Design and Evaluation of the Multilevel Mesh Generation Mode for Computational Electromagnetics

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Abstract ─ This work presents a multilevel technique developed for speeding-up efficiently the mesh generation process of large and complex bodies for electromagnetic analysis. The targets are meshed by generating intermediate meshes with elements of decreasing size edge until the desired size is reached. The technique minimizes the time of meshing any given geometry and ensures a high quality mesh, as the input geometry at any level is composed of simple surfaces that can be meshed easily. The times of meshing on several geometries with and without the proposed method are compared as well as the quality statistics of the meshes obtained. The Method-of-Moments is used to evaluate the accuracy of the resulting meshes.

Index Terms ─ Applied classical electromagnetism, meshers, Method-of-Moments, Radar Cross Section.

I. INTRODUCTION

The analysis of a wide set of problems in many technological areas such as electromagnetics, fluid dynamics, and heat transfer problems requires a good geometrical description of the objects to be analyzed. In most of these cases, the geometrical model must be discretized into elements with simple shapes and suitable sizes in the mesh generation process. Many algorithms of mesh generation have been developed to solve this problem, and most of them work with triangular or quadrilateral elements for surfaces and tetrahedra or hexahedra for volumes.

Optimizing a mesh generator designed for electromagnetic analysis is the main aim of this work. The Method-of-Moments [1-2] is the technique used for verifying with simulations the resulting meshes, so body-fitted quadrilateral or hexahedral elements are generated to reduce the number of unknowns to be analyzed.

In this paper, a method developed to accelerate the mesh generation by using a multilevel strategy is described. In particular, this work has been focused on a hybrid version of the paving algorithm [3-4], which generates meshes composed of quadrangular elements, and introduces some triangular elements in particular cases [5-6]. Anyway, the proposed technique may be applied on any mesh generation algorithm, both superficial and volumetric methods.

In the second section of the paper, the initial state of the considered mesh generator is evaluated as well as the already included optimization techniques and the lacks found for developing the multilevel mode instead of another method. The multilevel meshing mode is detailed in the third section. The fourth section is focused in the time reductions and mesh quality achieved when the multilevel mode is used or not. After evaluating the influence of the technique in the generated meshes, a set of simulations with the Method-of-Moments is included to verify that the results obtained are equivalent with and without using the multilevel mode in the fifth section. The main conclusions are summarized in last section.

II. ANALYSIS OF THE PROBLEM

Because of the strong influence of the geometrical models on the accuracy of results on EM simulations, an own mesh generator that works on NURBS surfaces [7] has been developed. It provides a high level detail with simple mathematical models, and allows to use less dense meshes than plane facets models.

The mesh generator is a hybrid version of the paving algorithm [3-4] optimized for electromagnetic simulation purposes that generates meshes of quadrilateral and triangular body-fitted elements. The use of hybrid meshes instead of only triangular or quadrilateral elements [8] provides an additional degree of freedom to electromagnetic solvers, as fewer unknowns are considered for quadrilateral elements but better features are achieved when some triangular elements are inserted in particular cases. Anyway, most of the elements of the generated meshes are quads, and the triangular elements are only inserted when they provide better quality (in terms of size homogeneity and angles quadrature of the elements) than the generation of quadrilateral irregular elements [5].

The mesh generator has been parallelized by using

Submitted On: May 27, 2014
Accepted On: March 29, 2015

1054-4887

the MPI paradigm. To avoid using complex geometry rasterization algorithms [9], the domain decomposition [10] is simply done by distributing the surfaces among the available processors to obtain a fair sharing out of the area to be meshed in each processor. The combination of a pre-processing stage that automatically detects the topologies between the surfaces to be distributed and the insertion of the common nodes between neighbouring surfaces before starting the mesh generation ensure the electrical continuity in the final meshes [5].

To optimize the combination of the mesh generator and the simulation kernels, several pre-processing and post-processing stages have been included and validated in different benchmark experiments [11-14].

To evaluate the dependence of the time of meshing on the number of elements generated, a square plate with an area of 1 m^2 has been meshed with different size edge. Every mesh has been generated with the same CPU, an Intel Core i7 – 740QM at 1.73 GHz with 8 GB of RAM and by using only one processor. The number of elements and the times of meshing are depicted in Fig. 1 with logarithmic axes. It should be noted that the times of meshing may be greater for more complex geometries due to the extreme simplicity of the square plate; as every generated element is a perfect quadrangle, less intersections need to be solved during the mesh generation, and then most of elements are generated by perfect rows in continuous steps. However, in more complex geometries, the time of meshing may be slightly greater due to the increasing number of intersections for which boundaries must be resolved. According to the Fig. 1, the time of meshing of the plate is almost quadratically increased with the number of elements.

Although a relatively low-power computer has been used for meshing the plate, such a machine is good enough for understanding the behavior of the algorithm in any computer. The same effects appear when more powerful machines are used.

In spite of parallelizing the mesh generator, the selected method for the load distribution may cause some bottleneck in particular cases such as in the square plate example; as the geometry has an only surface, the same time of meshing is required regardless of the number of processors used, as only a processor is really working. To solve this problem, additional techniques are required to accelerate the mesh generation process.

The slowness of the mesh generation method is solved by adopting a hierarchical strategy that provides two different benefits: the bottlenecks of the parallelization are dispelled; and the mesh generation is simplified in every step.

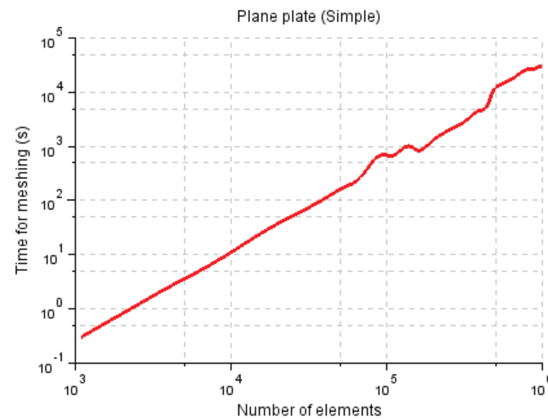

Fig. 1. Time of meshing of a square plate without the multilevel mode.

Consider a very simplified meshing algorithm as presented in Fig. 2, where the input parameters are the number of surfaces to be meshed `S`, the geometrical description in the `surfaces` array, and the total number of elements that should be generated in the whole geometry `N`. The main tasks of the mesh generation are the insertion of new elements on every `surfaces`, `InsertNewElement`, and the evaluation and resolution (if necessary) of intersections between the new element and the existing ones, `CheckIntersection`. To simplify the complexity analysis, the cost of both operations is considered as constant `K`, and the area of every surface is identical, so the same number of elements per surface is expected. With these assumptions, the overall cost associated to the meshing algorithm may be written as:

$$C = \sum_{i=0}^{S-1} \sum_{j=0}^{N/S-1} K \sum_{k=0}^{j-1} K = S \cdot \frac{\frac{N}{S} \cdot \left(\frac{N}{S} - 1 \right)}{2} K^2 , \tag{1}$$

that can be simplified for very electrically large cases (`N>>S`), as:

$$C \approx \frac{N^2}{2 \cdot S} K^2 . \tag{2}$$

However, reducing the computational order of the algorithm is possible by setting a good relation between `N` and `S`, so we need to change the S according to the magnitude of the problem that depends on `N`. This is the goal of the multilevel meshing mode, as the number of surfaces of the second and next levels is given by the number of elements generated in the previous levels. In particular, if the number of input surfaces of a given level is computed as:

$$S = \frac{N}{1 + 2 \cdot \log_2 N} , \tag{3}$$

the complexity of the meshing algorithm is reduced from N^2 to $N \cdot \log_2(N)$, and the size of the new input surfaces also tends to be homogenous, which may not happen in the original geometries.

```
SimplestMeshAlgorithm(S, surfaces, N)
{
   for (i=0; i<S; i++)
   {
      for (j = 0; j < N/S; j++)
      {
         InsertNewElement(surfaces[i],j)
         for (k = 0; k < j; k++)
         {
            CheckIntersection(j, k)
         }
      }
   }
}
```

Fig. 2. Pseudo-code of a simple mesh algorithm.

More detailed information about the multilevel mode is presented in the next section.

III. MULTILEVEL TECHNIQUE

The multilevel mode consists on dividing the geometry to be meshed into as many intermediate meshes as necessary. The process is simple, and the time of meshing is widely reduced in comparison with the conventional technique whenever the intermediate meshes are generated efficiently and the number of levels is optimal.

According to the scheme of the multilevel mode shown in Fig. 3, the complete process of meshing is described as follows:

1. First, the geometry is loaded and the size edges of the mesh elements are computed for every surface.
2. Thereafter, the geometry is evaluated. The shapes of the surfaces are analyzed and those that do not meet certain criteria of simplicity are split in more simple surfaces. For example, the meshing process for a cylinder built with four cylindrical surfaces connected together is easier than the one for the built with only a closed surface; therefore, at the preprocessing stage, the closed cylinder is divided into several simpler surfaces.
3. Sometimes, especially in electromagnetic applications such as the Method-of-Moments, the electrical continuity in the meshes is one of the most essential requirements; therefore, this stage must be included. Before starting the meshing, all the neighborhood relationships between the surfaces in electrical contact are established (topologies detection). This stage may be time-consuming, because it is based on the search of common points between near surfaces.
4. An estimation of the final number of elements in the mesh is performed taking into account the size edge of the elements, the area surfaces and the topologies between them.
   The number of levels required to generate the final mesh is evaluated by considering several parameters, such as the estimated number of elements, the number of surfaces to be meshed, the difference in size between the biggest surface and the smallest one, or the shapes of the most complex surfaces. For example, when a single surface is meshed with millions of elements, several levels are recommended; however, if there are hundreds of surfaces and all of them have similar dimensions, generating the mesh in a single level is faster.
5. If the estimation of the direct meshing mode is better than the multilevel one, the meshing algorithm is applied without any additional restriction.
6. Otherwise, the optimum size edge is calculated for each level to ensure that the same number of elements per surface is achieved in every level. The ratio between the size edge in a given level to the previous one should be neither too high, as too many levels with near densities of elements would waste time; nor too low, as if the number of elements to generate per surface in some level is too large, it would be the bottleneck of the algorithm. In the first level, the original meshing algorithm is applied with the biggest size edge. In the following levels, several preprocessing steps similar to the initial ones are applied just before meshing.
   6.1. In the first step, the input mesh is preprocessed. Because of the corresponding input geometry is a mesh, all the new surfaces generated are simple, and the quality evaluation is not required. To optimize the memory resources when there are too many input elements, the complete set of information about the original geometry (defined by its NURBS parameters) is deleted after the first level, and then the global description of the input mesh is only stored (indexed coordinates for the points and group of points indexes for the elements).
   6.2. The topology detection of continuous meshes is obtained with a simpler and faster method than of NURBS geometries. The search of common points between near surfaces is not necessary for finding the neighboring relations between the input elements. To identify all the relationships between the elements in the mesh, a simple search of shared points (in terms of indexes) is performed, as the input mesh has been generated with the criterion of sharing indexes of points when there is

electrical continuity between elements. The topologies of millions of elements may be computed in a few seconds with this method.

6.3. After preprocessing the input mesh, the desired meshing algorithm is applied to every surface until the mesh of the level is finished. As the meshing method works with NURBS surfaces, only the input element that is being meshed is parametrized as a NURBS entity, and after meshing it, its description is erased.

6.4. This process is repeated until the last level is reached, as indicated in the scheme in Fig. 3 by ellipsis.

7. When the whole geometry has been meshed, the information of final mesh is gathered, and the output files are generated.

This technique together with the parallelization of the mesh algorithm minimize the time of meshing, as shown in the efficiency results section.

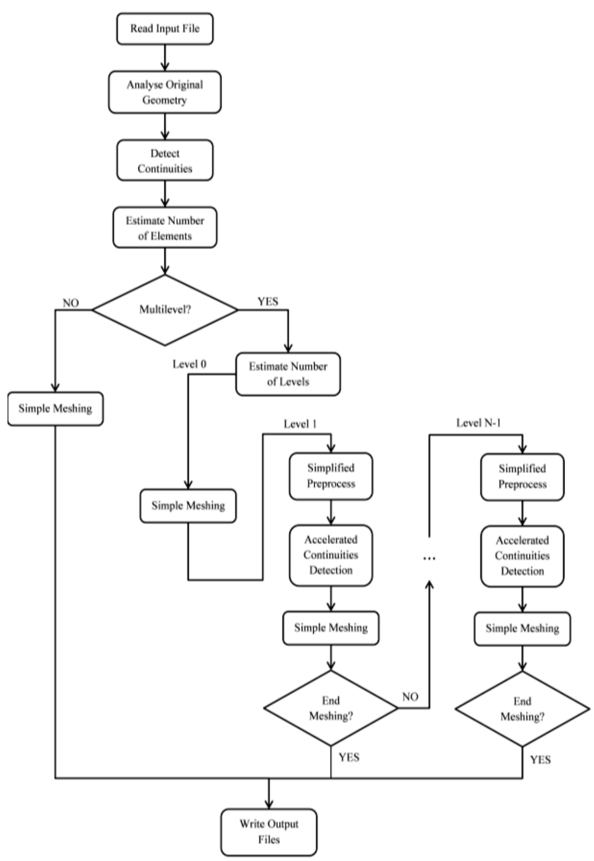

Fig. 3. Scheme for meshing with the multilevel mode.

IV. EFFICIENCY RESULTS

The results for the time of meshing of a simple and a complex geometry are presented in this section. The quality of the mesh obtained for the complex example is studied in detail.

The first example is the square plate of Section II to contrast the results obtained with and without the multilevel mode. Although this is one of the simplest cases to be analyzed, the conclusions obtained from it can be applied to any problem, as all the steps of the meshing algorithm represented in Fig. 3 are always applied. The variation on the time of each step may cause slight differences in performance on different problems. For example, the more surfaces in contact are found in a given geometry, the slower is the continuity detection step. On the other hand, another example such as a plane plate with tens of near holes may require more time for solving the intersections of confronted elements than the rest of the steps.

The times of meshing of the case of the square plate by using the multilevel mode are shown in Fig. 4.

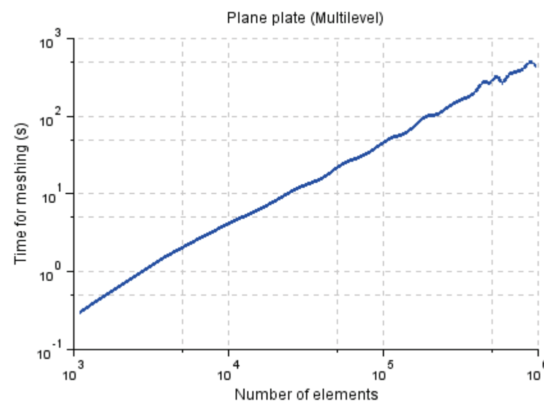

Fig. 4. Time of meshing of a square plate with the multilevel mode.

The number of elements for a given size edge by using the multilevel mode is not exactly equal to that used in the analysis without the multilevel mode, because this number depends on the input geometry of that particular level. If the size edge at a given level is not a multiple of the size edge at the next level, the size of the elements may vary slightly from the desired one, and the number of elements in the mesh may be different in consequence.

The shape of the curve with the multilevel mode in Fig. 4 is similar to the curve without the multilevel mode in Fig. 1, but the times have been minimized. Some oscillations may appear when intersections are computed, as also seen in the case of the simple meshing mode. To compare the times of meshing with and without the multilevel mode, the ratio between the times of meshing with the multilevel technique to the simple mode is represented in Fig. 5. This ratio, which we call the time relation, has a decreasing linear form, with the ratio reducing to a value of approximately 1 percent (less than 6 minutes) when we have more than half a million elements. According to the curve, the more elements are generated, the bigger is the time reduction provided by the multilevel method.

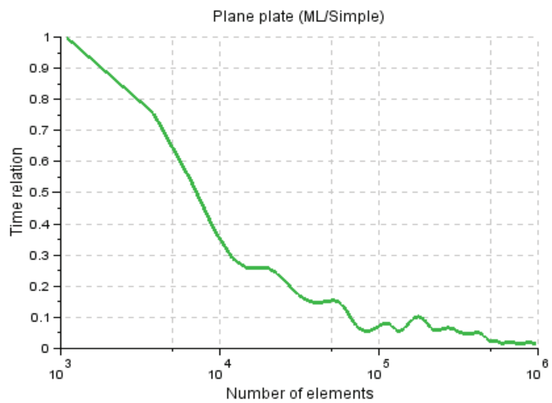

Fig. 5. Time of meshing relation of a square plate.

The proposed method for meshing is equally efficient for higher frequencies whenever a good relation between the number of levels employed and the density of elements per level is correctly distributed, as the number of elements generated per surface in each level must be large enough.

To evaluate the effect of the multilevel mode in a real geometry, the model of a vehicle has been meshed with a high density of elements by using several processors. It is composed of 363 curved surfaces with different shapes and sizes, and thus, the relative efficiency of the multilevel mode is enhanced. The original geometry and a mesh with a low density of elements are represented in Fig. 6.

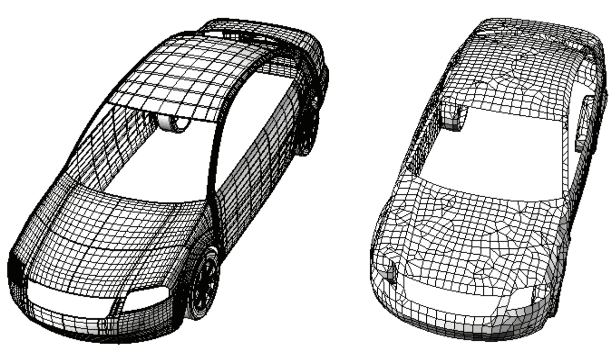

Fig. 6. Original (left) and meshed (right) vehicle.

The desired size edge for the elements is 1.5 mm, which leads to a high density mesh for the vehicle with six and a half million of elements. This case may be too large to be meshed on a personal computer; therefore, it has been meshed on a SUN X4000 Quad Opteron workstation with 32 cores at 2.4 GHz and 256 GB of RAM.

The vehicle has been meshed with the same size edge while varying the number of processors both with the simple meshing algorithm and with the multilevel mode by considering two levels.

The times of meshing in parallel of the vehicle without the multilevel mode are shown in Fig. 7 with linear axes. The more processors are used, the time of meshing is reduced. The expected shape of the curve is a decreasing logarithm. However, although the processing time is ideally decreased with the number of processors, the data distribution and synchronization between the processors require additional computation time. When multiple processors are used, the same number of surfaces per processor is distributed by trying to mesh the same area in each processor. The inconvenient of this type of load distribution may be observed when multiple large or complex surfaces are assigned to the same processor. The simplest example to explain this case is a geometry made with four identical surfaces to be meshed with thousands of elements: if four processors are used, only a quarter of the total time than with a processor is required; however, if two or three processors are used, the half of the total time is required because the processors with two surfaces assigned are the bottleneck. A similar problem may appear when several complex surfaces are assigned to the same processor. This effect is also depicted in Fig. 7, in particular when 13 and 17 processors are used.

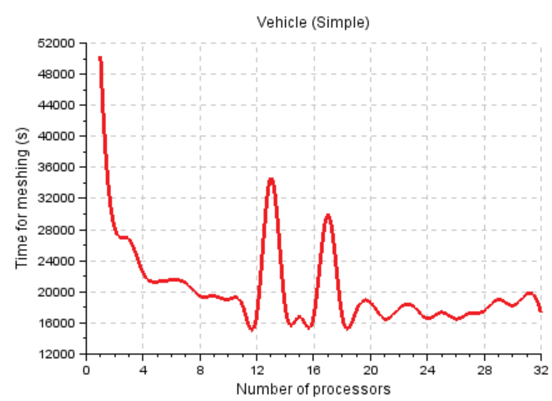

Fig. 7. Time of meshing of a vehicle without the multilevel mode and multiple processors.

If the multilevel mode is applied, the surfaces area of the input geometry is more homogeneous at every level, so these surfaces are meshed easily and the time of meshing of the whole geometry is reduced, as shown in Fig. 8. The curve of time of meshing versus the number of processors with the multilevel mode is reverted to the expected exponentially decreasing shape.

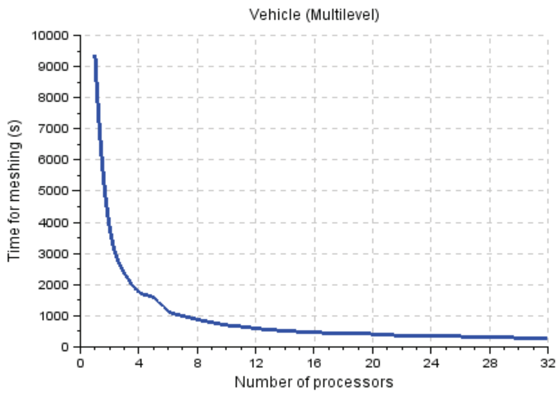

Fig. 8. Time of meshing of a vehicle with the multilevel mode and multiple processors.

The time relation between the times of meshing by using the multilevel mode to the simple mode with the number of processors (as defined previously) is shown in Fig. 9. At the beginning, the time relation decreases quickly; as the number of processors is increased, the curve tends to be stabilized around one percent.

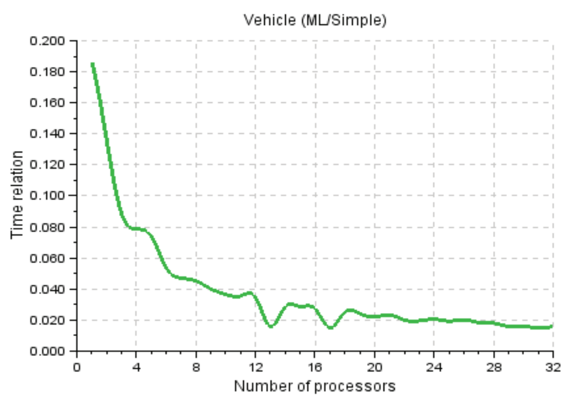

Fig. 9. Time of meshing relation of a vehicle with multiple processors.

The improvement in computation times when the multilevel mode is enabled can be explained clearly with an analysis of the areas in the input geometries.

The original geometry of the vehicle is composed of 363 surfaces with many small surfaces and a few large ones, as represented in the histogram in Fig. 10. As the histogram distribution is irregular, the mesh generation is limited by the time of meshing of the largest and the most complex surfaces, which have an area of 1.7 m^2 and require approximately 750,000 elements to be meshed.

When the first level is completely meshed, the areas of most of its elements are concentrated into the desired range of values for that level, as shown in Fig. 11. The histogram shape is not a perfect delta because of the existence of some surfaces in the original geometry smaller than the desired area elements for the first level and because of the appearance of triangular elements and imperfect quadrangles resulting from the intersections or unions performed during the mesh generation. The mean area of the elements is 7.5 cm^2 in this distribution; so 350 new elements will be generated approximately in the next level for each element in the current level.

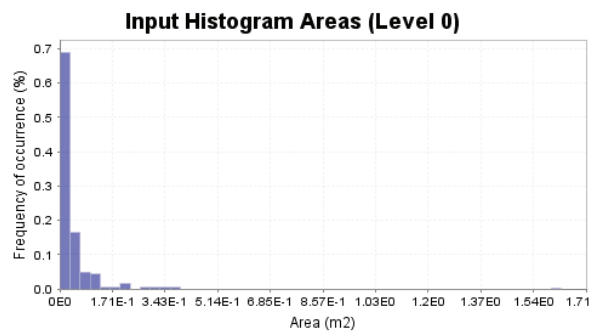

Fig. 10. Histogram of areas of the input surfaces of the vehicle.

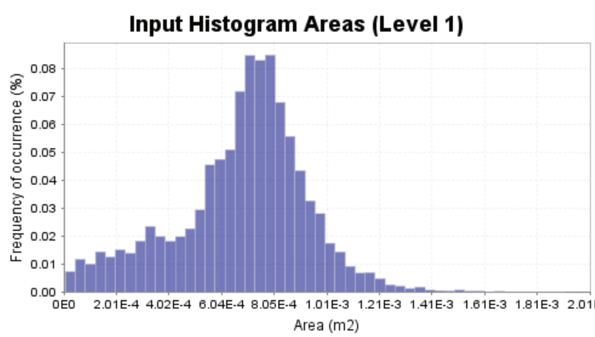

Fig. 11. Histogram of areas of the input elements of a vehicle at the second level.

After finishing the meshing process, the area of most of the elements is very close to the desired value (size edge of 1.5 mm, area of 2.25 mm^2), and the output histograms tend to be a delta function both with and without the multilevel mode, as shown in Fig. 12.

To verify that the final mesh corresponding to the multilevel mode is correct, the histograms of some statistics are studied in the following.

As the main objective of the implemented mesh generator is the generation of quadrilateral elements, verifying the quality by considering the size edge and the inner angles of these elements is required. In Fig. 13, the histogram of size edges of the elements generated is represented while comparing the results from the mesh obtained with the simple meshing algorithm to the obtained with the multilevel mode. The distribution of lengths of the elements is better in the multilevel mesh, as it is more concentrated at the desired length of 1.5 mm.

The histogram of inner angles has a similar distribution both for the simple and the multilevel meshes, with most of the angles clustered near 90 degrees, as represented in Fig. 14.

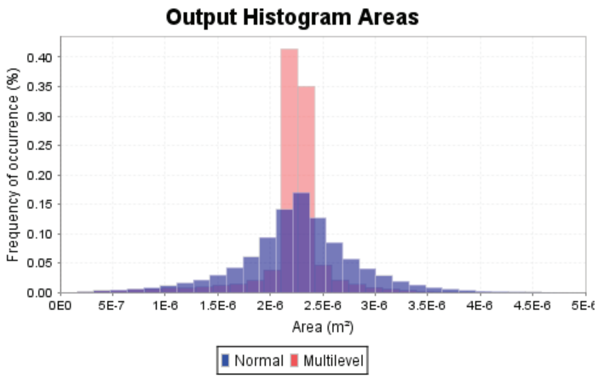

Fig. 12. Histogram of areas of the final output elements of a vehicle.

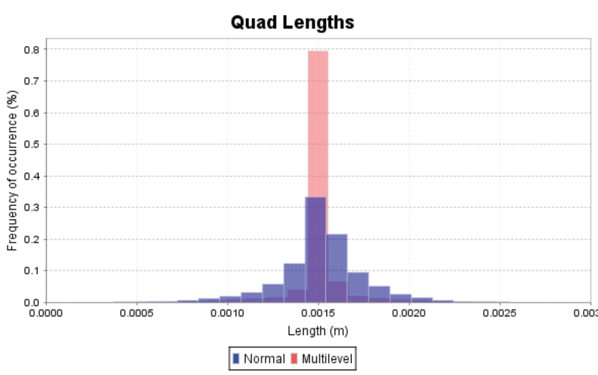

Fig. 13. Histogram of lengths of the final output quads of a vehicle.

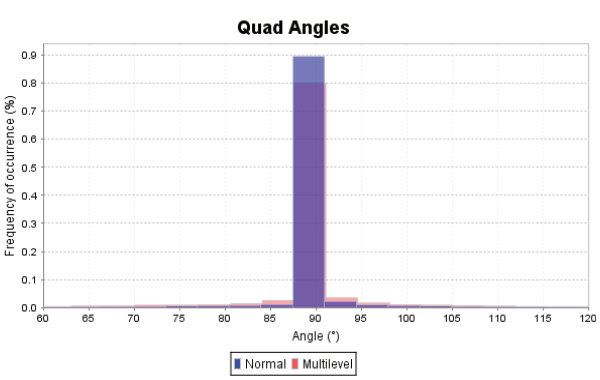

Fig. 14. Histogram of inner angles of the final output quads of a vehicle.

With these results, it is verified that most of the elements in the mesh are perfect quads.

The modified version of the paving algorithm developed inserts triangular elements only when it is the best solution or when the size of the rows is adjusted during the mesh generation. Therefore, the triangles do not satisfy the criterion of having all the borders with the same size edge or the same angle. It is likely that the triangles have two borders with the same length when they are generated for a perfect quad, and a third one that may be different because it is the union between the other two borders. This explanation is verified with the histogram shown in Fig. 15, where the size edge of the triangles is represented, and the histogram of the inner angles shown in the Fig. 16. The angles are concentrated between 30 and 90 degrees, and the size edges are clustered near 1.5 mm for the two meshing modes. The distribution is wider than of the quads histograms because of the uncontrolled generation of the triangles.

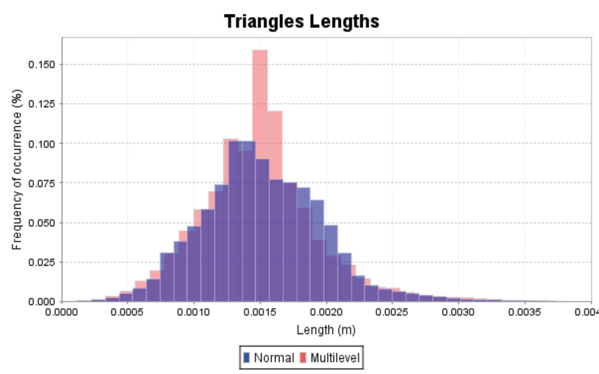

Fig. 15. Histogram of lengths of the final output triangles of a vehicle.

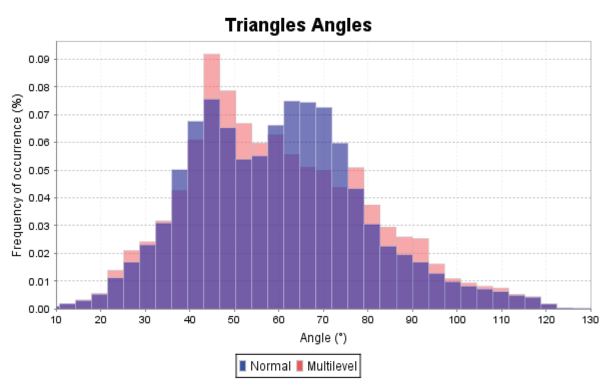

Fig. 16. Histogram of inner angles of the final output triangles of a vehicle.

The mesh of a ship with a very high density (150 million of elements) has been generated in only three hours by using 16 processors and the multilevel mode. The ship has dimensions of 160x30x30 m (length x width x height) and an area of 9,885.52 m^2, but with an irregular distribution of surfaces. A resume of some parameters of interest for meshing that ship by using two levels with a size edge of 8.3 mm is represented in Table 1.

Table 1: Statistics of areas by levels for a ship

Level	Number of Surfaces	Mean Area (m^2)	Max. Area (m^2)	Min. Area (m^2)
0	162	64.41	934.3	0.43
1	19,482	0.51	1.41	0.00482

The parameters of the level 0 in Table 1 correspond to the original geometry of the ship, which is composed of 162 surfaces and has a factor of 2,000 between the largest and the smallest area surfaces. The area of the largest surface in the ship exceeds 900 m^2, so it requires approximately 13 million elements.

However, in the second level, the factor between the largest and the smallest area surface is reduced to 300, the surfaces are smaller and more homogeneous, and the largest surface is meshed with only 20.000 elements approximately, so the meshing is easier and faster at this level.

V. ACCURACY RESULTS

Several electromagnetic problems are solved in order to show that the new features of the mesher keep the accuracy of results. All analyses are performed with 8 processors in a SUN 2 QUAD Core Intel Xeon 2.27 GHz machine, with 12 GB of RAM, by applying the Method-of-Moments.

In every case there is a slightly difference between the meshes obtained with the traditional single level mesh algorithm and with the new one. Even though the original geometry to be meshed is the same, in the multilevel approach the input geometry at the next levels to the first one are simplified models after meshing the original one, so the mesh generation process is simpler and the final mesh is different.

The first case of the study is the parabolic reflector fed with a pyramidal horn located at its focus shown in Fig. 17. The diameter of the aperture is 5 m and the focal length is 2.5 m. When this geometry is meshed with a single level 233,692 elements are generated, whereas 212,920 elements are generated by considering two meshing levels. For both meshes, the radiation pattern is obtained at the frequency 3 GHz.

Figure 18 presents the comparison of the results by considering both meshes for the cut ϕ=0°. As the final mesh in both cases come from the same input geometry with the same size edge for elements, the gain of them are almost identical and the minimum differences are due to the fact that the simulated meshes with the Method-of-Moments are slightly different.

The geometrical model of the second case of study is the *Tabarca* ship shown in Fig. 19. The number of elements that compose the mesh when it is obtained by applying the multilevel mode is 268,943, and 267,645 when this technique is not considered.

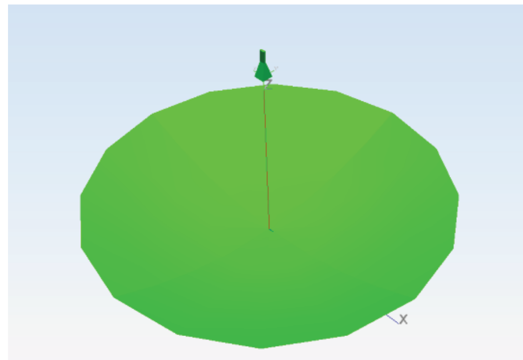

Fig. 17. Geometrical model of a reflector.

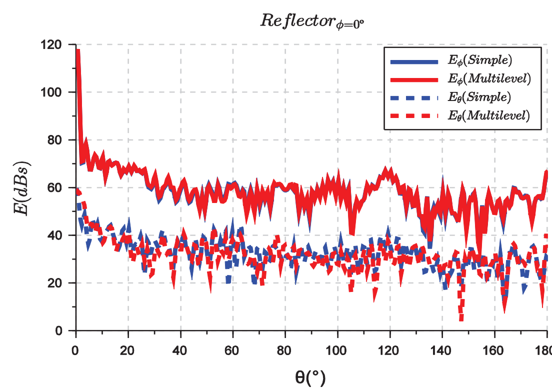

Fig. 18. Comparison between gain results of a reflector for both meshes, cut ϕ=0°.

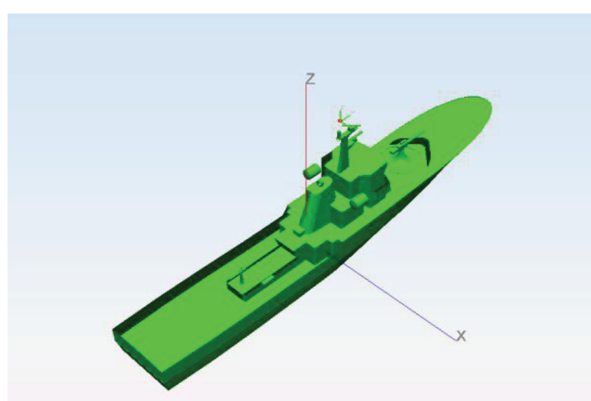

Fig. 19. Geometrical model of a ship.

A electric dipole is located at (0.0 m, 4.0 m, 11.0 m), represented in Fig. 19 with a secondary reference system on top of the ship, in order to obtain the far field at the cut ϕ=0° and a sweep from θ=0° to θ=180°at 1 GHz. Figure 20 shows the comparison of the radiation pattern by considering both meshes, and the results are almost identical again.

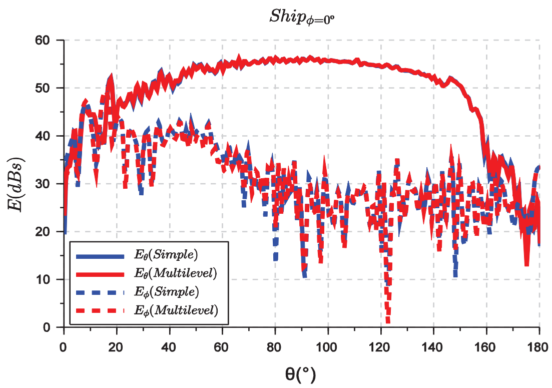

Fig. 20. Comparison between the far field results of a ship for both meshes, ϕ=0°.

Finally, the Radar Cross Section (RCS) of the airplane shown in Fig. 21 has been calculated at 2 GHz. The number of elements of the two meshes is slightly different as in the previous cases: 252,537 elements when this geometry is meshed by applying the multilevel approach are obtained, and 231,420 elements are generated without this technique.

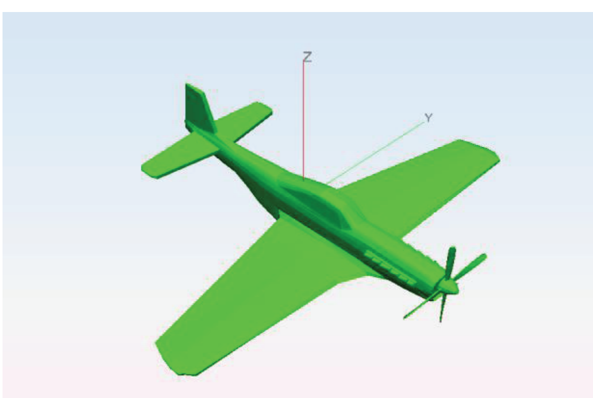

Fig. 21. Geometrical model of an airplane.

The results of the bistatic RCS for the cut ϕ=0° and a sweep from θ=0° to θ=180° are plotted in Fig. 22. Although the meshes are different, the theta-component of the electrical field, that is the strongest (co-polar) component, is almost equal. The differences between the two curves of the phi-component are not very significant because it is the weakest (cross-polar) component and therefore the dependency on the mesh is stronger, so both results are valid.

Table 2 shows the consumed time using the normal and the multilevel procedure for the test cases analyzed before.

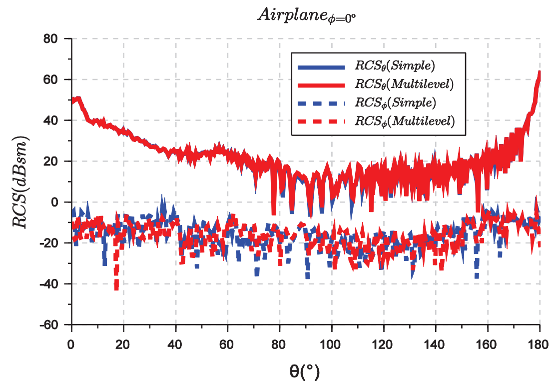

Fig. 22. Comparison between the RCS components of an airplane for both meshes, ϕ = 0°.

Table 2: Time of meshing of different cases

Case 1: Reflector	Time (s)	Number of Elements
Multilevel	66	211,616
Normal	431	233,866
Case 2: Ship	Time (s)	Number of Elements
Multilevel	58	268,889
Normal	148	267,433
Case 3: Airplane	Time (s)	Number of Elements
Multilevel	36	252,568
Normal	124	231,415

VI. CONCLUSIONS

Mesh generation is crucial for the convergence and quality of results in many numerical methods, and therefore, the meshes must be accurate and homogeneous.

There are many algorithms for mesh generation available in the literature, each one with its advantages and disadvantages. Although triangulation is one of the fastest methods, the number of unknowns to be analyzed may be excessive. Thus, in electromagnetic studies, meshes of quadrilateral elements are preferred, but the generation of these elements may be very complex.

One of the best methods of generating quadrilateral meshes is the paving algorithm, which consists of generating elements in an advancing front and ensuring that most of the elements in the mesh are perfect quadrilaterals. However, this algorithm may be very slow for cases in which the input surfaces are extremely complex or when the number of elements to be generated is too high.

In addition to the parallelization of the meshing algorithms, the generation of meshes in progressive steps is a good solution for minimizing the times of meshing, especially in cases with a very large number of elements in the same surface, when this technique is clearly advantageous.

The computation time reduction with the multilevel mode is demonstrated in the simple case of a square plate as well as in real geometries, such as a vehicle or a ship. Furthermore, when the multilevel mode is used, the quality of the meshes is equal to or better than the ones obtained by using the conventional technique because most of the elements are close to perfect quads.

Despite of designing and validating the proposed algorithm with a superficial mesh generator, the technique may be extended to volumetric mesh generators. Due to the computational complexity of the most of volumetric mesh algorithms, especially the advancing front methods, the efficiency of this technique should be widely enhanced.

ACKNOWLEDGMENT

This work has been supported in part by the Spanish Department of Science and Technology Project TEC 2013-46587-R and by University of Alcalá Project CCG2014/EXP-089.

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