A New Closed-Form Expression for Dispersion Characteristics of Fundamental Mode of SIW by Least Squares Method

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Abstract **─** A new and accurate closed-form expression is introduced using least squares method (LSM) to calculate propagation constant of substrate integrated waveguide (SIW) at its fundamental mode of operation. The derived equation is a function of geometrical parameters of the structure and accurately estimates cutoff frequency of the dominant mode. The LSM is used to determine the effective width of the SIW structure. A review and comparisons with recently published simulation and measurement results are also provided, which verify the accuracy of the proposed method.

Index Terms **─** Dispersion, least squares method (LSM), substrate integrated waveguide (SIW).

I. INTRODUCTION

In recent years the SIW technology has been widely used in implementation of microwave devices and antennas [1-5] due to their attractive features such as simple and planar structure. A variety of numerical methods including method of moments (MOMs), finite difference time domain (FD-TD), boundary integral resonant mode expansions (BI-RME), method of lines (MOL), and mode matching have been reported in literature to study the dispersion characteristics of these structures. In spite of the accuracy of those methods, they consume lots of time and need large amount of memory.

In this paper, an accurate closed-form expression is derived using least squares method to calculate the effective width of the SIW structures. The effects of geometrical parameters of the SIW structure on propagation constant and cutoff frequency are investigated for four specific SIW structures. It is shown that the proposed method accurately estimates dispersion characteristic of the SIW and an excellent agreement is obtained between the results of the proposed method with those obtained by measurement.

II. THEORY OF THE PROPOSED METHOD

A linear system of m equations in n unknowns $x_1,\ldots, x_n$ is a set of equations of the form:

$$\begin{cases} a_{11}x_1 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + \ldots + a_{2n}x_n = b_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ a_{m1}x_1 + \ldots + a_{mn}x_n = b_m \end{cases}. \quad (1)$$

The system is called linear because each variable x_j appears in the first power only, just as in the equation of a straight line. $a_{11},\ldots, a_{mn}$ are given numbers, called the coefficients of the system. $b_1,\ldots, b_m$ on the right are also given numbers. A solution of (1) is a set of numbers $x_1,\ldots,x_n$ that satisfies all m equations. From the definition of matrix multiplication we see that m equations of (1) may be written as a single vector equation **Ax**=**b,** where the coefficient matrix **A** = $[a_{jk}]$ is $m\times n$ matrix, **x** and **b** are column vectors.

It is assumed that the coefficients a_{jk} are not all zero, so that **A** is not a zero matrix. Note that x has n components, whereas b has m components. If $m=n$ and **A** is nonsingular, the answer is simply $\mathbf{x}=\mathbf{A}^{-1}\mathbf{b}$. But if $m>n$ so that we have more equations than unknowns, the problem is called over determined, and generally no **X** vector satisfies **Ax**=**b** exactly. Given an *m-by-n* matrix **A** and an *m*-by-1 vector **b**, the least squares problem is to find an *n*-by-1 vector **X** which minimize **Ax**=**b** [6]. The norm of the vector $\mathbf{Ax}-\mathbf{b}$ is defined as $\|\mathbf{Ax}-\mathbf{b}\|=\sqrt{(\mathbf{Ax}-\mathbf{b})^T(\mathbf{Ax}-\mathbf{b})}$, in which $(\mathbf{Ax}\text{-}\mathbf{b})^T$ is transpose of the **Ax**-**b** matrix. The least squares solution is $\hat{\mathbf{x}}$ vector which minimize error expression $E=\|\mathbf{Ax}-\mathbf{b}\|^2$. This is the sum of squares of the errors in m equations ($m>n$). The best $\hat{\mathbf{x}}$ comes from the normal equations $\mathbf{A}^T\mathbf{A}\hat{\mathbf{x}}=\mathbf{A}^T\mathbf{b}$. If marix **A** is left-invertible, then $\hat{\mathbf{x}}=(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b}$ is the unique solution of the least squares problem [6].

III. SIW STRUCTURE

The geometry of the SIW structure with its physical parameters is shown in Fig. 1. It consists of two rows of conducting cylinders implanted in a

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dielectric substrate which connects two conductor parallel plates at the top and bottom of the substrate. Therefore, a synthetic rectangular waveguide filled with dielectric material is made in planar form. The diameter of cylindrical posts is d and they are separated in transverses and axial plane by S and W respectively.

Experimental and numerical methods reveal that the propagation characteristics of dominant mode of the SIW structures are equivalent to those of an equivalent metallic rectangular waveguide with the effective width of W_{eff}. So, it is assumed that the effective width is generally related to the geometrical parameters. Thus, two expressions using unknown coefficients are defined for evaluating the effective width of SIW structures [7-10].

$$W_{eff} = \gamma_1 W + \gamma_2 \frac{d}{S} W + \gamma_3 \frac{d^2}{S} + \gamma_4 \frac{d^2}{W}, \qquad (2)$$

$$W_{eff} = \sigma_1 W + \sigma_2 S + \sigma_3 d + \sigma_4 \frac{d^2}{S} + \sigma_5 \frac{d^2}{W}. \qquad (3)$$

Unknown coefficients can be calculated using LSM procedure. Table 1 shows the calculated unknown coefficients.

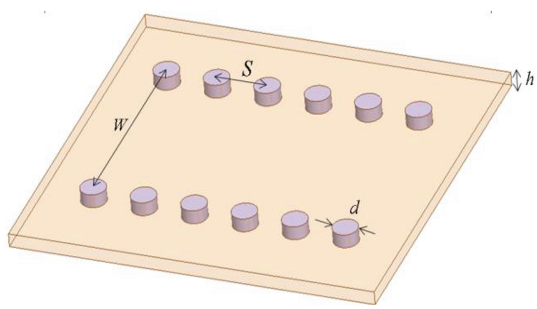

Fig. 1. Geometry of the SIW structure.

Table 1: Calculated unknown coefficients using LSM

i	γ_i	σ_i
1	1.3	1.103
2	-1.026	0.552
3	7.957	-3.222
4	-22.015	4.553
5	0	-10.974
E(error)	2.9×10^{-4}	2.3×10^{-4}

IV. RESULTS VERIFICATION

To verify the accuracy of the proposed method, four examples are presented.

A. Example I

In the first one, a specific SIW structure with geometrical parameters of W=3.97 mm, d=0.635 mm, S=1.016 mm and relative permittivity of 9.9 is considered. Numerical results for cutoff frequency of TE_{10} mode of this structure is shown in Fig. 2. It can be seen that a very good agreement is obtained between the measured results and those obtained by the proposed W_{eff} using Equation 3. A small deviation between results can be seen, but still proposed W_{eff} method using Equation 2 in this paper accurately predicts propagation constant of the structure.

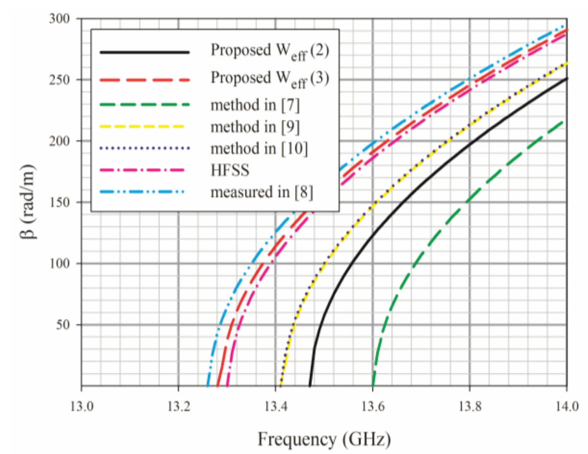

Fig. 2. Propagation constant of TE_{10} mode versus frequency for SIW structure I.

B. Example II

In the second example, another specific SIW structure with geometrical parameters of W=7.2 mm, S=2 mm and relative permittivity of 2.33 is considered. Cutoff frequency of TE_{10} mode of this SIW structure versus via diameter d, is shown in Fig. 3. These results indicate that both proposed W_{eff} using Equations 2 and 3 predicts same dispersion characteristics and agree well with other published methods.

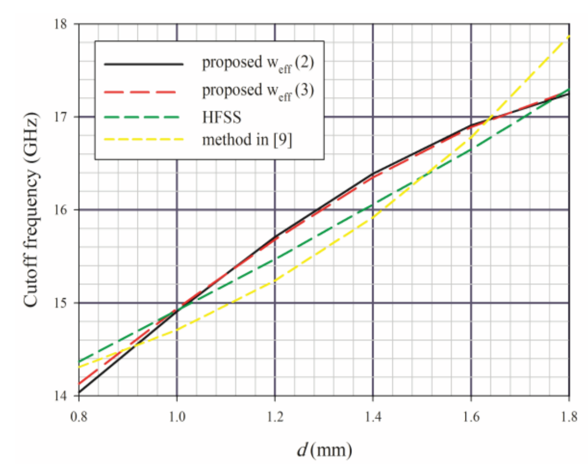

Fig. 3. Cutoff frequency of TE_{10} mode versus via diameter d for the 2^d SIW structure.

C. Example III

A SIW structure with parameters W=5.25 mm, d=0.8 mm, S=1.5 mm and ε_r=2.2 is considered in the third example. Numerical results of the propagation constant of the mentioned structure at TE_{10} mode versus frequency of the presented method in this paper are shown in Fig. 4. It shows that our results and measured results in [10] agree very well.

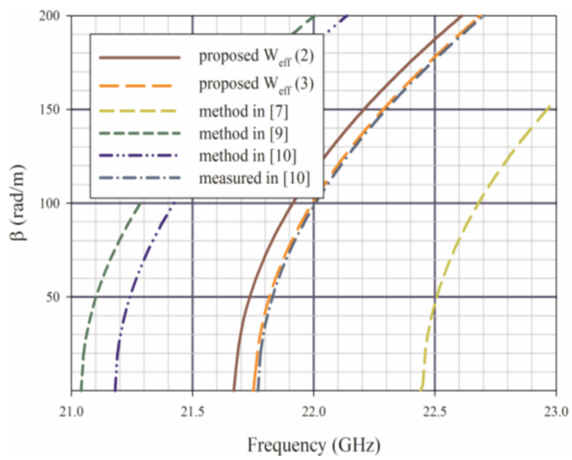

Fig. 4. Propagation constant of TE_{10} mode versus the 3[d] SIW structure.

D. Example IV

In the fourth example, a specific SIW structure with geometrical parameters of S=1.5 mm and relative permittivity of 2.2 is considered. Numerical results of cutoff frequency for this structure at TE_{10} mode are shown in Fig. 5 versus W, width of the structure for different values of via diameter d. It can be concluded that the results of the proposed method in this paper agree very well with those presented in [9].

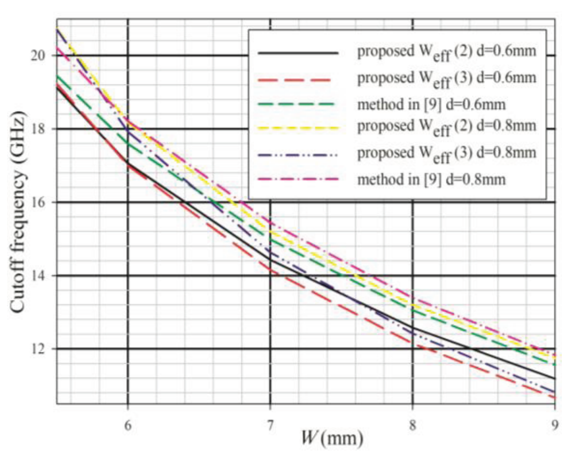

Fig. 5. Cutoff frequency of TE_{10} mode versus W for different values of d for 4[th] SIW structure.

V. CONCLUSION

In this paper, an accurate closed-form expression is introduced using least squares method (LSM) to calculate the effective width of the substrate integrated waveguide (SIW). The effects of geometrical parameters of the structure on propagation constant and cutoff frequency are investigated for four specific SIW structures. The results for propagation constant and cutoff frequency of the dominant mode of the SIW structures are in a very good agreement with other reported simulation and measured results. The proposed method using LSM accurately predicts the dispersion characteristics of the SIW for a wide range of structure parameters. Therefore, it could be used for designing wide variety of SIW structure.

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