x plane [1]. The tangential electric field must be zero on the scatterer for a PEC object. To guarantee the unicity of the solution, the reduced scattered fields in equation (3) are coupled through both the boundary conditions on the surface S_1 and the divergence-free condition [1], as shown in equation (13):

$$\begin{cases} n_x u_y^s(P) - n_y u_x^s(P) = \\ \qquad -e^{-ikx}\left(n_x E_y^i(P) - n_y E_x^i(P)\right) \\ n_x u_z^s(P) - n_z u_x^s(P) = \\ \qquad -e^{-ikx}\left(n_x E_z^i(P) - n_z E_x^i(P)\right) \\ n_y u_z^s(P) - n_z u_y^s(P) = \\ \qquad -e^{-ikx}\left(n_y E_z^i(P) - n_z E_y^i(P)\right) \\ \frac{i}{2k}\left(\frac{\partial^2 u_x^s(P)}{\partial y^2} + \frac{\partial^2 u_x^s(P)}{\partial z^2}\right) + \\ \qquad iku_x^s(P) + \frac{\partial u_y^s(P)}{\partial y} + \frac{\partial u_z^s(P)}{\partial z} = 0 \end{cases} \qquad \forall P\in S_1, \tag{13}$$

where p is a point on the surface of the scatterer, (E_x^i,E_y^i,E_z^i) is the field components of the incident wave and (n_x,n_y,n_z) is the outer normal to the surface at p.

For the last transverse (y-z) plane, the boundary conditions for the equation (3) on both the PEC surface S_1 and the aperture S_2 are obtained by equation (13) and (12), respectively. The computation for equation (3) is taken from one plane to another, which begins just before the PEC scattering object and stops beyond it. Once the scattered electric fields are obtained in the last transverse plane, the RCS can be calculated by Fourier transform of them [1].

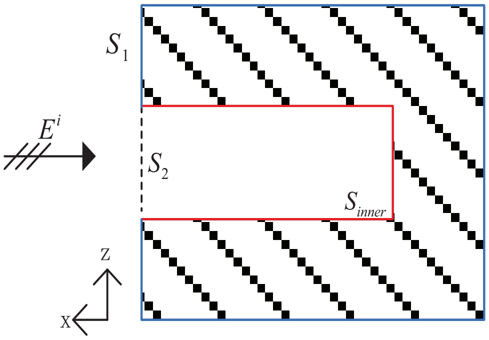
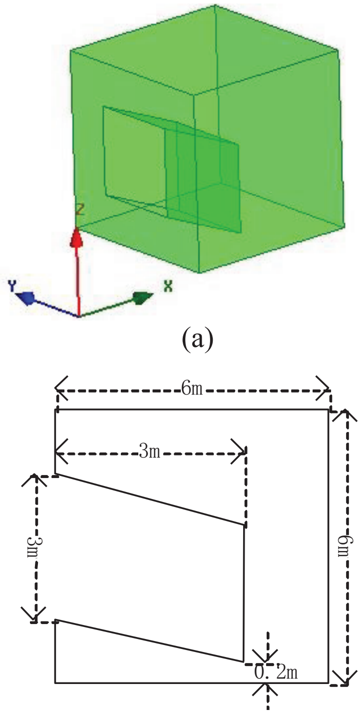


Fig. 2. A PEC object with cavity in free space.

III. NUMERICAL RESULTS

As shown in Fig. 3, a slant cavity nested in a block is presented to demonstrate the efficiency of the proposed method. The incident field is a plane wave, vertical polarized and propagating along -x direction. The RCS curves are compared between the proposed method and the MLFMM in Fig. 4 for a wide band from 200 to 600 MHz. It can be observed that the proposed method is in good agreement with the MLFMM. As shown in Fig. 5, the bi-static RCS results at the frequency of 300 MHz are compared among the proposed method, the conventional PE method and the MLFMM. Moreover, the amplitude and the phase of the scattering electric fields for the cavity's middle line paralleled to the y axis in the last transverse plane are given in Fig. 6. It can be found that the proposed method is more accurate than the conventional PE method when compared with the MLFMM. As listed in Table 1, both the CPU time and the memory requirement are compared between the proposed method and the MLFMM at the frequency of 300 MHz. It can be found that memory requirement and CPU time can be reduced 71.9% and 43.6% for the proposed method, respectively.



(a)

(b)

Fig. 3. The model of the slant cavity nested in a block: (a) stereogram, and (b) side view.

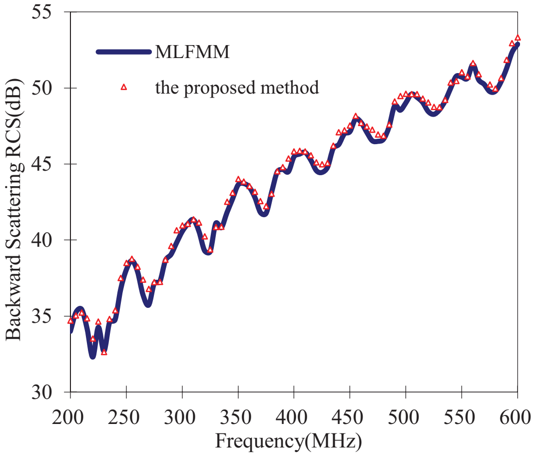


Fig. 4. The RCS from 200 MHz to 600 MHz.

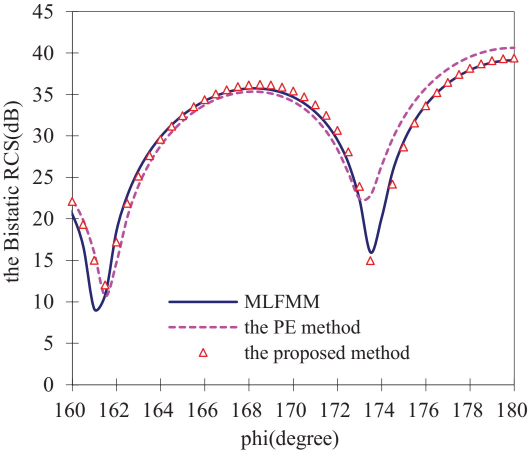
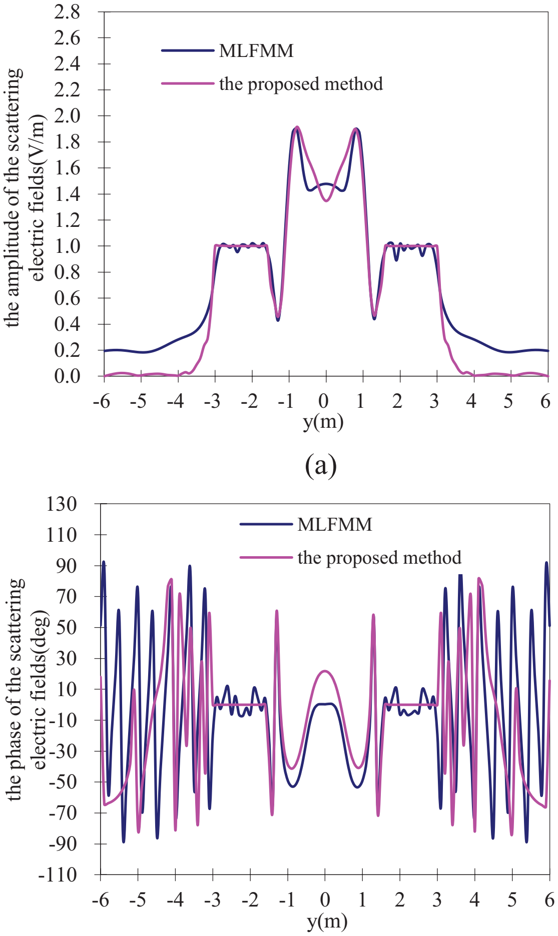


Fig. 5. The bi-static RCS of the cavity at 300 MHz.



(a)

(b)

Fig. 6. (a) The amplitude of the scattering electric fields for the cavity's middle line, and (b) the phase of the scattering electric fields for the cavity's middle line.

Table 1: Comparison for the CPU time and the memory requirement between the proposed method and the MLFMM at the frequency of 300 MHz

	CPU Time (s)	Memory Requirement (MB)
MLFMM	1184	448
The proposed method	668	126

IV. CONCLUSION

In this paper, a vector parabolic equation method combined with the MLFMM has been proposed to analyze the electromagnetic scattering from a PEC cavity. The MLFMM is introduced to compute the scattered currents on the inner side of the cavity and the electric fields in the aperture induced by these scattered currents are obtained. The scattered fields from the cavity are calculated by the vector PE method with the corresponding boundary conditions. It can be found that both the CPU time and the memory requirement are significantly reduced when compared with the MLFMM.

ACKNOWLEDGMENT

We would like to thank the support of Natural Science Foundation of 61431006, Jiangsu Natural Science Foundation of BK2012034, Natural Science Foundation of 61271076, 61171041, 61371037, Ph.D. Programs Foundation of Ministry of Education of China of 20123219110018; the Fundamental Research Funds for the central Universities of No. 30920140111003, No. 30920140121004.

REFERENCES

[1] A. A. Zaporozhets and M. F. Levy, "Bistatic RCS calculations with the vector parabolic equation method," *IEEE Trans. Antennas and Propagation*, vol. 47, pp. 1688-1696, 1999.

[2] Z. Nie, H. Wang, and J. Wang, "A combined field solution with single operator for electromagnetic scattering from conductive targets with open cavities," *IEEE Trans. Antennas and Propagation*, vol. 56, pp. 1734-1741, 2008.

[3] C. F. Wang and Y. B. Gan, "2-D cavity modeling using method of moments and iterative solvers," *J. Electromagn. Waves*

Appl., vol. 17, no. 12, pp. 1739-1741, 2003.
[4] T.-T. Chia, R. J. Burkholder, and R. Lee, "The application of FDTD in hybrid methods for cavity scattering analysis," *IEEE Trans. Antennas and Propagation*, vol. 43, 1082-1090, 1995.
[5] J.-M. Jin and K. Donepudi, "Electromagnetic scattering from large, deep, and arbitrarily-shaped open cavities," *Antennas and Propagation Society International Symposium*, vol. 4, 2186-2189, 1998.
[6] J. Liu and J. Jin, "A special higher order finite-element method for scattering by deep cavities," *IEEE Trans. Antennas and Propagation*, vol. 48, 694-703, 2000.
[7] J. P. Bergener, "Three-dimensional perfectly matched layer for the absorption of electromagnetic waves," *J. Comp. Phys*., no. 127, pp. 363-379, 1996.
[8] F. Collino, "Perfectly matched absorbing layers for the paraxial equations," *J. Comp. Phys*., 94:1-29, 1991.
[9] J. P. Bergener, "A perfectly matched layer for the absorption of electromagnetic waves," *J. Comp. Phys*., 114:185-200, 1994.

Zi He received the B.Sc. degree in Electronic Information Engineering from the School of Electrical Engineering and Optical Technique, Nanjing University of Science and Technology, Nanjing, China, in 2011. She is currently working towards the Ph.D. degree in Electromagnetic Fields and Microwave Technology at the School of Electrical Engineering and Optical Technique, Nanjing University of Science and Technology. Her research interests include antenna, RF-integrated circuits, and computational electromagnetics.

Zhenhong Fan was born in Jiangsu, China, in 1978. He received the M.Sc. and Ph.D. degrees in Electromagnetic Field and Microwave Technique from Nanjing University of Science and Technology (NJUST), Nanjing, China, in 2003 and 2007, respectively.

During 2006, he was with the Center of wireless Communication in the City University of Hong Kong, Kowloon, as a Research Assistant. He is currently an Associate Professor with the Electronic Engineering of NJUST. He is the author or co-author of over 20 technical papers. His current research interests include computational electromagnetics, electromagnetic scattering and radiation.

Dazhi Ding was born in Jiangsu, China, in 1979. He received the B.S. and Ph.D. degrees in Electromagnetic Field and Microwave Technique from Nanjing University of Science and Technology (NUST), Nanjing, China, in 2002 and 2007, respectively.

During 2005, he was with the Center of Wireless Communication in the City University of Hong Kong, Kowloon, as a Research Assistant. He is currently an Associate Professor with the Electronic Engineering of NJUST. He is the author or co-author of over 30 technical papers. His current research interests include computational electromagnetics, electromagnetic scattering, and radiation.

Rushan Chen (M'01) was born in Jiangsu, China. He received the B.Sc. and M.Sc. degrees from the Department of Radio Engineering, Southeast University, China, in 1987 and 1990, respectively, and the Ph.D. degree from the Department of Electronic Engineering, City University of Hong Kong, in 2001.

He joined the Department of Electrical Engineering, Nanjing University of Science and Technology (NJUST), China, where he became a Teaching Assistant in 1990 and a Lecturer in 1992. Since September 1996, he has been a Visiting Scholar with the Department of Electronic Engineering, City University of Hong Kong, first as Research Associate, then as a Senior Research Associate in July 1997, a Research Fellow in April 1998, and a Senior Research Fellow in 1999. From June to September 1999, he was also a Visiting Scholar at Montreal University, Canada. In September 1999, he was promoted to Full Professor and Associate Director of the Microwave and Communication Research Center in NJUST, and in 2007, he was appointed Head of the Department of Communication Engineering, NJUST. He was appointed as the Dean in the School of Communication and Information Engineering, Nanjing Post and Communications University in 2009. And in 2011, he was appointed as Vice Dean of the School of Electrical Engineering and Optical Technique, Nanjing University of Science and Technology. His research interests mainly include microwave/millimeter-wave systems, measurements, antenna, RF-integrated circuits, and computational electromagnetics. He has authored or co-authored more than 200 papers, including over 140 papers in international journals.

Chen received the 1992 third-class science and technology advance prize given by the National Military Industry Department of China, the 1993 third-class science and technology advance prize given by the National Education Committee of China, the 1996 second-class science and technology advance prize given by the National Education Committee of China, and the 1999 first-class science and technology advance prize given by Jiangsu Province, as well as the 2001 second-class science and technology advance prize. He is the recipient of the Foundation for China Distinguished Young Investigators presented by the National Science Foundation (NSF) of China in 2003. In 2008, he became a Chang-Jiang Professor under the Cheung Kong Scholar Program awarded by the Ministry of Education, China. Besides, he was selected as a Member of Electronic Science and Technology Group by Academic Degree Commission of the State Council in 2009. Chen is a Senior Member of the Chinese Institute of Electronics (CIE), Vice-Presidents of Microwave Society of CIE and IEEE MTT/APS/EMC Nanjing Chapter. He serves as the Reviewer for many technical journals such as IEEE Trans. on AP and MTT, Chinese Physics etc., and now serves as an Associate Editor for the International Journal of Electronics.